Non-linear terms in Delta Scuti stars power spectra.

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Scientific motivation

Non-linear effects explained special features of some variable stars light curves:

- RR- Lyrae and Cepheids (Christy 1962, 1964, 1966, 1967)
- White Dwarfs (Brickhill 1992, Brassard 1995, Wu 2001, Montgomery 2005)
- SPB, Be and γ-Dor stars (Kurtz et al., 2015)

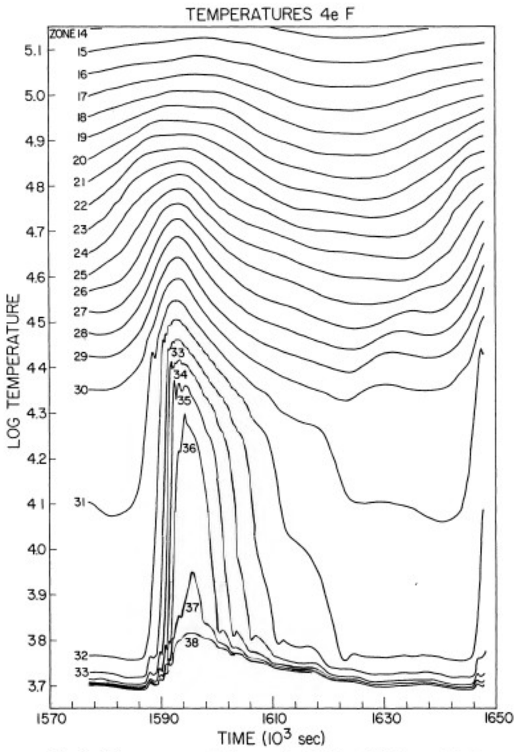


Fig. 1.—The temperature variation at various depths to 10^4 °K for model 4e F

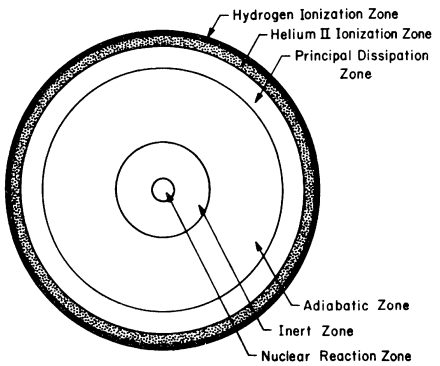
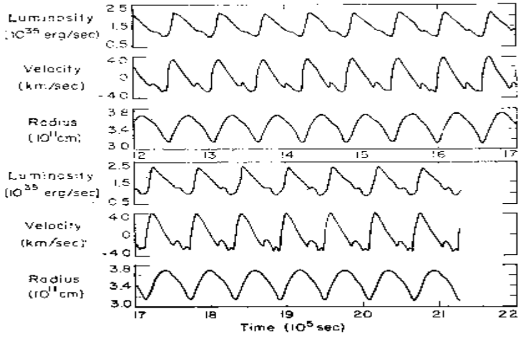


Fig. 1. Section through a typical pulsating variable showing the location of the various regions contributing to the excitation and dissipation.



Scientific motivation

Non-linear models for δ Sct stars

- The “Main Sequence Catastrophe” (Stellingwerf 1980)
- Amplitude Equation formalism (AE) (Dziembowski 1984, Buchler 1997)
- White Dwarfs non-linear models failed when applied to δ Sct stars (Balona 2012)

Scientific motivation

Open issues in δ Sct studies that can be solved by non-linear models

- Density of their power spectra: identify non-linear terms from pulsation modes.
  - Match with linear pulsation models
- Amplitudes in their power spectra: driving and damping mechanisms and pulsating mode selection
  - Why some stars in the δ Sct instability strip do not pulsate?
  - Why stars with similar stellar parameters have so different frequency distribution?

PhD outline

Is it possible to characterize non-linear effects of δ Sct stars observationally?

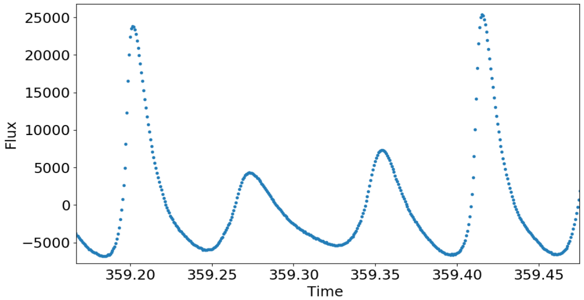
1) Set a general non-linear model (Garrido, 1996)
2) Identify non-linear terms in δ Sct stars power spectra.
3) Search for any possible characterization

Non-linear effects in δ Sct stars:

- Interaction between the oscillation and the variation of the depth of the convective zone in the outer layer of a pulsating star (Brickhill, 1992; Wu, 2001)
- The non-linear response of the emergent flux to changes of the local temperature at the stellar surface. (Brassard et al., 1995)
- Resonance (Dziembowski, 1982)

Coupled modes

Non-linear distortion processes



Extract of the GSC00144-03031 light curve observed by CoRoT. Steep ascends and slow descends move away from the characteristic sinusoidal shape.

Harmonics and cross-frequency terms

Theoretical framework

- **General non-linear theory for pulsating stars**
  - Second order perturbations
  - Simple development of these eqs. in Kurtz, 2015

    showing pulsations at combination frequencies as solutions.

$$\frac{\partial^2 \xi}{\partial t^2} + L(\xi) + N^{(2)}(\xi, \xi) + N^{(3)}(\xi, \xi, \xi) + \dots = 0;$$

$$\frac{\partial^2 \xi^{(2)}}{\partial t^2} + L(\xi^{(2)}) = -N^{(2)}(\xi^{(1)}, \xi^{(1)});$$

- **Modeling light curves**
  - Expression for the light variation ΔL/L, obtained by integrating the relative flux variation ΔF/F over the complete stellar disk, considering limb darkening effects.

$$\frac{\Delta F}{F} = \frac{\epsilon_i}{2\pi}\Re\{(V_1 + V_2)Y_{\ell_i}^{m_i}(\Theta, \Phi)Y_{\ell_j}^{m_j}(\Theta, \Phi)e^{i((\omega_i+\omega_j)t+\varphi_i+\varphi_j)}\}$$

- **Modeling Fourier parameters**
  - The Simple model
  - The Mode Coupling model (Breger and Montgomery, 2014; Bowman et al., 2016)
  - The Volterra expansion model (Garrido and Rodríguez, 1996)

Modeling Fourier parameters

- The Simple model

$$Y_1 = A_1 \cos(2\pi\nu_1 t + \varphi_1)$$
$$Y_2 = A_2 \cos(2\pi\nu_2 t + \varphi_2)$$

$$Y_c(t) \propto Y_1(t) Y_2(t)$$
$$Y_c(t) \propto A_1 \cos(2\pi\nu_1 t + \varphi_1) A_2 \cos(2\pi\nu_2 t + \varphi_2)$$
$$Y_c(t) \equiv A_c \cos(2\pi\nu_{+c} t + \varphi_{+c}) + A_c \cos(2\pi\nu_{-c} t + \varphi_{-c})$$

$$\boxed{\begin{aligned} \nu_{\pm c} &= \nu_1 \pm \nu_2 \\ \varphi_{\pm c} &= \varphi_1 \pm \varphi_2 \\ A_c &= \frac{A_1 A_2}{2} \end{aligned}}$$

- The Mode Coupling model

(Dziembowski, 1982)

(Breger and Montgomery, 2014)

(Bowman et al., 2016)

  - Resonance conditions:

$\omega_c \approx \omega_1 + \omega_2$ or $\omega_c \approx 2\omega_1$

$$\boxed{\begin{aligned} \nu_{\pm c} &= \nu_1 \pm \nu_2 \\ \varphi_{\pm c} &= \varphi_1 \pm \varphi_2 \\ A_c &= \mu_c A_1 A_2 \end{aligned}}$$

$$\mu_c = \frac{H}{2\sigma_c \gamma_c I_c}$$

Coupling factor

Modeling Fourier parameters

- The Volterra expansion model

Monoperiodic

$$Y(t) = A_0\,\Gamma_1(\omega_0)\,e^{i\omega_0 t+\phi_0} + A_0^2\,\Gamma_2(\omega_0,\omega_0)\,e^{2i\omega_0 t+2\phi_0} + A_0^3\,\Gamma_3(\omega_0,\omega_0,\omega_0)\,e^{3i\omega_0 t+3\phi_0} + \dots,$$

$$Y(t) = \tilde{A}_1\,e^{i\omega_0 t+\tilde{\phi}_1} + \tilde{A}_2\,e^{2i\omega_0 t+\tilde{\phi}_2} + \tilde{A}_3\,e^{3i\omega_0 t+\tilde{\phi}_3} + \dots,$$

where

$$\tilde{A}_1 = A_0\,|\Gamma_1(\omega_0)|,\quad \tilde{A}_2 = A_0^2\,|\Gamma_2(\omega_0,\omega_0)|,\quad \tilde{A}_3 = A_0^3\,|\Gamma_3(\omega_0,\omega_0,\omega_0)|$$

and

$$\tilde{\phi}_1 = \phi_0 + \arg\{\Gamma_1(\omega_0)\},\quad \tilde{\phi}_2 = 2\,\phi_0 + \arg\{\Gamma_2(\omega_0,\omega_0)\},\quad \tilde{\phi}_3 = 3\,\phi_0 + \arg\{\Gamma_3(\omega_0,\omega_0,\omega_0)\}.$$

Double-mode

$$Y(t) = A_0\,\Gamma_1(\omega_0)\,e^{i\omega_0 t+\phi_0} + A_1\,\Gamma_1(\omega_1)\,e^{i\omega_1 t+\phi_1} + A_0^2\,\Gamma_2(\omega_0,\omega_0)\,e^{2i\omega_0 t+2\phi_0} + A_1^2\,\Gamma_2(\omega_1,\omega_1)\,e^{2i\omega_1 t+2\phi_1} + A_0\,A_1\,\Gamma_2(\omega_0,\pm\omega_1)\,e^{i(\omega_0\pm\omega_1)t+(\phi_0\pm\phi_1)} + A_1\,A_0\,\Gamma_2(\omega_1,\pm\omega_0)\,e^{i(\omega_1\pm\omega_0)t+(\phi_1\pm\phi_0)} + \dots.$$

$$Y(t) = \tilde{A}_1\,e^{i\omega_0 t+\tilde{\phi}_1} + \tilde{A}_2\,e^{i\omega_1 t+\tilde{\phi}_2} + \tilde{A}_3\,e^{2i\omega_0 t+\tilde{\phi}_3} + \tilde{A}_4\,e^{2i\omega_1 t+\tilde{\phi}_4} + \tilde{A}_5\,e^{i(\omega_0\pm\omega_1)t\pm\tilde{\phi}_5} + \dots,$$

where

$$\tilde{A}_1 = A_0|\Gamma_1(\omega_0)|,\quad \tilde{A}_2 = A_1|\Gamma_1(\omega_1)|,\quad \tilde{A}_3 = A_0^2|\Gamma_2(\omega_0,\omega_0)|,\quad \tilde{A}_4 = A_1^2|\Gamma_2(\omega_1,\omega_1)|,\quad \tilde{A}_5 = A_0A_1|\Gamma_2(\omega_0,\omega_1)|,\ \text{or}\ \tilde{A}_5 = A_1A_0|\Gamma_2(\omega_1,\omega_0)|^\dagger$$

and

$$\tilde{\phi}_1 = \phi_0 + \arg\{\Gamma_1(\omega_0)\},\quad \tilde{\phi}_2 = \phi_1 + \arg\{\Gamma_1(\omega_1)\},\quad \tilde{\phi}_3 = 2\,\phi_0 + \arg\{\Gamma_2(\omega_0,\omega_0)\},\quad \tilde{\phi}_4 = 2\,\phi_1 + \arg\{\Gamma_2(\omega_1,\omega_1)\},\quad \tilde{\phi}_5 = \phi_0 \pm \phi_1 + \arg\{\Gamma_2(\omega_0,\pm\omega_1)\},\ \text{or}\ \tilde{\phi}_5 = \phi_1 \pm \phi_0 + \arg\{\Gamma_2(\omega_1,\pm\omega_0)\}^\dagger.$$

† Because no condition of symmetry is yet imposed

Methodology

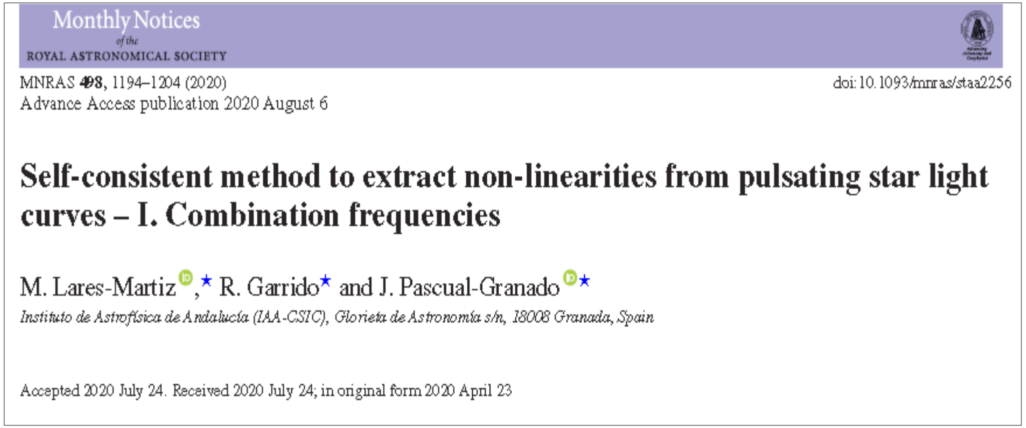
Frequency Relation

$$\boxed{\omega_c = \pm n\,\omega_i \pm m\,\omega_j \pm \ldots}$$

- *Best Parent* Method (BPM):

➔ Non-linear least-squares fit of the *family* of combination frequencies (*Best parents* and their children) which best described the signal (minimum variance of the fit residual light curve).

➔ Finds the *Best parents* **exhaustively**



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Self-consistent method to extract non-linearities from pulsating star light curves – I. Combination frequencies

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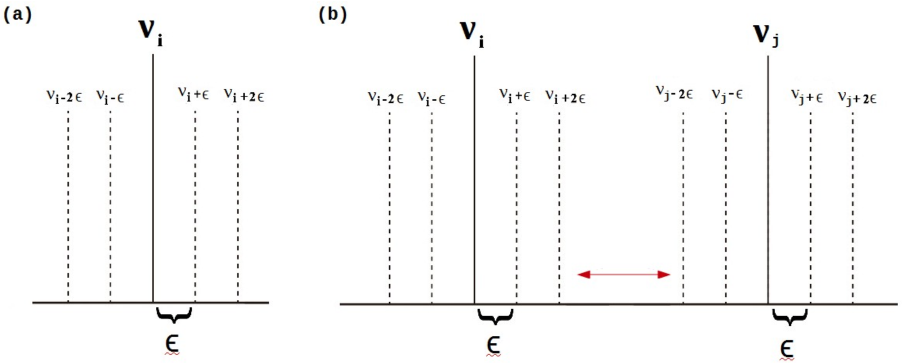
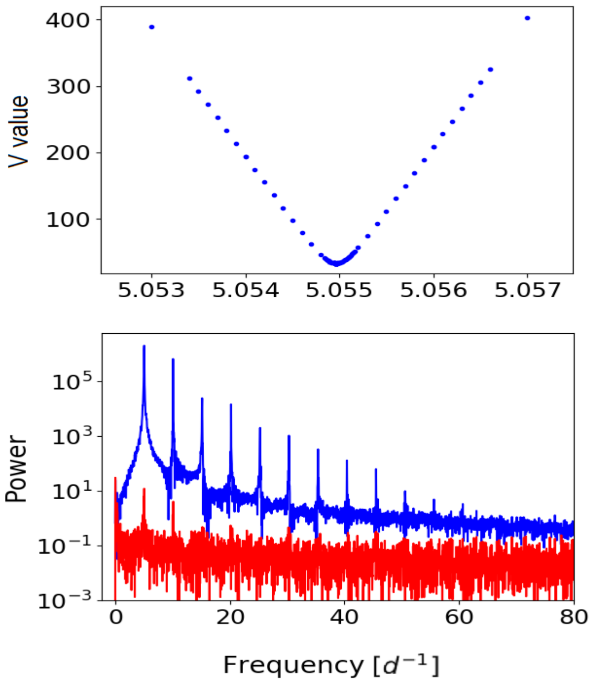


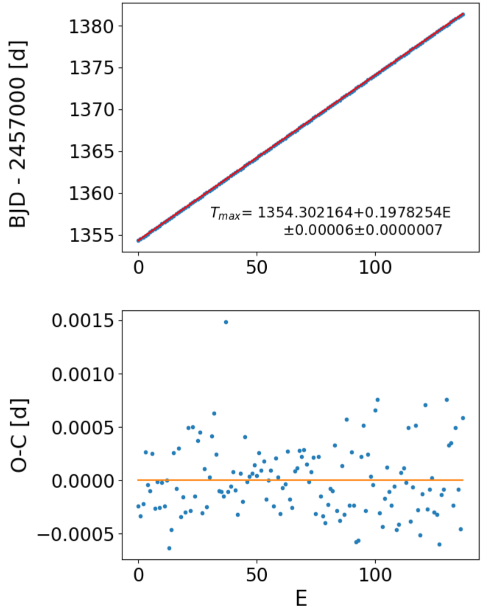
Results

- TIC 9632550 (monoperiodic δ Sct star)

No. of statistically significant frequencies	V value	Frequency[d^{-1}]
1	3155.844405793707201	5.0
5	948.937738175725485	5.05
14	33.629889361388315	5.055
14	33.629889361388315	5.055
14	**32.871889584474623**	**5.05496**
14	32.862102488776905	5.054964
14	32.862102488776905	5.054964
14	32.862101660828912	5.05496404
14	32.862101656257245	5.054964037
14	32.862101656225789	5.0549640372
14	32.862101656224368	5.05496403722
14	32.862101656224311	5.054964037229
14	32.862101656223409	5.0549640372274
14	32.862101656222627	5.05496403722726
14	32.862101656222627	5.05496403722726

TIC 9632550			
Tag	'best' parent [d^{-1}]	Combinations extracted	%CF
f0	5.05496	13 Harmonics	98.98





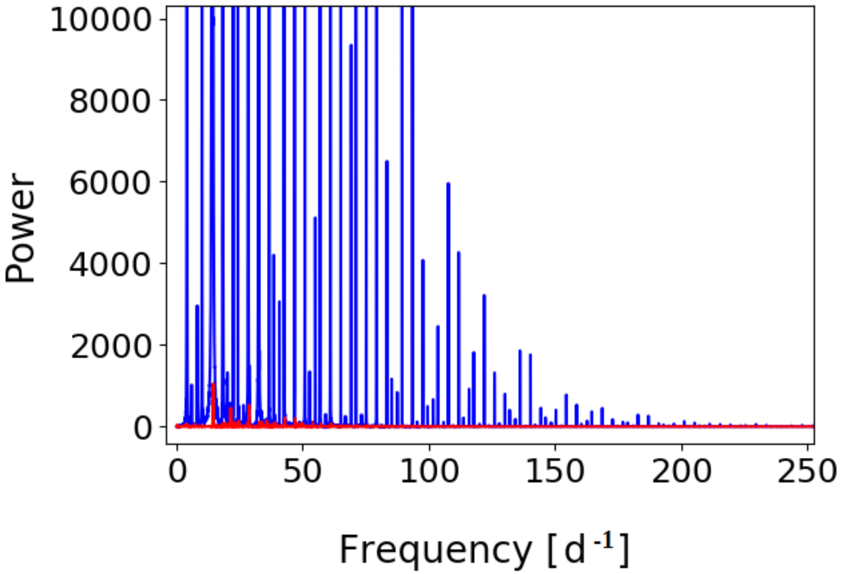
ω_0 = (5.05496±0.00002) d^{-1}

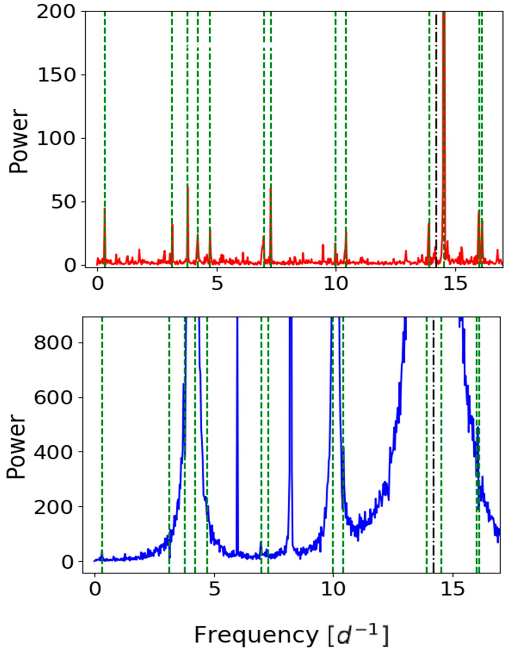
(Lares-Martiz, M. et al., 2020)

Results

- KIC 5950759 (double mode HADS star)

KIC 5950759			
Tag	'best' parents [d^{-1}]	Combinations extracted	%CF
f0	14.22136	177 in total:	97.48
f1	18.33722	17 harmonics	
		92 sums	
		and 68 differences	



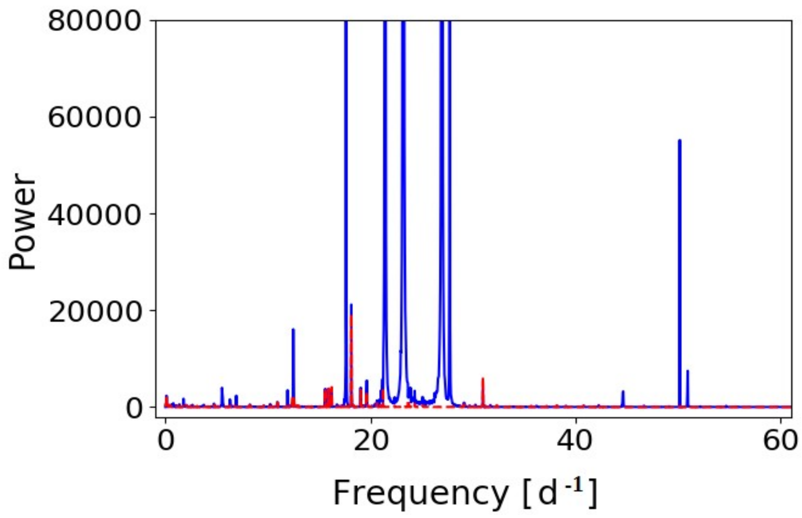


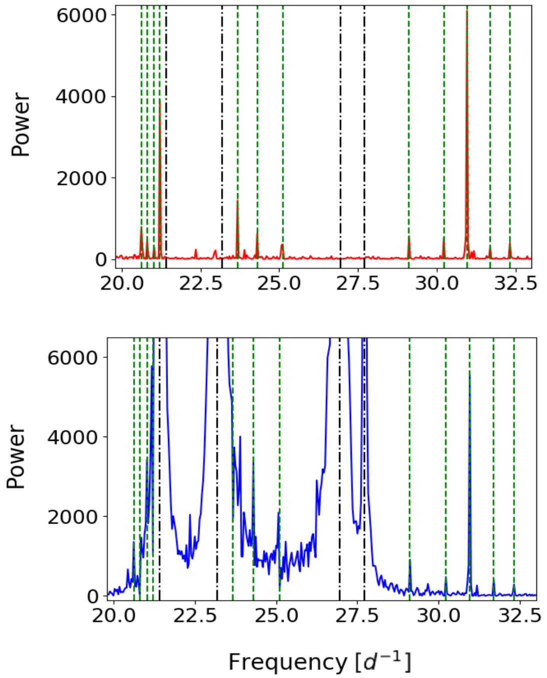
(Lares-Martiz, M. et al., 2020)

Results

- HD 174966 (multiperiodic δ Sct star)

HD 174966			
Tag	'best' parents [d^{-1}]	Combinations extracted	%CF
f0	17.6230	118 in total:	93.01
f1	23.1948	1 harmonic	
f2	21.4210	11 sums	
f3	26.9585	106 differences	
f4	27.7150		





(Lares-Martiz, M. et al., 2020)

Summary 1

- BPM is essentially a ‘standard’ approach of fitting sinusoids to the light-curve, in this cases of the exact combination values resulting from the frequency relation between parents and children.
- BPM assures an exhaustive search for the 'best' parents, which can not be accomplished by any other algorithm of non-linear least-squares available at the moment.
- BPM do not add information to the residual light curve and guarantees that any remaining variance is not caused by the parent mode frequencies and their associated children frequencies.
- For mono-periodic stars, it achieves precision in frequencies approximately equal to those achieved by the O-C method.
- For double-mode and multi-periodic stars, it allows frequency structures to emerge from what was previously considered as noise.

Theoretical framework

Phase and amplitude relations

Considering complex generalized transfer functions,

$$\Gamma_o(\omega_i) = |\Gamma_o(\omega_i)| e^{i arg\{\Gamma_o(\omega_i)\}} ,$$

from the Volterra expansion:

- **Phase relation:**

$$\boxed{\begin{aligned} arg\{\Gamma_o\} = \Delta\phi &= \phi_{obs} - \phi_{calc} \\ &= \phi_{obs} - (\pm n\phi_i \pm m\phi_j) \end{aligned}}$$

- **Amplitude relation:**

$$\boxed{|\Gamma_o| = A_r = \frac{A_{obs}}{A_i^n A_j^m}}$$

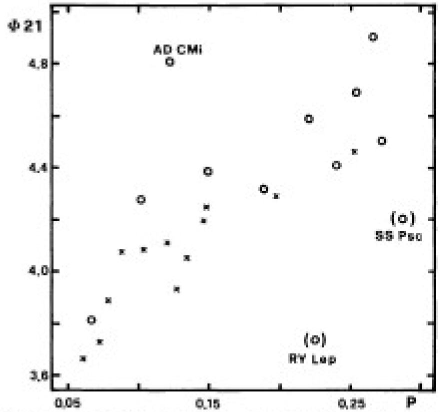


Fig. 9. The phase differences ϕ_{21}, ϕ_{31} and ϕ_{41} (radians) versus the period (days)

Source:
Antonello, 1986

- **Previous studies of these relations:**
  Cepheids:
  Simon and Lee (1981)
  HADS and SX Phe stars:
  Antonello (1986)
  HADS and LADS:
  Garrido and Rodriguez (1996)
  Balona (2012a,2012b,2016)

Theoretical framework

Hypothesis:

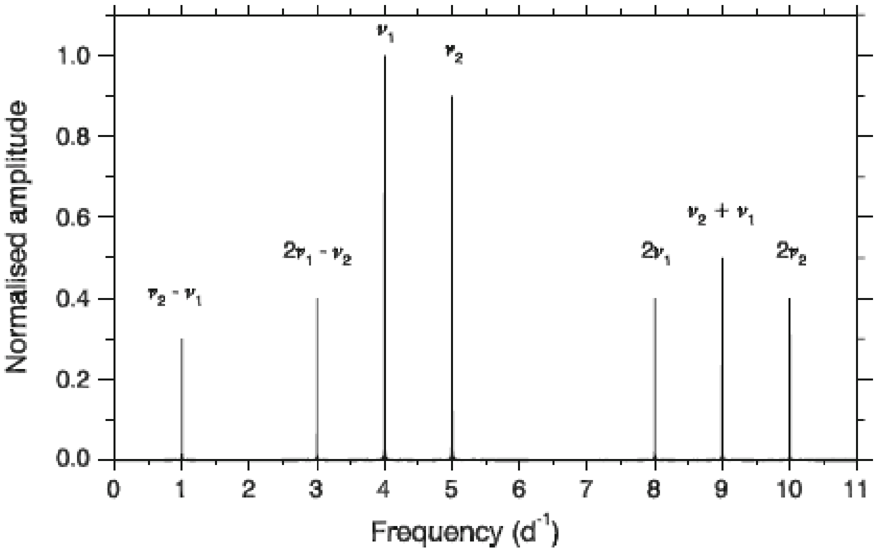
Combination frequency of HADS stars are from non-linear distortion processes

- There are too many combination frequencies to be eigenmodes themselves.
- Normally pulsate in radial modes. Assuring that the children frequencies are cross-terms of clear independent modes as parents.
- Mode coupling induce amplitude modulation of the modes. HADS do not show amplitude variability, supporting the assumption of their combination frequencies being of a non-linear distortion nature

Theoretical framework

Discriminate combination frequencies of non-linear distortion processes from...

- Modulation of instrumental origin (Scargle, private communication)
- Rotational splittings (Bowman, 2017)
- Resonantly excited modes (Breguer and Montgomery, 2014. Bowman, 2016)
- Independent modes



Source: Bowman, 2017

Methodology

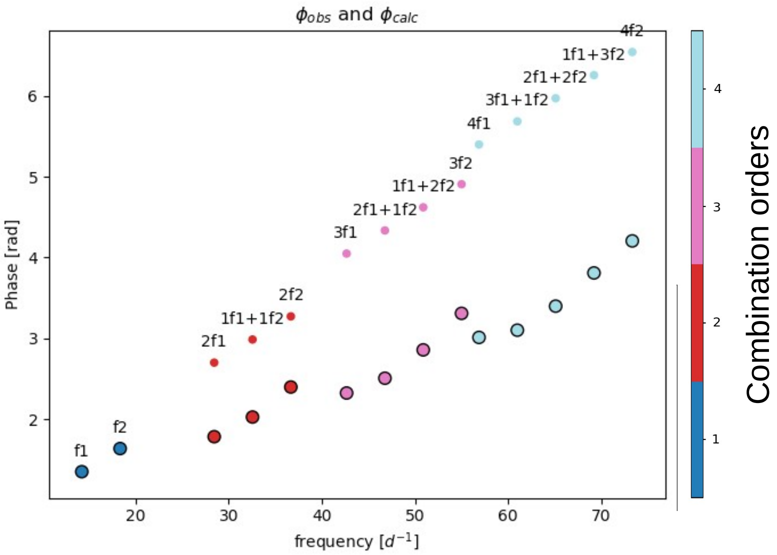
- Compute the BPM to a set of HADS:
  - Two-termed combinations: $$\omega_k = \left|\pm n \cdot \omega_i \pm m \cdot \omega_j\right|$$
  - Order of the combination (O) < Nyquist frequency: $$O = |\pm n| + |\pm m|$$
  - 15 HADS from TESS Sector 1 and 2 (Antoci, 2019.)

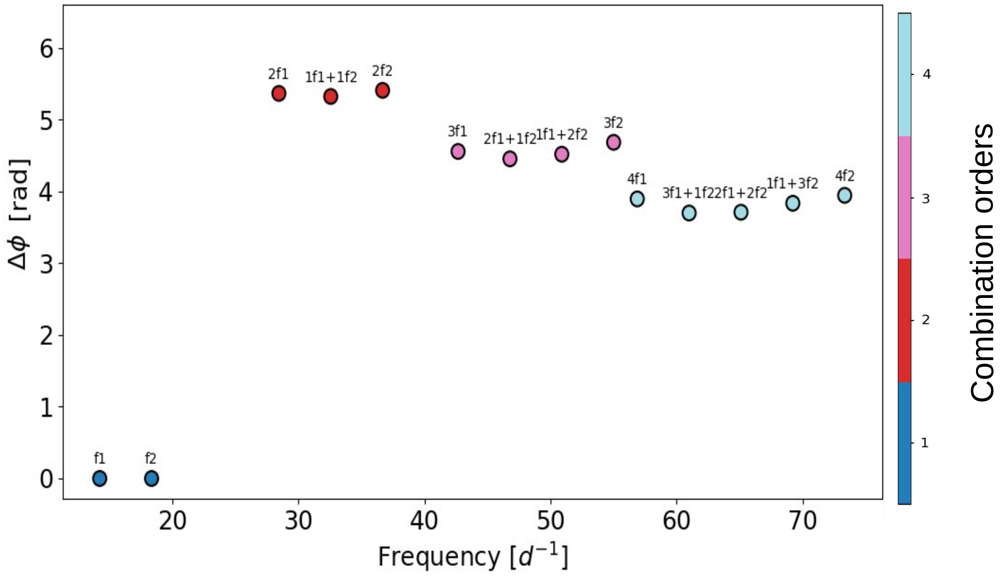
    2 HADS from Kepler

    1 HADS from CoRoT

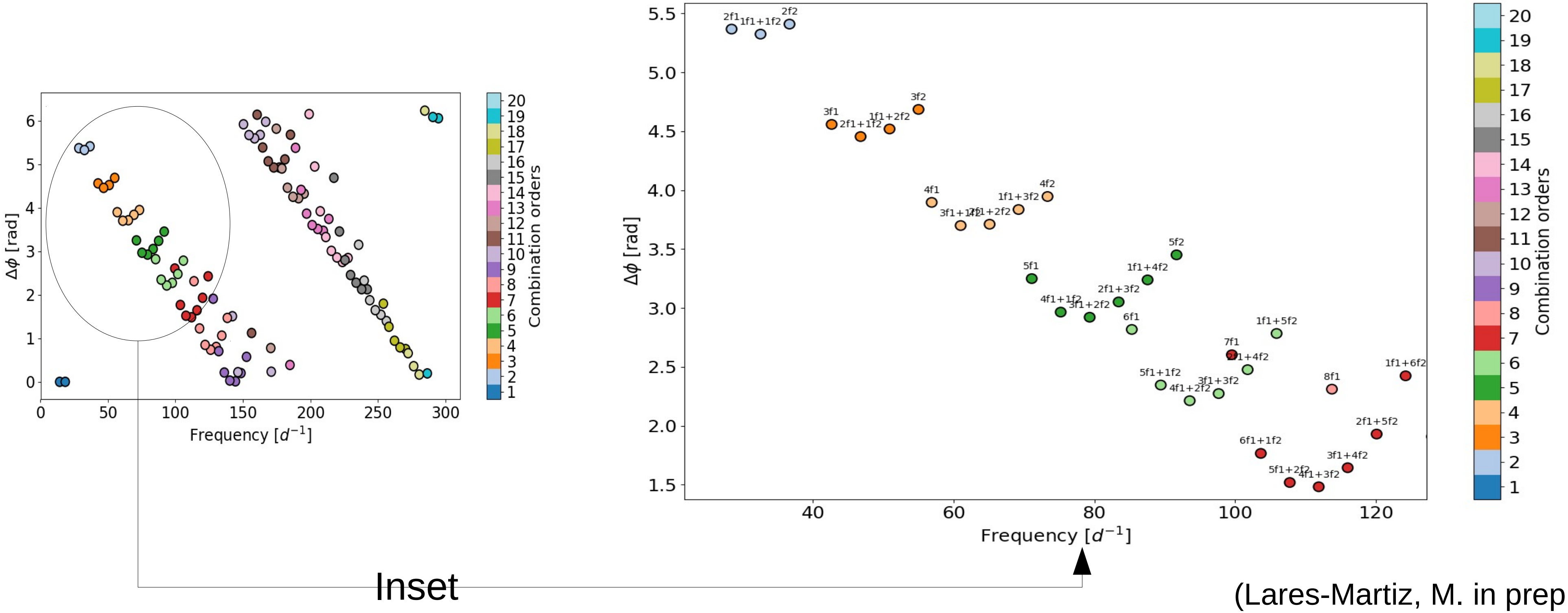
- Build the Phase differences (ΔΦ) and Amplitude ratio plots (Ar) with the BPM Fourier parameters for the statistically significant combination frequencies.

Case study KIC 5950759: Phase differences plot





Case study KIC 5950759: Phase differences plot

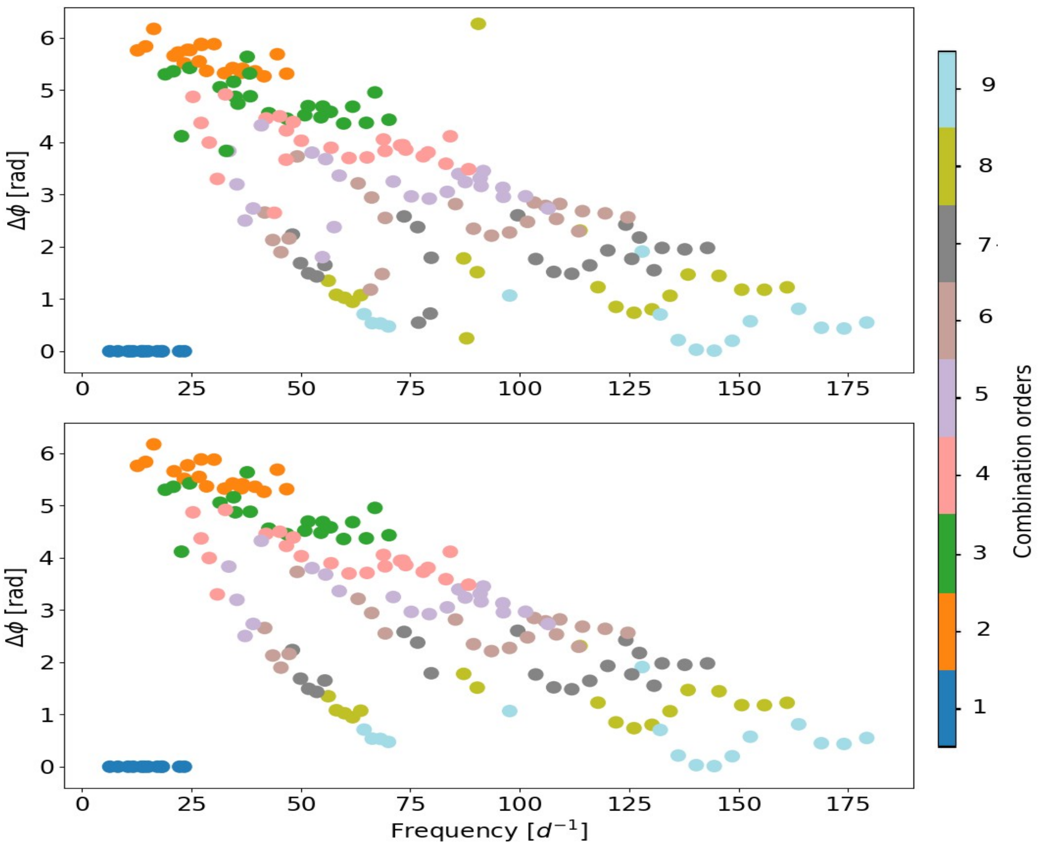


(Lares-Martiz, M. in prep.)

Results

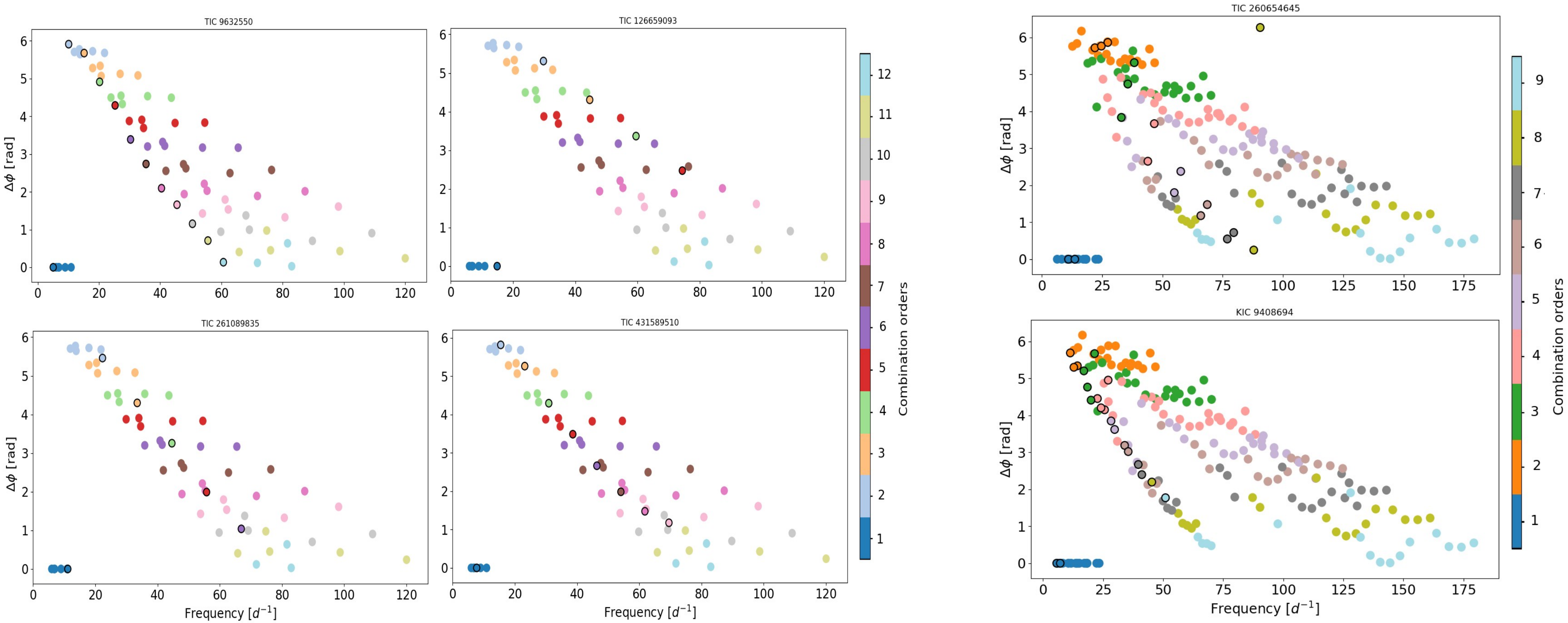
Phase difference plot of 17 HADS:

- The arguments of the Γ_O functions seem to decrease with higher combination orders
- Each combination order has it own *band* of possible phase difference values
- 6 peculiar HADS that distorts the pattern:
  - High rotation values for a HADS
  - Parents do not obey strictly P1/P0 period relation.



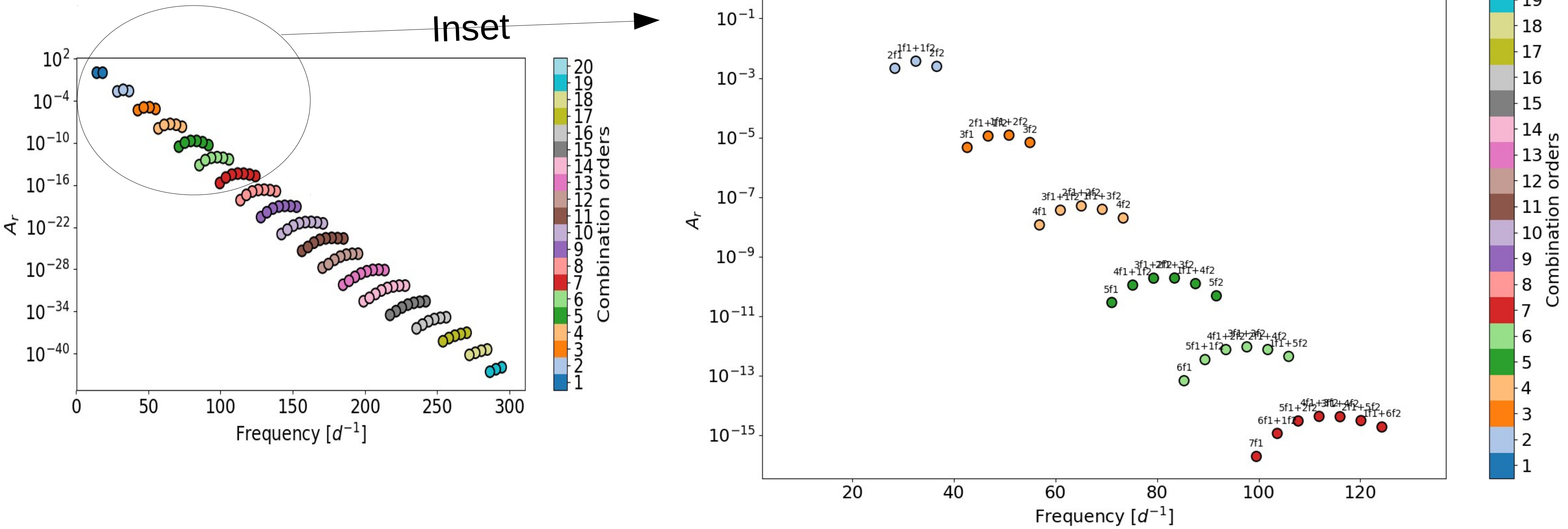
Results

Phase difference plot of peculiar HADS:



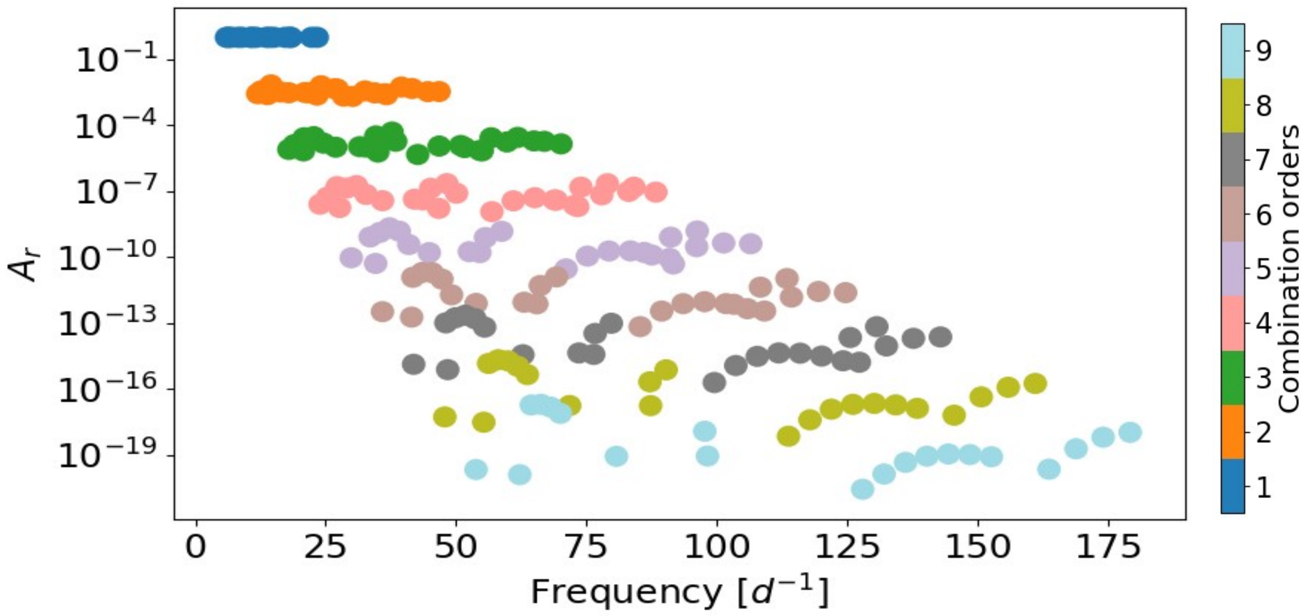
Results

Case study KIC 5950759: Amplitude ratios plot

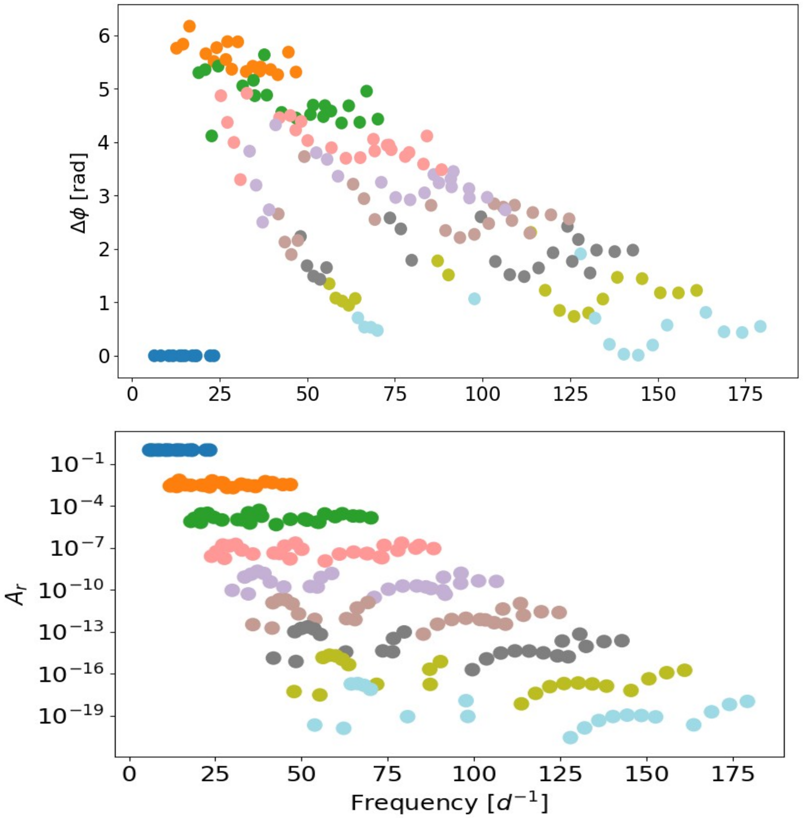


Results

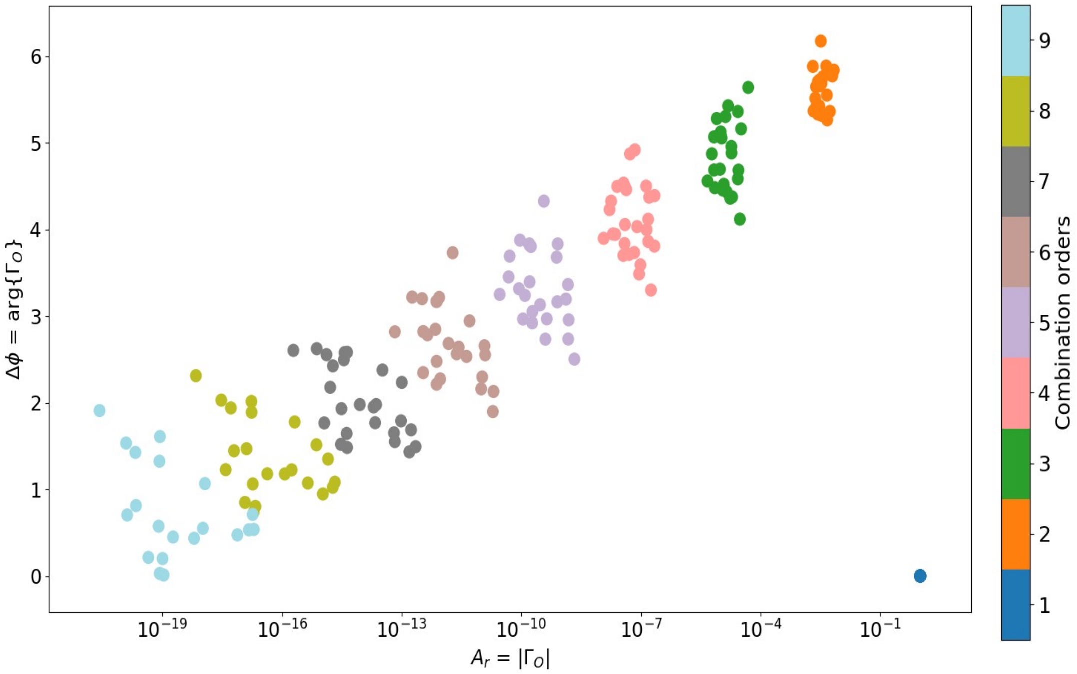
Amplitude ratios plot of the 11 (non-peculiar) HADS:
(Fourier amplitude parameters in ppt)



Applicability examples



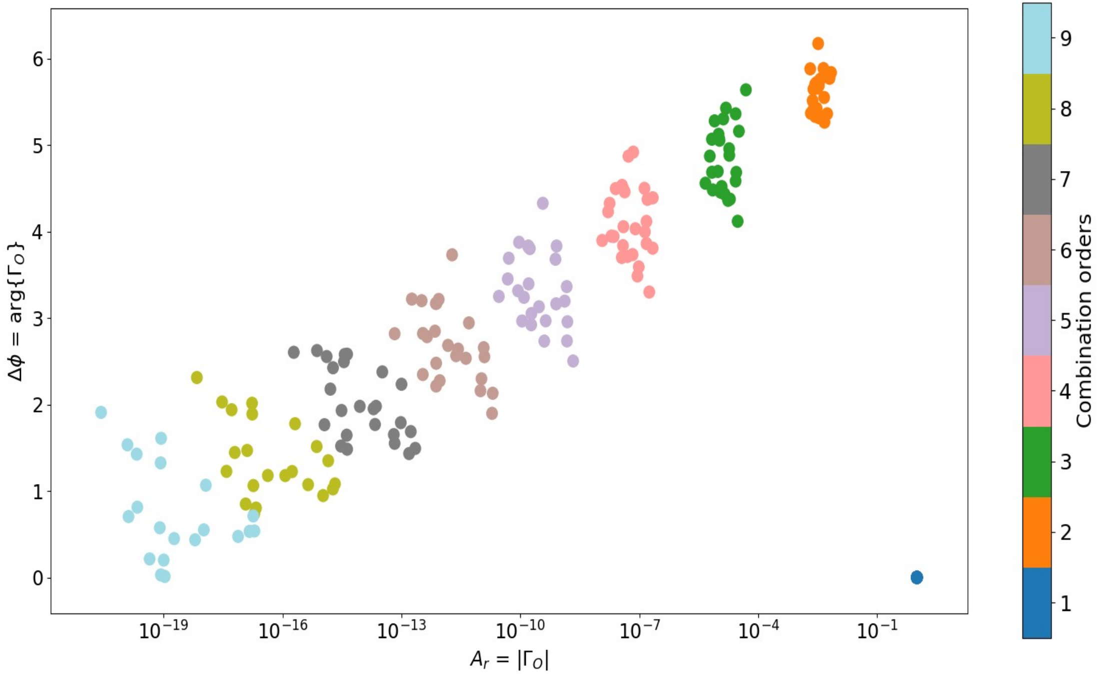
Template



Applicability examples

Identify non-linearity nature

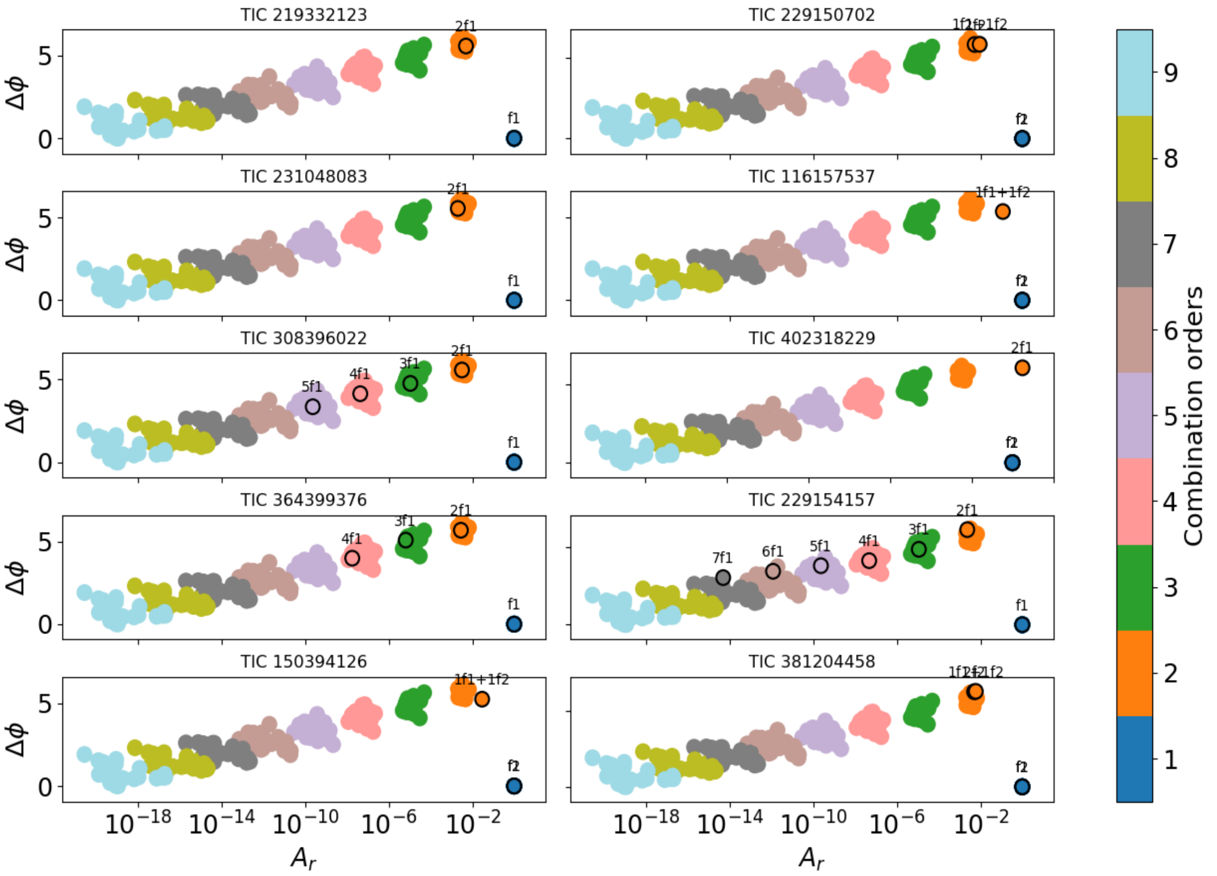
- BPM of 74 LADS stars of TESS Sector 1 and 2
- *Diagnostic plots:* ΔΦ and Ar of each LADS over the alleged non-linear distortion template



Applicability examples

Match examples:

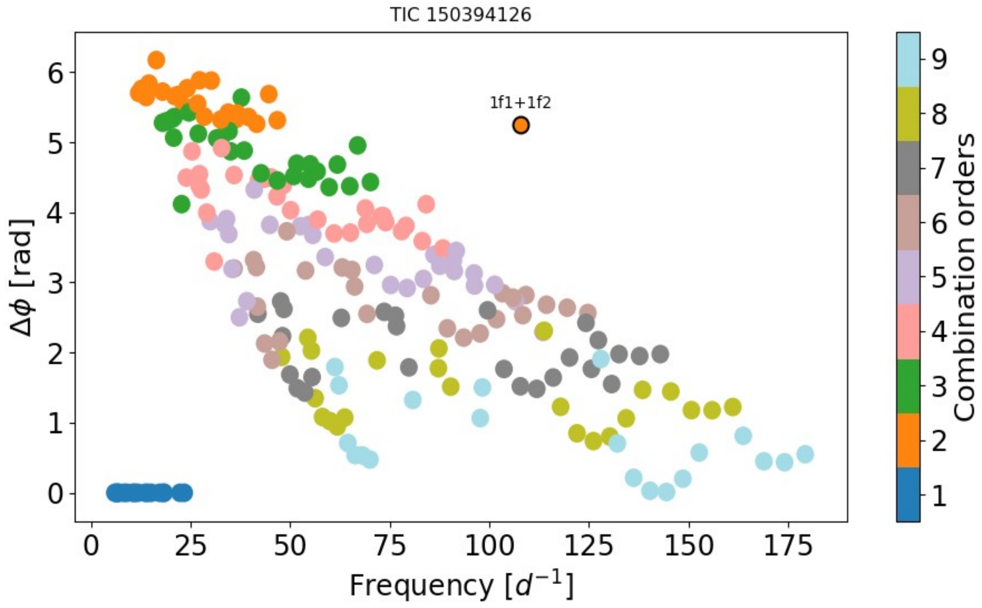
- Combination frequency of a non-linear distortion processes nature
- The parents are radial modes



Applicability examples

Possible mode identification:
TIC 150394126

$$f_1 = \nu_n = 49.08808\, d^{-1}$$
$$f_2 = \nu_{n+1} = 58.58333\, d^{-1}$$
$$f_1 + f_2 = 107.67141\, d^{-1}$$
$$\Delta\nu = 9.49525\, d^{-1}$$
$$\frac{\nu_n}{\nu_{n+1}} = 0.838$$

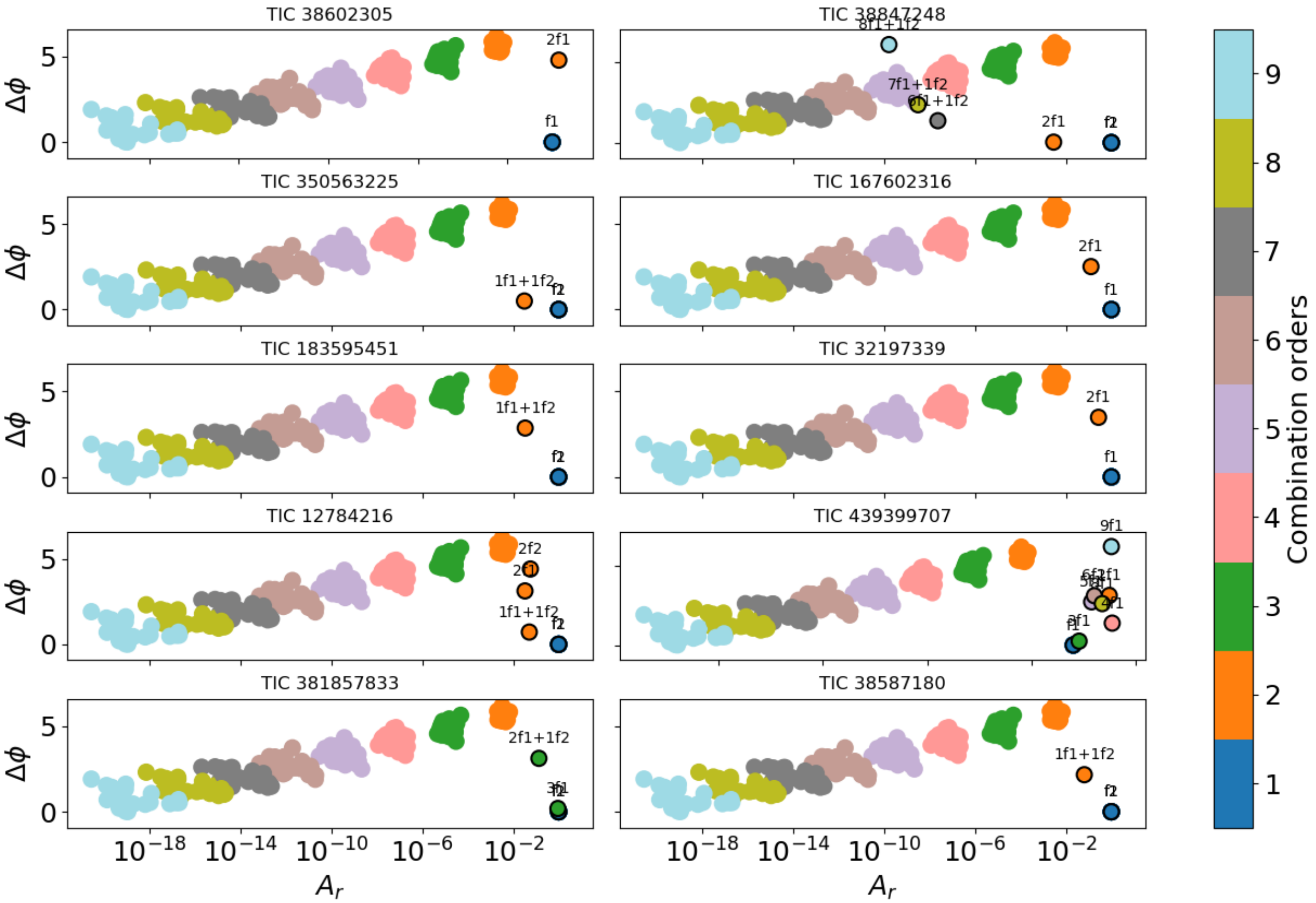


- From Stellingwerf's 1979 period relations, one can derive:

$$0.818 \leq \frac{\nu_2}{\nu_3} \leq 0.831 \longrightarrow \begin{array}{l} f_1 \sim radial\, mode\, with\, n=2 \\ f_2 \sim radial\, mode\, with\, n=3 \end{array}$$

No-Match examples

- Non-linear distortion process non-linearity of non-radial parents (whose template pattern is still unknown)
- A resonantly excited mode.
- An instrumental systematic that is happen to be modulating the spectra.
- An independent mode.

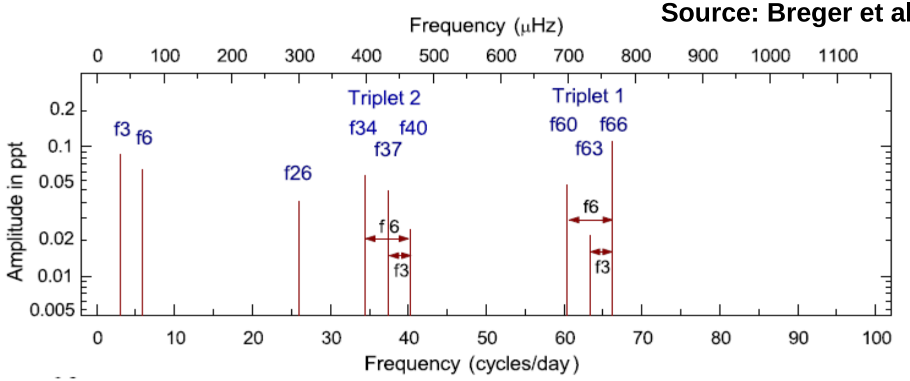


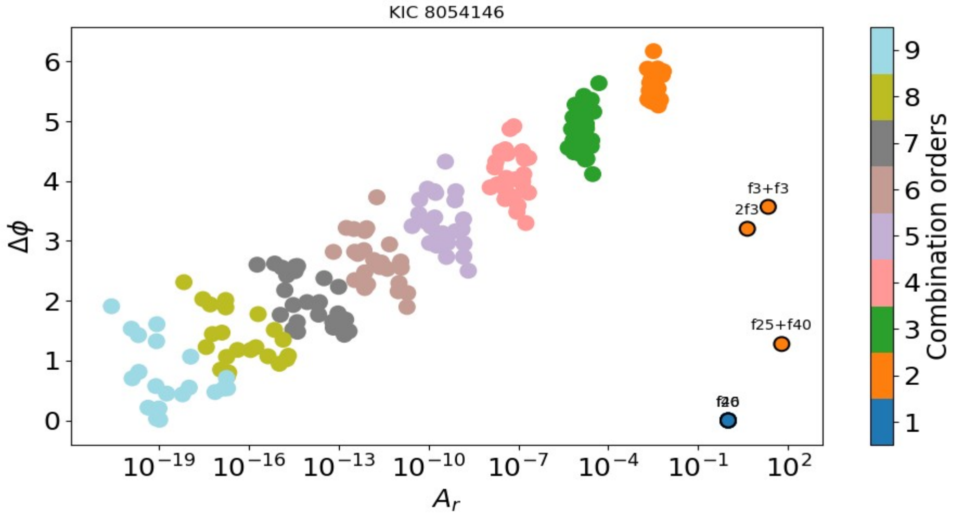
Applicability examples

Source: Breger et al., 2014.

Identify non-linearity nature

- Breger and Montgomery, 2014
  - F_{66} is most likely to be a couple mode, whose parents are F_{40} and F_{26}
  - F_6 is most likely to be a couple mode, either from the sum combination $F_{3a}+F_{3b}$ or the harmonic $2F_{3a}$
- Diagnostic plot
  - Confirms that F_{66} is not from a non-linear distortion process.
  - Confirms that F_6 is not from a non-linear distortion process, neither from the sum $F_{3a}+F_{3b}$ or the harmonic $2F_{3a}$





Summary 2

- The study enable to empirically characterize the non-linear behavior of HADS stars.
- Assuming that combination frequencies in HADS stars are owe to non-linear distortion processes, one can associate their Γ_o functions with this nature.
- The study expose the possibility of mode identification and non-linearity nature identification.

Conclusions and Future work

- Most relevant conclusions
  - A proper non-linear terms extraction from δ Sct stars power spectra can reveal frequency structure previously hidden that could match with the linear models
  - Diagnostic plots would be a faster method to identify the non-linearities nature (no need of tracking the variation between all the signal components) , and with no previous condition needed to apply it (no need of amplitude modulation).

- Future work
  - Building a HADS Catalog of space observations
  - Spectroscopy follow up of δ Sct stars (HADS and LADS)
  - Time frequency analysis
  - Development of the analytical equations
  - Statistically strengthen the Diagnostic plots procedure
  - Study of the physical meaning of the Γ_o functions