PDF Lagrangian models of turbulence for the modelling of convection/oscillation coupling

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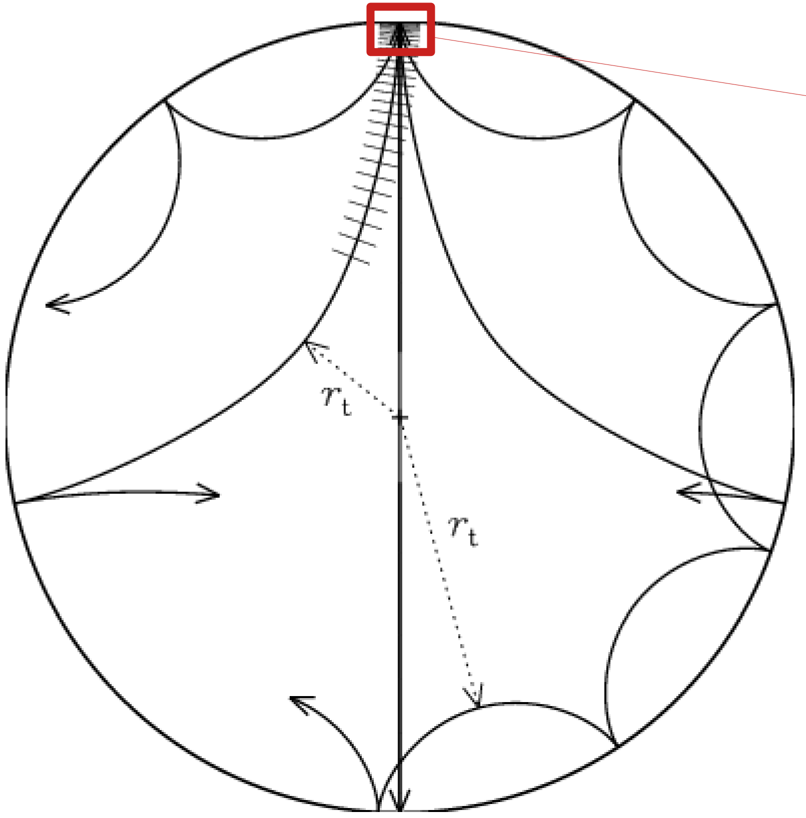
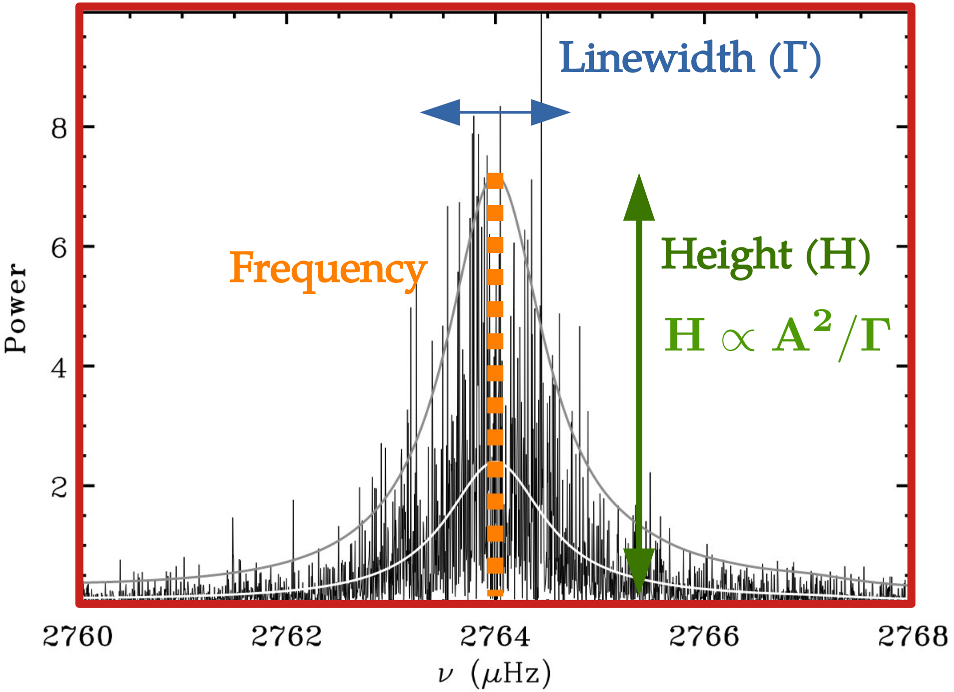
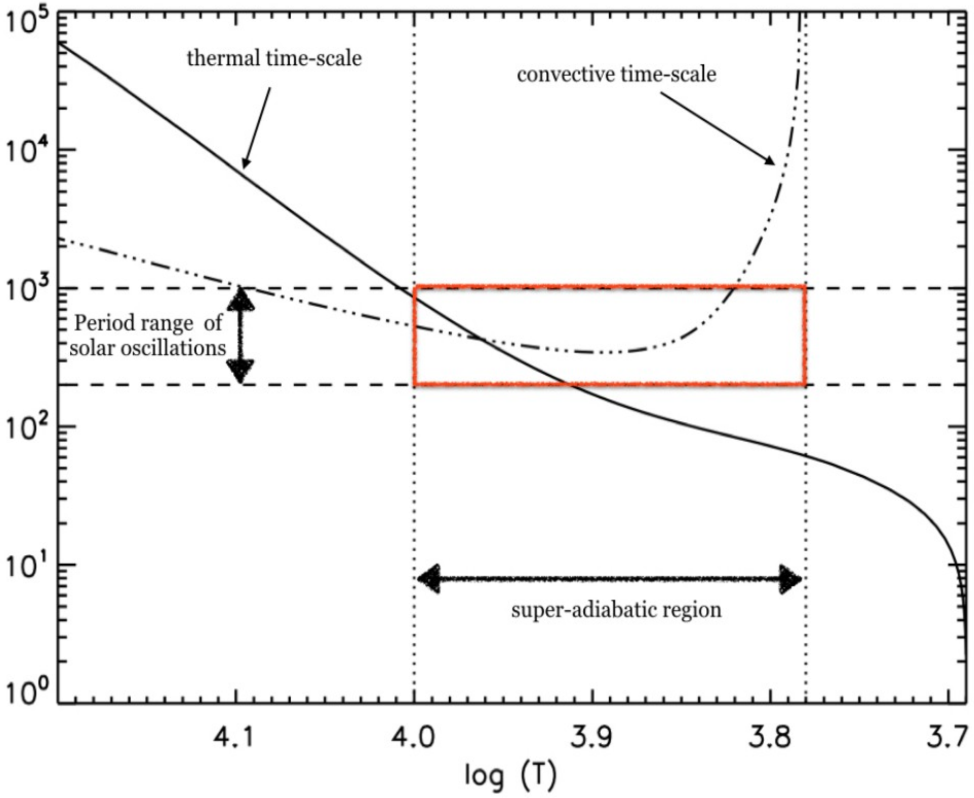
(sur l'aimable invitation de Kevin Belkacem)

P-modes: propagation in turbulent medium
→ effect on the frequency
→ driving
→ sink of energy = damping



Localised near the surface





- Coupling near surface because
  - coincidence time scales

$$\tau_{\rm turb} \sim \tau_{\rm th} \sim \Pi \sim 5 \text{ minutes}$$

  - coincidence spatial scales

$$\lambda \sim l_{\rm conv} \sim 1 \text{ Mm}$$

- Good news: seismic observables can be used to constrain turbulent convection
- Bad news: theoretical modelling is very complicated

- Today: done with MLT, Reynolds-stress models. Also valuable insight from simulations
- But not ideal: full effect of turbulent cascade can't be included → turbulent dissipation?

- Core idea → represent turbulent flow with set of individual fluid parcels
- Turbulent flow → particle evolution follows stochastic differential equations

$$\mathrm{d}f^\star = \underbrace{a(f^\star, t)\ \mathrm{d}t}_{\text{Deterministic}} + \underbrace{b(f^\star, t)\ \mathrm{d}W}_{\text{Random}}$$

- ω_t = turbulent frequency → inverse of turbulent energy decay rate
- C_0 = Kolmogorov constant → universal value
- G_{ij} = drift tensor → given by equivalent Reynolds-stress model

- Simple model

$$\mathrm{d}X_i^\star = U_i^\star \mathrm{d}t$$

$$\mathrm{d}U_i^\star = \underbrace{-\frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x_i}\mathrm{d}t + g_i}_{\text{Mean pressure force + gravity}} + \underbrace{G_{ij}(U_j^\star - \widetilde{U}_j)\mathrm{d}t + \sqrt{C_0 \widetilde{u_i'' u_i''}\,\omega_t}\,\mathrm{d}W_i}_{\text{Fluctuating pressure force + buoyancy + turbulent dissipation}}$$

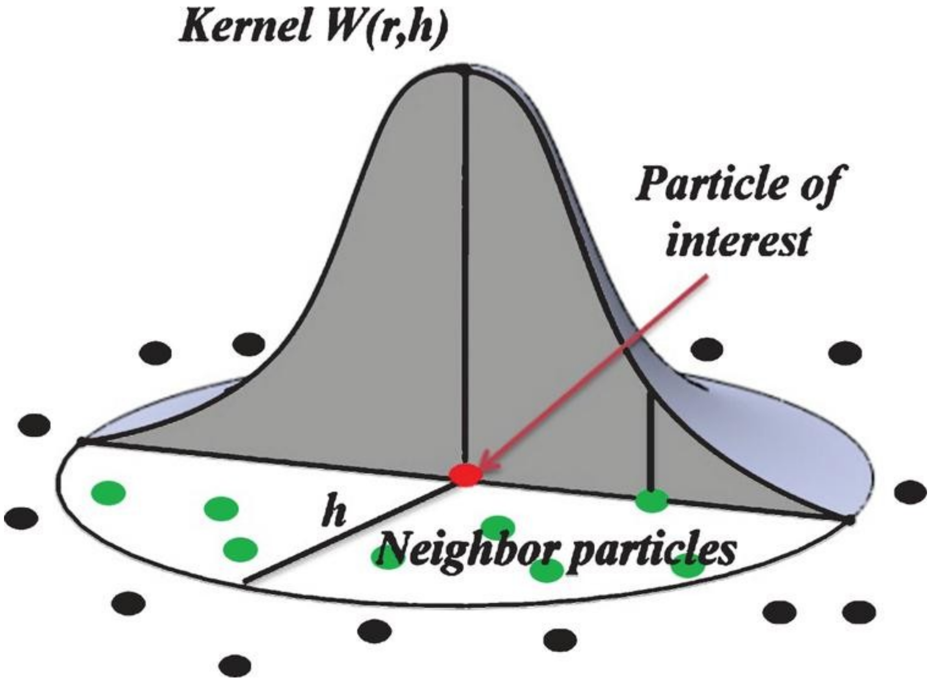
Pope (1983)

But: necessary to evaluate the mean fields at the particle position

In general, extraction from DNS/LES

How to estimate the mean fields ?

Here, impossible because
- we need
- that would decouple oscillations and turbulence



Solution: estimation directly from fluid parcels
→

$$\langle Q(\boldsymbol{x},t)\rangle_{N,h} \equiv \frac{V}{N}\sum_{i=1}^{N} W\left[\boldsymbol{x}^{(i)}(t)-\boldsymbol{x},h\right] Q\left(\boldsymbol{U}^{(i)}\right)$$

Example for density

$$\overline{\rho}(\boldsymbol{x},t) = \frac{V}{N}\sum_{i=1}^{N} W\left[\boldsymbol{x}^{(i)}(t)-\boldsymbol{x},h\right]$$

Credits: Yi et al. (2017)

Linearisation:

$$\boxed{\frac{\partial \boldsymbol{\xi}_{\rm osc}}{\partial t} - \mathbf{u}_{\rm osc}} - \boxed{(\boldsymbol{\xi}_{\rm osc} \cdot \boldsymbol{\nabla})\mathbf{u}_t} = \boxed{\mathbf{0}}$$

$$\boxed{\frac{\partial \mathbf{u}_{\rm osc}}{\partial t} - \mathbf{L}_1^d} - \boxed{\mathbf{L}_1^s} = \boxed{\mathbf{L}_0}$$

Deterministic propagation
→ p-modes without turbulence

Stochastic perturbation of wave propagation
→ damping + surface effects

Stochastic inhomogeneous term
→ driving

Philidet et al. (2021)

Expansion over normal modes:

$$|z(t)\rangle = \sum_i A_i(t) \exp^{\mathrm{j}(\omega_i t + \Phi_i(t))} |z_i\rangle$$

Unperturbed eigenfunction
Perturbed mode amplitude
Perturbed mode phase

Time-dependent perturbation → linear coupling between modes
But for now, let us focus on single mode case

Stochastic perturbation has memory
→ effect on mean amplitude and mean phase must be accounted for
→ simplified amplitude equation formalism (Stratonovitch 1965, Buchler et al. 1993)

$$\mathrm{d}A = \left(A\left(\kappa + \mathrm{Re}(\alpha_3)\right) + \frac{\mathrm{Re}(\alpha_1)}{2A} \right) \mathrm{d}t + \left(A^2 \left(2\alpha_2^R + \mathrm{Re}(\alpha_3)\right) + \mathrm{Re}(\alpha_1) \right)^{1/2} \mathrm{d}W_A$$

$$\mathrm{d}\Phi = \mathrm{Im}(\alpha_3)\, \mathrm{d}t + \left(\frac{1}{A^2} \mathrm{Re}(\alpha_1) + 2\alpha_2^I + \mathrm{Re}(\alpha_3) \right)^{1/2} \mathrm{d}W_\Phi$$

Philidet et al. (2022)

Physics contained in complex α_i → autocorrelation spectrum of stochastic perturbation

Mean energy $E_m = \langle A^2 \rangle$ and
mean phase $\Phi_m = \langle \Phi \rangle$

$$\frac{\mathrm{d}E_m}{\mathrm{d}t} = -2\eta E_m + \mathcal{P}$$

$$\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = \delta\omega$$

Damping rate

Excitation rate

Modal surface effects

$$\mathcal{P} = \frac{1}{2\mathcal{I}} \int \mathrm{d}^3\mathbf{X}\ \rho_0^2 k_j k_l \mathrm{Re}\left(\xi_{\mathrm{osc,i}} \xi^\star_{\mathrm{osc,k}} \phi^{(4b)}_{ijkl}(\mathbf{k},\omega)\right)$$

Depends on equilibrium structure + mode structure
Depends on spectrum of turbulent fields

$$\eta = -\frac{1}{4\mathcal{I}^2} \int \mathrm{d}^3\mathbf{X}\ \rho_0^2 \mathrm{Re}\left(F^{(1)}_i F^{(1)\star}_j \phi^{(2)}_{ij}(2\mathbf{k},2\omega) + F^{(2)}_{ij} F^{(2)\star}_{kl} \phi^{(4b)}_{ijkl}(2\mathbf{k},2\omega) + F^{(3a)}_{ijm} F^{(3a)\star}_{kln} \phi^{(4c)}_{ijklmn}(2\mathbf{k},2\omega) \right.$$
$$\left. +2F^{(1)}_i F^{(3b)\star}_{jkl} \phi^{(4a)}_{ijkl}(2\mathbf{k},2\omega) + 2F^{(3a)}_{ijm} F^{(2)\star}_{kl} \phi^{(4d)}_{ijklm}(2\mathbf{k},2\omega)\right)$$

$$\delta\omega = \frac{1}{4\mathcal{I}^2} \int \mathrm{d}^3\mathbf{X}\ \rho_0^2 \mathrm{Im}\left(F^{(1)}_i F^{(1)\star}_j \phi^{(2)}_{ij}(2\mathbf{k},2\omega) + F^{(2)}_{ij} F^{(2)\star}_{kl} \phi^{(4b)}_{ijkl}(2\mathbf{k},2\omega) + F^{(3a)}_{ijm} F^{(3a)\star}_{kln} \phi^{(4c)}_{ijklmn}(2\mathbf{k},2\omega)\right)$$

Example turbulent spectra

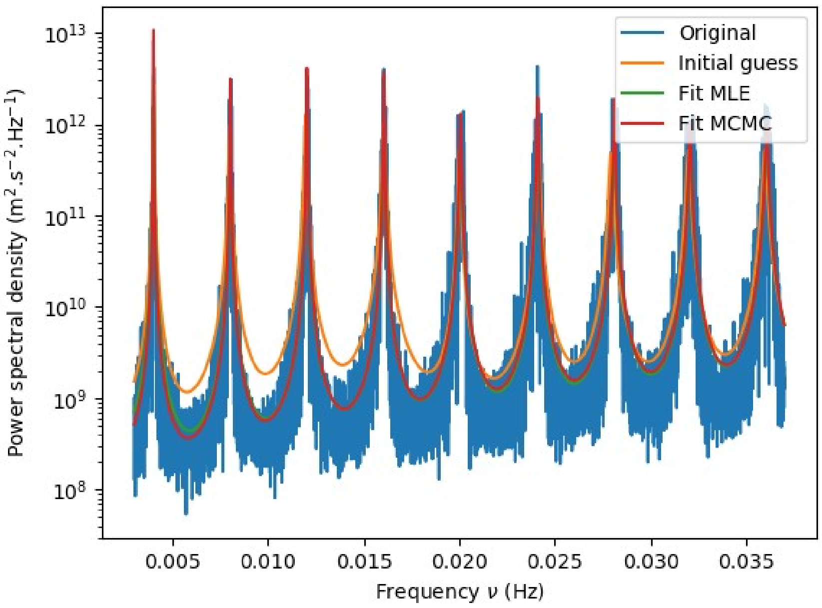
$$\phi^2_{ij}(\mathbf{k},\omega) \equiv \int_{-\infty}^{0} \mathrm{d}\tau \int \mathrm{d}^3\delta\mathbf{x}\ \langle u_{t,i}(\mathbf{X},t)\ u_{t,j}(\mathbf{X}+\delta\mathbf{x},t+\tau)\rangle \exp^{j(\mathbf{k}.\delta\mathbf{x}+\omega\tau)}$$

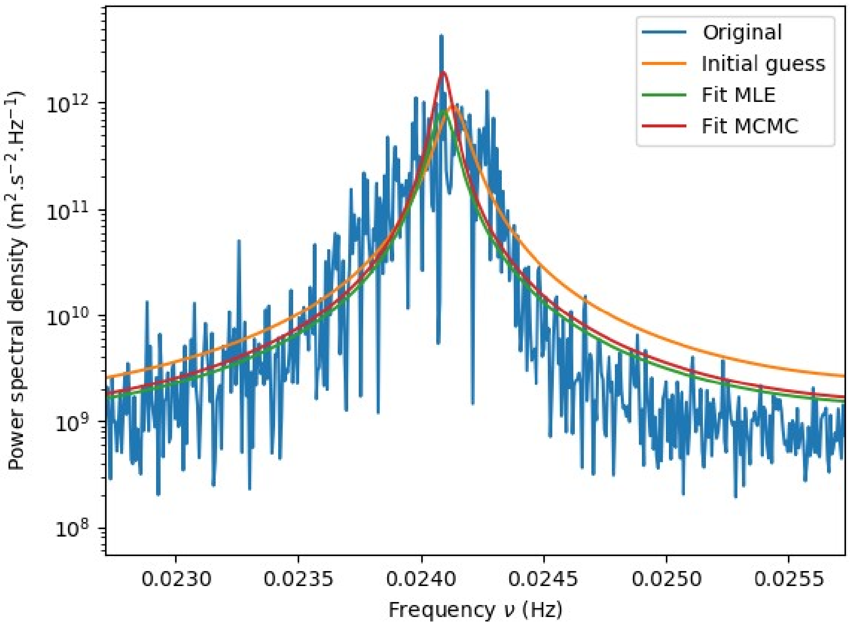
- Consistent with known expression of excitation rate (e.g Samadi & Goupil 2001)
- Damping rate and modal surface effects are real and imaginary part of same complex quantity
- Allows for the injection of any prescription for turbulent velocity spectrum

That was for very simple Lagrangian model: for more complete model, need to go numerical

- Very simple to implement: no spatial grid or scheme needed
- Time scheme adapted to stochastic nature of equations
- Output of simulation = kernel averages computed from set of particles

For the moment, 1D implementation: test theoretical formalism





2D and 3D implementations are under way. Investigation of:

- Coupling with gravity waves
- Lagrangian formalism → perfectly adapted to Lagrangian tracers → transport (chemical, angular momentum, ...)

Still very much pending though

Danke sehr!!