Impacts of central mixing and nuclear reactions network on the size of convective cores

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Convective cores in low-mass stars

- Found in the center of stars $M \gtrsim 1.2\, M_\odot$
- Limits classically defined by the Schwarzschild [Ledoux] criterion:

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- Local criteria, one dimensional...
- Cores are actually bigger than their Schwarzschild limit, due to:
  - Overshooting
  - Rotation mixing
  - Semi-convection

The question of the size of convective cores

- Artifical core extent in stellar evolution codes:
  - "Step" overshooting: $d_{\rm ov} = \alpha_{\rm ov} H_p$
  - Diffusive overshooting: $D_{\rm ov} = D_{\rm conv} \exp\left[-\frac{2(r-R_{\rm cc})}{f_{\rm ov} H_p}\right]$

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- Observation constraints through:
  - Cluster color-magnitude diagrams (e.g., Maeder+ 81)
  - Binary stars modeling (e.g., Claret+ 2018)
  - Asteroseismology (e.g., Silva-Aguirre+ 2011, Deheuvels+ 2016, Noll+ 2021)

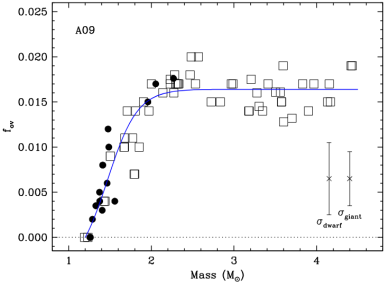

Figure 1: From Claret+ 2018

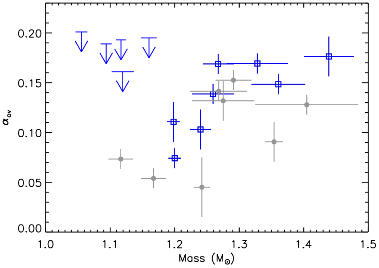

Figure 2: From Deheuvels+ 2016

However...

- All those methods rely on stellar evolution codes...
- ...which make assumptions on physical processes in the core, especially:
  - Nuclear reactions
  - Central convective mixing

- What are the impacts of such assumptions on the size of convective cores?
- Does it have an impact on the parameters retrieved through seismic modeling of MS stars?

Nuclear Reactions

Characteristics

- Provides energy during the MS, pp-chain and CNO cycle
- For $\lesssim 1.5\,M_\odot$, pp-chain produces most of the energy

Usual assumptions

- Assume equilibrium of lithium, beryllium and deuterium
  - `NOM_NUC = 'ppcno9'`
- Take into account all reactions
  - `NOM_NUC = 'ppcno12'`

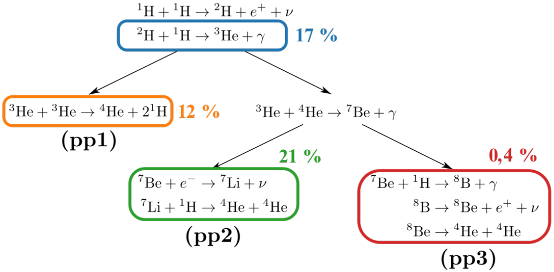

Figure 3: pp-chain and share of total energy production for a $1.5\,M_\odot$ star during the MS

Convective mixing

Features

- Elements are mixed by fluid movement
- Very efficient mixing (timescale $\ll$ evolution timescale)

Usual assumptions

- Instant mixing (all elements are homogeneous, e.g. non-diffusive CESTAM)
  ou
- Diffusive mixing:
  - MESA: diffusion coefficent computed with MLT
  - CESTAM with micro. diffu.: high *ad-hoc* coefficent ($10^{13}\,\mathrm{cm}^2.\mathrm{s}^{-1}$)

Typical timescales

Convective mixing

- Convective turn-over timescale

$$\tau_{\rm conv} = \int_0^{R_{cc}} \frac{\mathrm{d}r}{v_{\rm conv}} \sim 30\,\mathrm{days}$$

Nuclear reactions

- Time for the element to reach equilibrium

$$\tau_i = -n_i \left(\frac{\mathrm{d}n_i}{\mathrm{d}t}\right)^{-1}_{\rm nucl}$$

Element	τ
^{3}He	2.84×10^4 yr
^{7}Be	108 days
^{7}Li	178 s
^{8}B	1.11 s
^{2}H	0.889 s

Table 1: Nuclear timescales for a 1.5 $M_\odot$

Comparisons between the two timescales

If $\tau_{\mathrm{nucl}} < \tau_{\mathrm{conv}}$

- Elements have time to reach their equilibrium abundance
- Non-homogeneous in the core
- Case of $^{7}\mathrm{Li}, ^{8}\mathrm{B}, ^{2}\mathrm{H}$

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If $\tau_{\mathrm{nucl}} > \tau_{\mathrm{conv}}$

- Elements are efficiently mixed by convection
- Homogeneous in the core
- Case of $^{1}\mathrm{H}, {}^{3}\mathrm{He}, {}^{4}\mathrm{He}, {}^{7}\mathrm{Be}$

What if we assume instant mixing?

- All elements are homogeneous
- Not correct for $^7\mathrm{Li}, ^8\mathrm{B}, ^2\mathrm{H}$

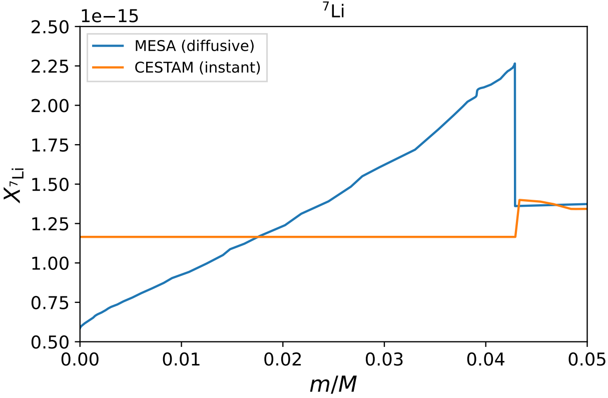


Figure 4: Lithium mass fraction in the core

How does this impact the stellar structure?

- $^{7}\mathrm{Li}$ involved in:

$$^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + \nu$$
$$^{7}\mathrm{Li} + {}^{1}\mathrm{H} \rightarrow 2\,{}^{4}\mathrm{He}$$

- The composition impacts the energy production:

$$\epsilon_{\mathrm{pp2}} \sim X_{^{7}\mathrm{Li}} X_{^{7}\mathrm{Be}} \langle \sigma v \rangle$$

- For stars $< 1.5\,M_{\odot}$, they represent $\sim 20\%$ of the total energy production
- The energy production impacts the luminosity profile : $L = \partial\epsilon/\partial m$

Impact on the central radiative gradient

- The luminosity profile impacts the radiative gradient

$$\nabla_{\mathrm{rad}} \propto \frac{\kappa \textcolor{red}{L} P}{m T^4}$$

- This is indeed verified in the models

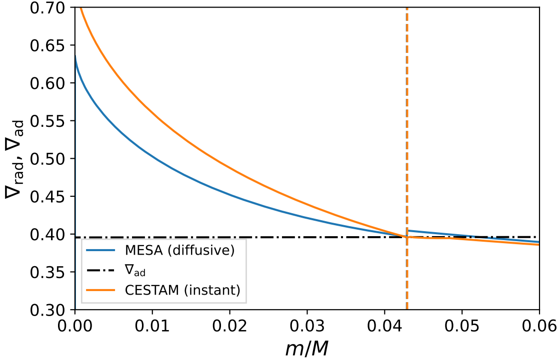
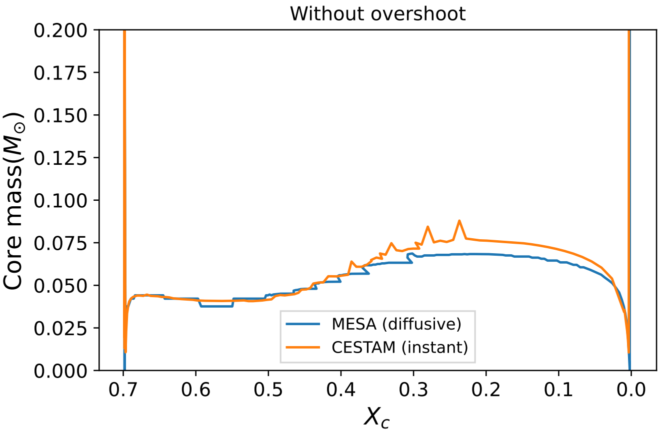


Figure 5: ∇_{rad} in MESA (diffusive) and CESTAM (instant) models

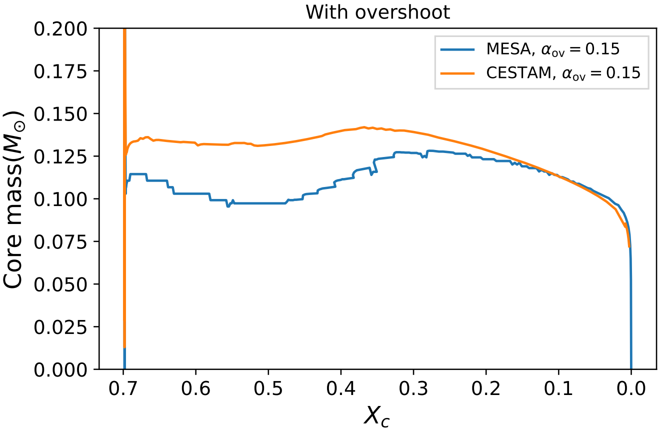
Impact of the mixing on the core structure with no overshoot

- No difference for models without overshoot. . .



Impact of the mixing on the core structure with overshooting

- ... but strong differences when overshoot is included!



What if we use a simple nuclear network?

- $^7\mathrm{Li}, ^7\mathrm{Be}$ and $^8\mathrm{B}$ at the equilibrium
- Not the case for $^7\mathrm{Be}$!
- Erroneous composition profile

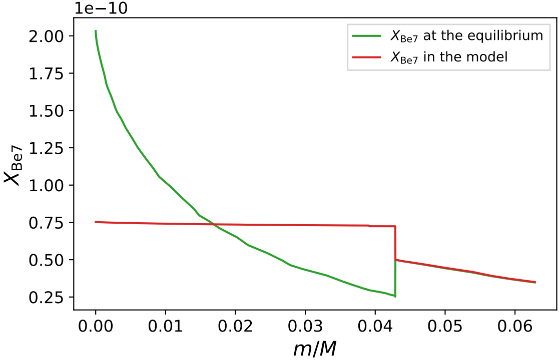


Figure 6: Beryllium mass fraction both at equilibrium and with full network

What impact on the stellar structure?

- $^7\mathrm{Be}$ involved in:

$$^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu$$

- Very low energy production, but then $X_{^7\mathrm{Be}}$ impacts $X_{^7\mathrm{Li}}$ as $^7\mathrm{Li}$ is at equilibrium:

$$X_{^7\mathrm{Li,eq}} = \frac{r_1 n_e}{r_2 X_{^1\mathrm{H}}} X_{^7\mathrm{Be}}$$

- Same impact as the mixing on ∇_{rad}

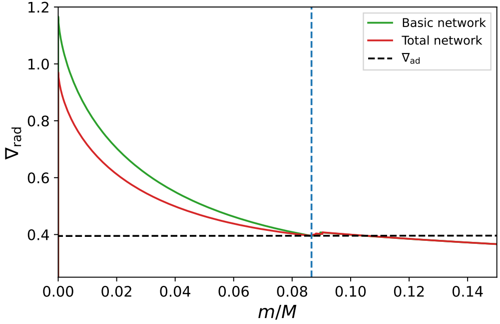
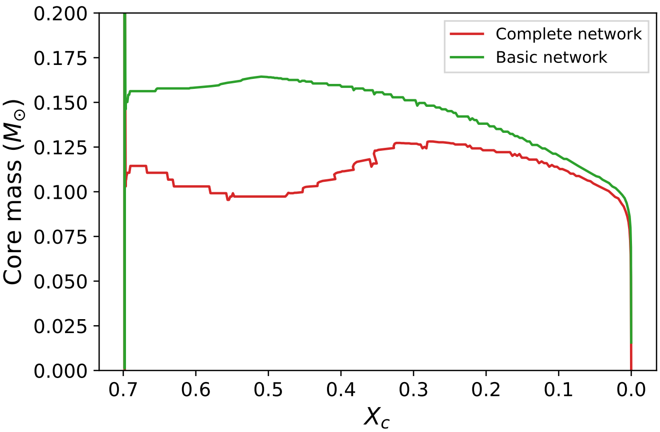


Figure 7: ∇_{rad} for a model with a basic and full nuclear network

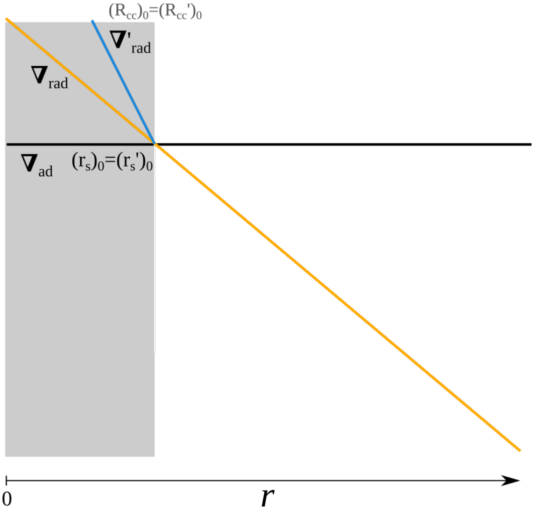
What impact on convective cores?

- Similar core mass differences within models with overshoot



Why this only appears with overshooting?

- Toy model to understand what's happening:



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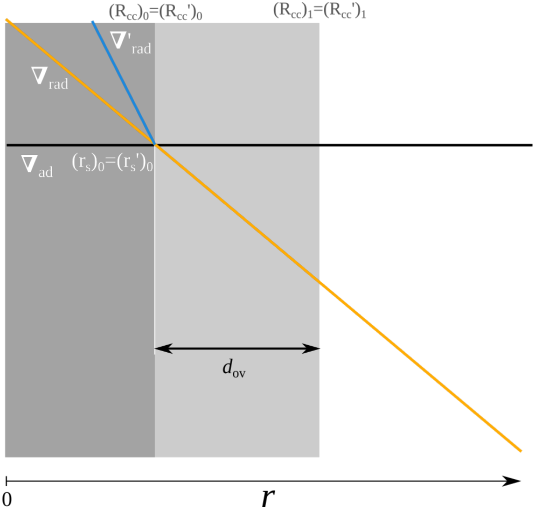

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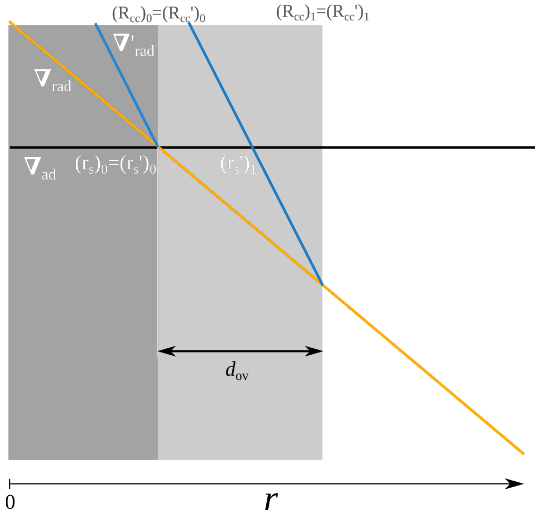

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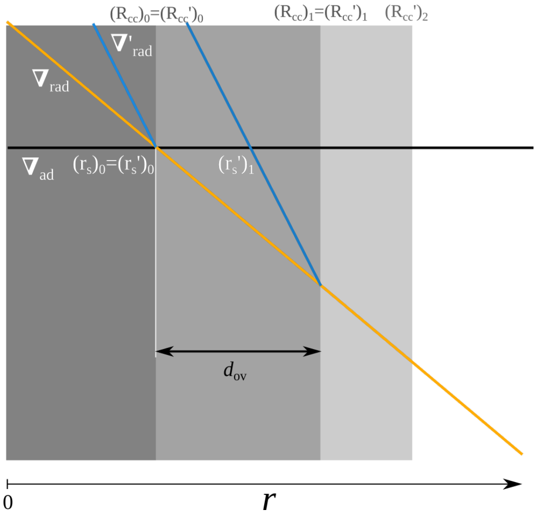

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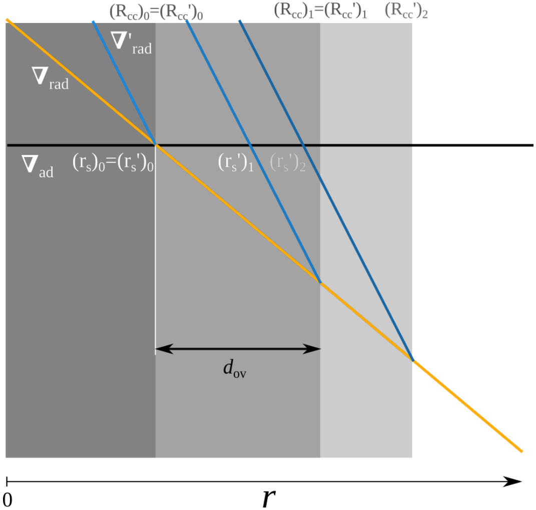

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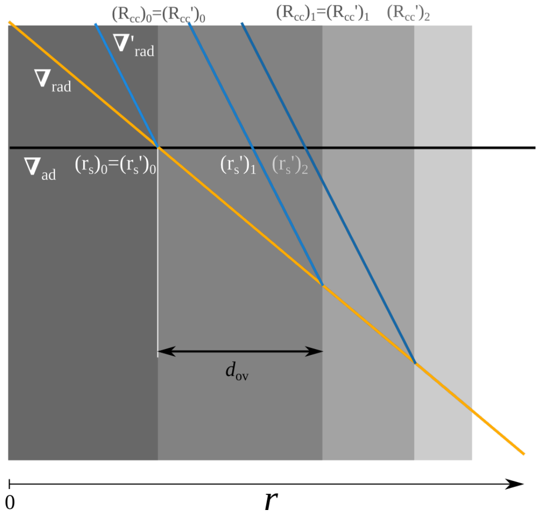

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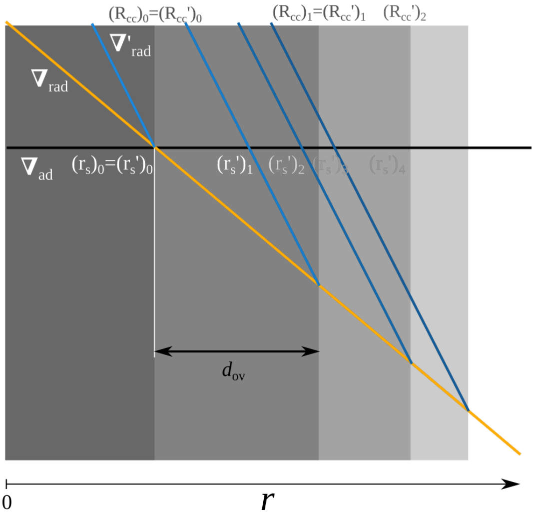

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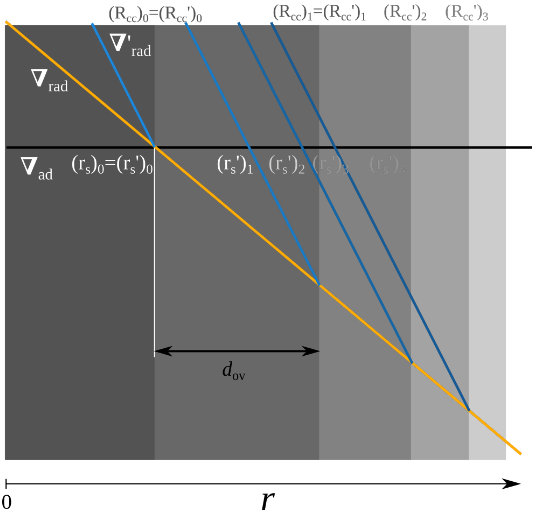

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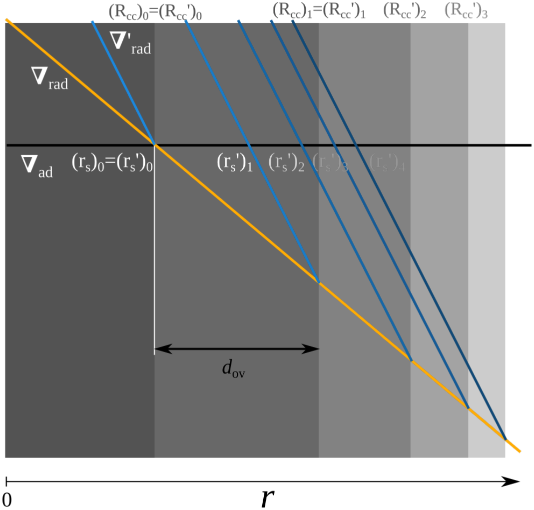

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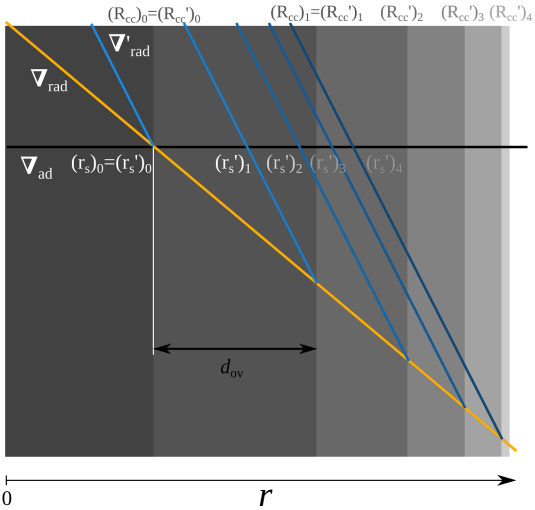

Why this only appears with overshooting?

- Directly observed in models!

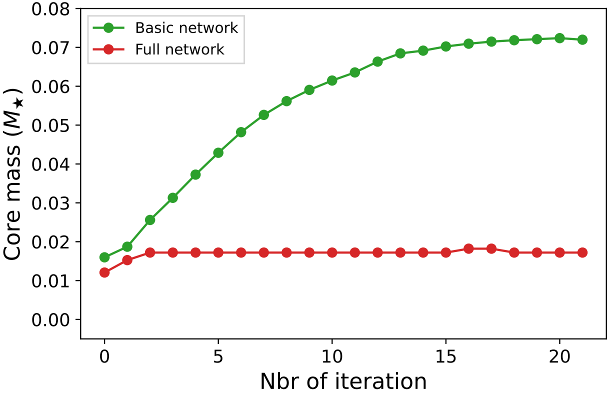

Figure 8: Hydrogen profiles for every evolution step after adding overshooting

What stars are the most sensitive to this?

- From the toy-model, we can find that:

$$\Delta r_s \to d_{\rm ov}\left(\frac{a'}{a} - 1\right)$$

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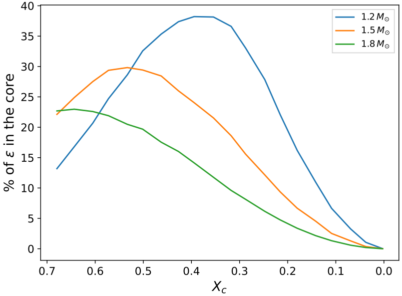

Figure 9: Evolution of the pp2 part for a $1.2, 1.5$ and $1.8\,M_\odot$

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- $\frac{a'}{a}$: depends on the % of pp2, so depends on mass and evolution
- d_{ov}: depends on the mass

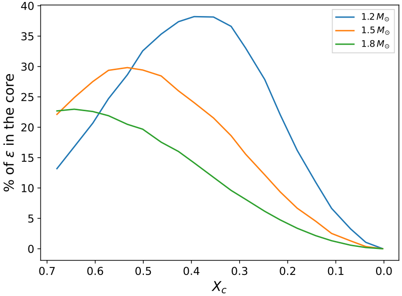

Figure 9: Evolution of the pp2 part for a $1.2, 1.5$ and $1.8\ M_\odot$

Solar like-oscillators

- Low-mass stars (between 1.15 and 1.5 $M_\odot$): solar-like oscillators
- Exhibit numerous p-modes, stochastically excited by the convective envelope
- Low-amplitude, but good quality data from the *Kepler* satellite

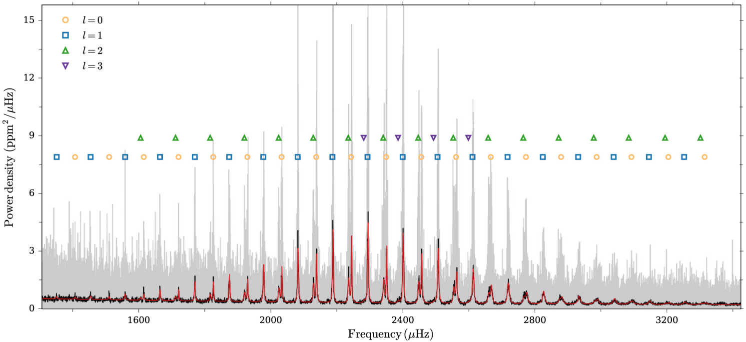

Figure 10: PSD of KIC6225718 from Lund+ 2017

Impact on seismic modeling

- Solar-like oscillators exhibit p-modes, highly sensitive to the near-surface regions of the star

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- Use of r_{01} ratios:
  - Less sensitive to surface effects (Roxburgh & Vorontsov 2003)
- Use of coefficients a_0, a_1, a_2 of 2nd degree polynome fit as seismic observables (Popielski+ 2005, Deheuvels+ 2016)

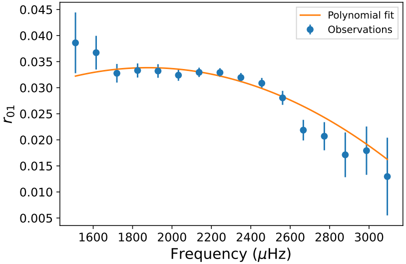

Figure 11: Ratios and polynomial fit of KIC6225718

Impact on the seismic modeling

- Comparisons with MESA (Paxton+ 2011) and ADIPLS (JCD 2008) models
  - Basic networks, assuming $^7\mathrm{Li}$ and $^7\mathrm{Be}$ at the equilibrium
  - Full networks
- Grid computed by varying M, Z/X, Y_0 and α_{ov}

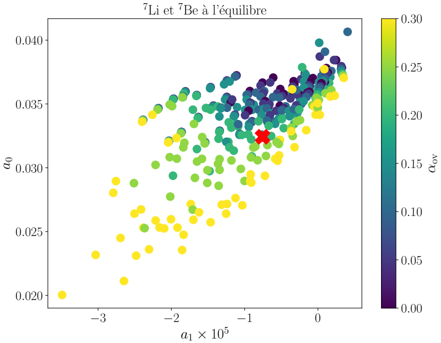

Figure 12: Basic network

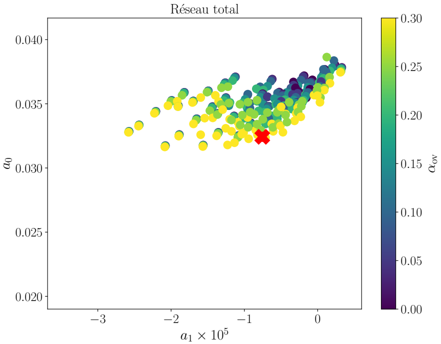

Figure 13: Full network

Impact on the retrieved parameters

- Significant difference in the retrieved $\alpha_{\rm ov}$ parameter
- No significant difference for the others parameters
- Results for KIC6225718:

M ($M_\odot$)	1.2780 ± 0.029
R ($R_\odot$)	1.2736 ± 0.012
Age (Gyr)	1.6522 ± 0.38
$[Z/X]$ (dex)	0.1595 ± 0.069
Y_0	0.2611 ± 0.017
$\alpha_{\rm ov}$	0.2013 ± 0.032

Table 2: Parameters for basic network

M ($M_\odot$)	1.2764 ± 0.029
R ($R_\odot$)	1.2733 ± 0.010
Age (Gyr)	1.7333 ± 0.33
$[Z/X]$ (dex)	0.1436 ± 0.060
Y_0	0.2565 ± 0.015
$\alpha_{\rm ov}$	0.2836 ± 0.035

Table 3: Parameters for total network

Impact on the mass/overshoot relation

- Use of stars in the LEGACY (Lund+ 2017) sample with $M \gtrsim 1.15\,M_\odot$
- No impact on the trend
- High values of $\alpha_{\rm ov}$ in low-mass models with full network

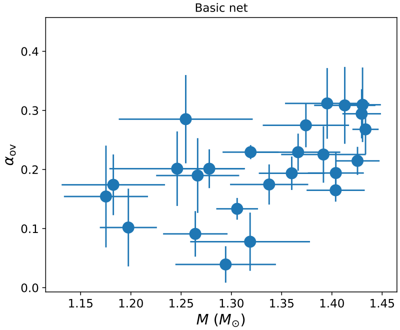

Figure 14: Basic network

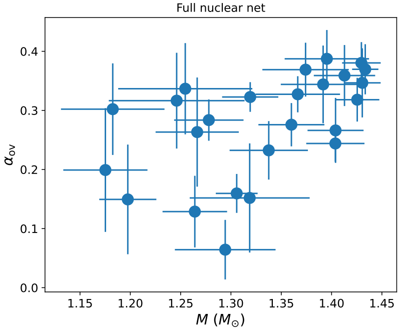

Figure 15: Full network

Conclusion

- In order to have a self-consistent determination of convective core boundaries:
  - The mixing must not be taken as instantaneous
  - Full reactions network must be taken into account
- Else...
  - Wrong $^7\mathrm{Li}$ and $^7\mathrm{Be}$ abundances in low-mass stars convective cores
  - Bigger cores in models with overshooting
- Impact on the α_{ov} value retrieved with MS stars seismic modeling
- Apparently no impact on the other parameters