3D Time-dependent convection model for asteroseismology

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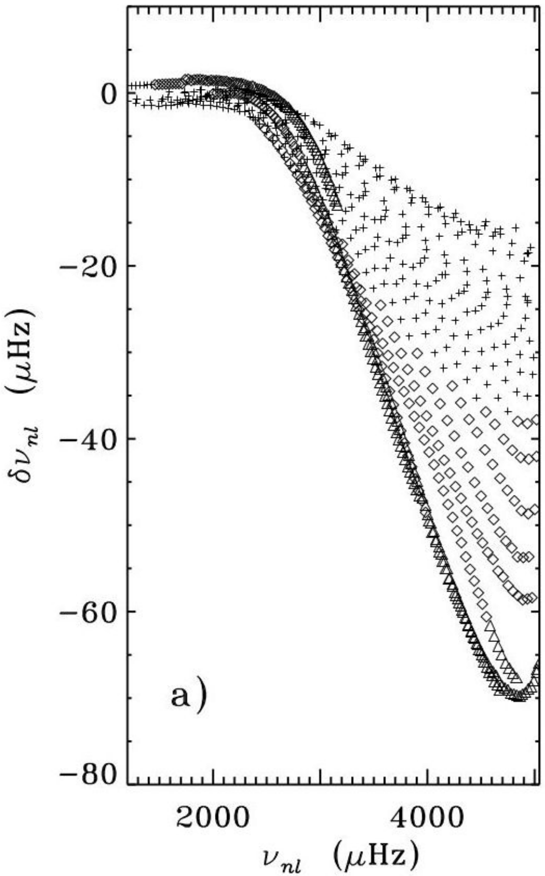
Asteroseismology and Surface effect

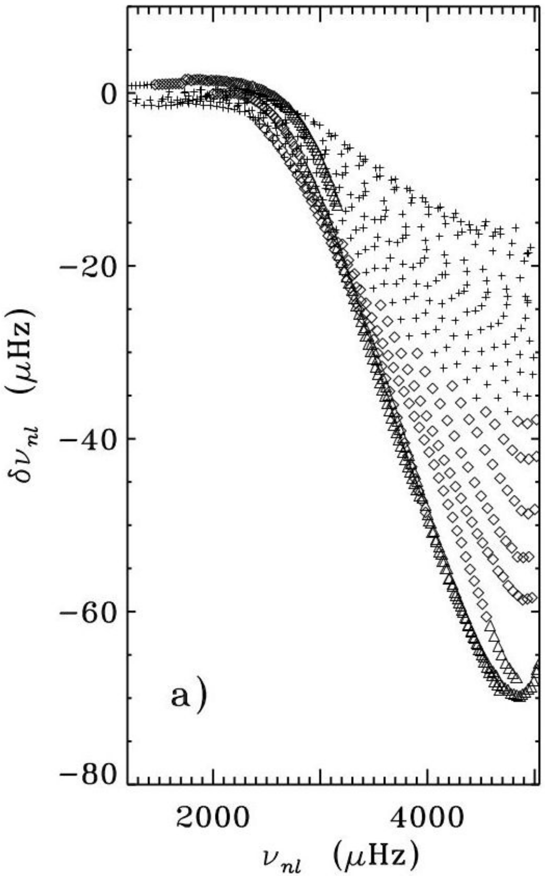
- Current pulsation and structure models
  - Mainly adiabatic approximation
  - 1D models
- Drift from observations (solar-like)
  - High frequencies



Christensen-Dalsgaard et al. (1996)

Asteroseismology and Surface effect

- Current pulsation and structure models
  - Mainly adiabatic approximation
  - 1D models
- Drift from observations (solar-like)
  - High frequencies
- Surface effects
  - Inaccurate representation of superficial layer
  - Interaction: Turbulence & Oscillations
  - Currently still an issue



Christensen-Dalsgaard et al. (1996)

Turbulence Challenges

Structure

- Very difficult to model
  - Large spectrum of space and time-scales
    - Stellar to centimeter
  - Extremely turbulent flow
    - $Re \sim 10^{10}$
- Large Eddy Simulations

Modal effect

- Superficial layer
  - $\tau_{th} \sim P \sim \tau_{conv}$
  - Highly non-adiabatic
  - System of coupled equations
    - Dynamical
    - Thermal
  - Convective zone ($M_\star \lesssim 1.5\, M_\odot$)
- Coupling: Convection & Oscillation

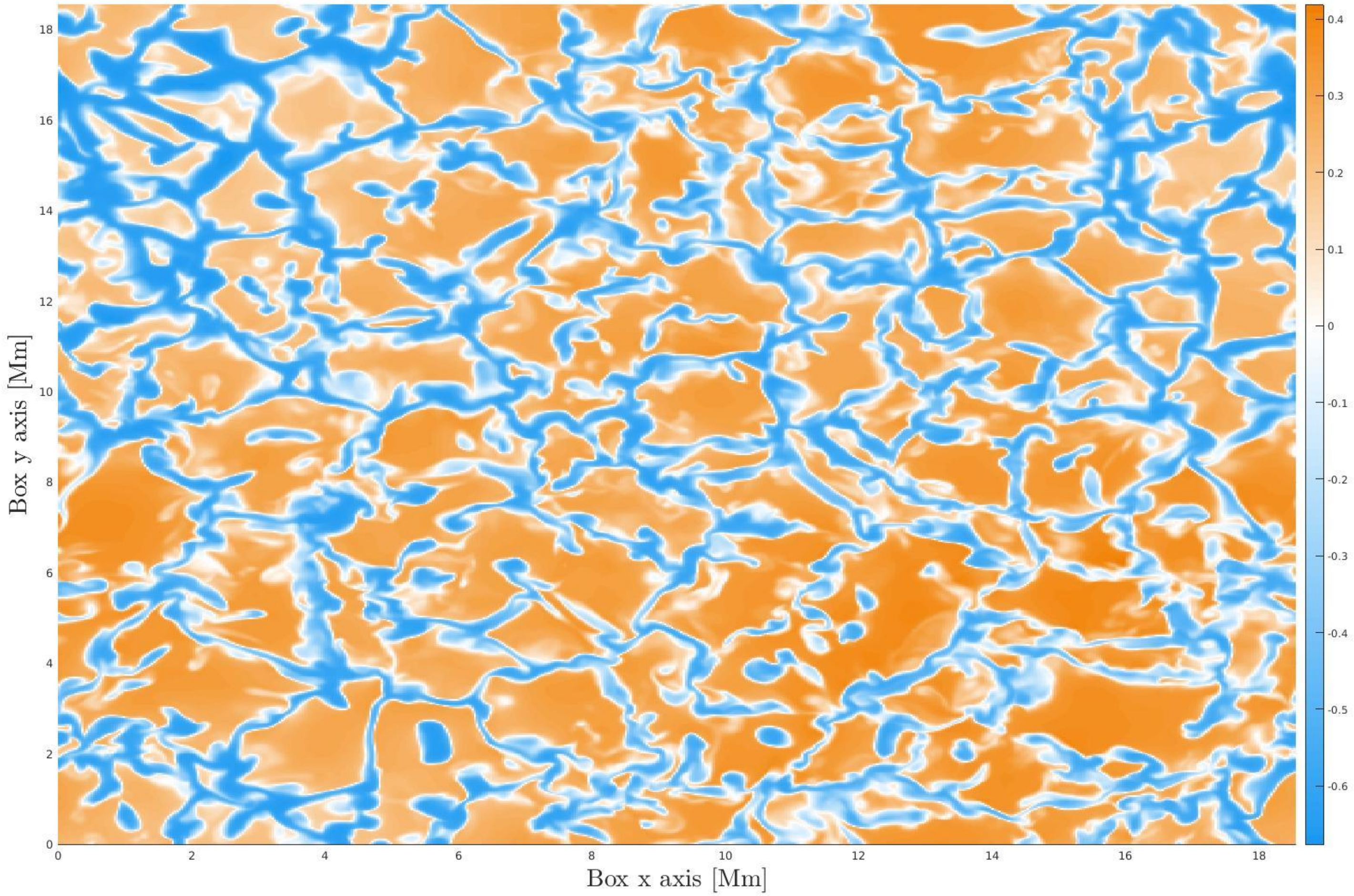
Turbulence Challenges

- Our approach
  - 3D space
  - 3D hydrodynamic simulation
  - Perturbative approach
    - Applied to 3D medium

3D Hydrodynamic simulation: CO^5BOLD

- Target: Sun
- Grid nodes: 189x189x150
- Box size: 18.5x18.5x8.4 (Mm)
- Duration: 3 min
- Time step: 5s

Normalized internal energy ($\frac{U-\overline{U}}{\overline{U}}$ [/])
Horizontal cut at z =-0.02104[Mm]



TDC Model

TDC model - Starting point

- Navier-Stokes equations

$$\text{Continuity:}\ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

$$\text{Momentum:}\ \frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = -\frac{\nabla P}{\rho} + \underline{g}$$

$$\text{Energy:}\ \rho T \frac{Ds}{Dt} = -\nabla \cdot \underline{F}$$

- Radiative transport

$$\underline{F} = -\chi \nabla T$$

TDC model - Starting point

- Eulerian perturbation

$$\text{Continuity:}\ \frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho' \underline{v} + \rho \underline{v}') = 0$$

$$\text{Momentum:}\ \frac{\partial \underline{v}'}{\partial t} + \underline{v}' \cdot \nabla \underline{v} + \underline{v} \cdot \nabla \underline{v}' = -\frac{\nabla P'}{\rho} - \nabla P \left(\frac{1}{\rho}\right)' + \underline{g}'$$

$$\text{Energy:}\ \left(\frac{\rho'}{\rho} + \frac{T'}{T}\right)\left(\rho T \frac{Ds}{Dt}\right) + (\rho T)\left(\frac{Ds}{Dt}\right)' = -\nabla \cdot \underline{F}'$$

$$\text{Radiation:}\ \underline{F}' = -\chi' \nabla T - \chi \nabla T'$$

TDC model - Characteristics

1. No horizontal averages
   - Convective flux, Reynold stress
2. No closing equation
   - Turbulence modelling
3. No approximation on the equations
   - Ad hoc prescriptions

$\boxed{\text{Innovative approach in 3D}}$

TDC model - Developments

- Oscillation mode
  - Decomposition
    - Time component
    - Space component

$$A'(\underline{r},t) = \boxed{a'(\underline{r})}\,\boxed{\mathrm{e}^{i\,\sigma t}}$$

TDC model - Developments

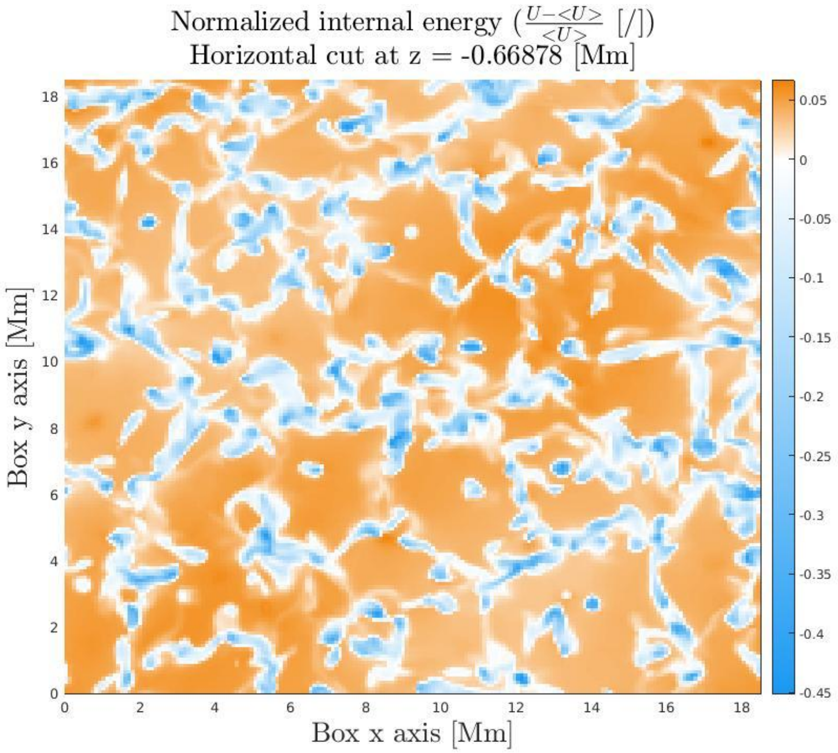
- Oscillation mode
  - Decomposition
    - Time component
    - Space component

$$A'(\underline{r},t) = \boxed{a'(\underline{r})}\,\boxed{\mathrm{e}^{i\,\sigma t}}$$

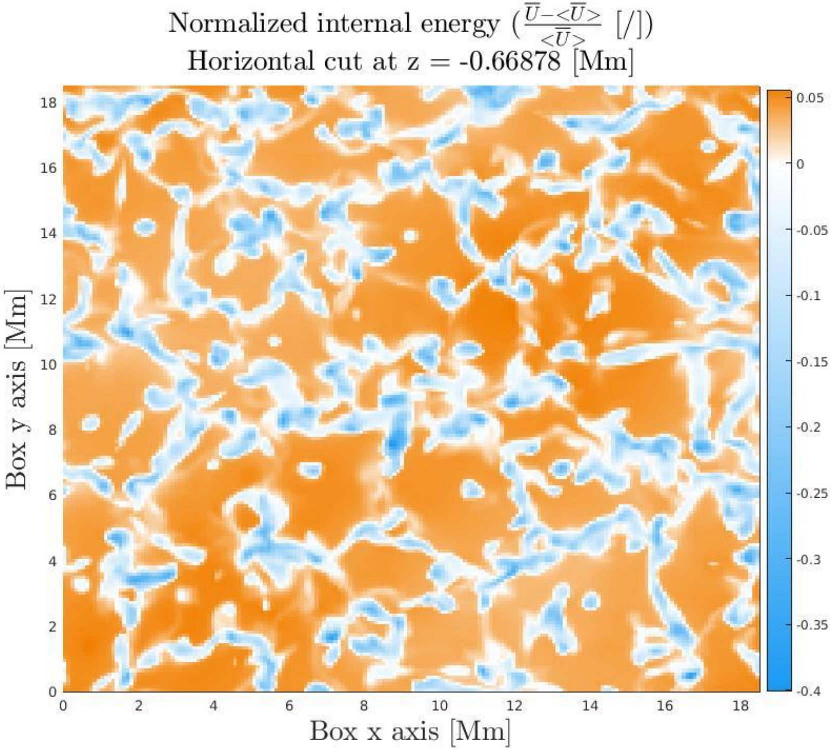
- Fourier transform at the oscillation frequency

$$\frac{1}{\Delta t}\int_t A'(\underline{r},t)B(\underline{r},t)\mathrm{e}^{-i\,\sigma t}dt = \frac{a'(\underline{r})}{\Delta t}\int_t B(\underline{r},t)\mathrm{e}^{i\,\sigma t}\mathrm{e}^{-i\,\sigma t}dt = a'(\underline{r})\,\overline{B}(\underline{r})$$

TDC model - Time step



Snapshot



Time average
(3 min)

TDC model - Developments

- Fourier transform (2D)
  - Each horizontal plane

$$A(\underline{x}) \rightarrow A(\underline{k}) = \iint A(\underline{x})\, \mathrm{e}^{-i\underline{k}\cdot\underline{x}}\, dS$$

TDC model - Developments

- Fourier transform (2D)
  - Each horizontal plane

$$A(\underline{x}) \to A(\underline{k}) = \iint A(\underline{x})\, \mathrm{e}^{-i\underline{k}\cdot\underline{x}}\, dS$$

- Main advantage
  - Natural distinction
    - Horizontal average equation ($\underline{k}=\underline{0}$)
    - Different scales of turbulence ($\underline{k}\neq\underline{0}$)
  - $\boxed{\nabla_h x' \Longrightarrow i\underline{k}\, x'}$
  - Assess impact of different scales
  - Real space easily accessible

TDC model - Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

Eulerian perturbation

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho' \underline{v} + \rho \underline{v}') = 0$$

Time processing

$$i\sigma\, \rho' + \nabla \cdot (\rho' \overline{\underline{v}} + \overline{\rho}\, \underline{v}') = 0$$

2D Fourier transform at <u>k</u>

$$\boxed{i\sigma\, \rho'_k + \sum_{k_1} \left\{ i\, \underline{k} \cdot \rho'_{k_1}\, \underline{v}_{h,k-k_1} + i\, \underline{k} \cdot \rho_{k_1}\, \underline{v}'_{h,k-k_1} + \frac{\partial}{\partial z}\left(\rho'_{k_1}\, v_{z,k-k_1} + \rho_{k_1}\, v'_{z,k-k_1}\right) \right\} = 0}$$

TDC model - Hypothesis

$$i\sigma\,\rho'_k + \sum_{k_1}\left\{ i\,\underline{k}\cdot\rho'_{k_1}\,\underline{v}_{h,k-k_1} + i\,\underline{k}\cdot\rho_{k_1}\,\underline{v}'_{h,k-k_1} + \frac{\partial}{\partial z}\left(\rho'_{k_1}\,v_{z,k-k_1} + \rho_{k_1}\,v'_{z,k-k_1}\right)\right\} = 0$$

- Distinction based on <u>k</u>
  - k = 0
    - Horizontal average
    - Every term of the sum kept
  - k ≠ 0
    - Turbulent fluctuations
    - Keep dominant terms
      - $\|x_0'\|,\|x_0\| \gg \|x_k'\|,\|x_k\|$

$$\boxed{i\sigma\,\rho'_0 + \sum_{k_1}\left\{\frac{\partial}{\partial z}\left(\rho'_{k_1}\,v^*_{z,k_1} + \rho^*_{k_1}\,v'_{z,k_1}\right)\right\} = 0}$$

$$\boxed{\begin{aligned} i\sigma\,\rho'_k + i\underline{k}\cdot\left(\rho'_0\,\underline{v}_{h,k} + \rho_0\,\underline{v}'_{h,k} + \rho'_k\,\underline{v}_{h,0} + \rho_k\,\underline{v}'_{h,0}\right) \\ + \frac{\partial}{\partial z}\left(\rho'_0\,v_{z,k} + \rho_0\,v'_{z,k} + \rho'_k\,v_{z,0} + \rho_k\,v'_{z,0}\right) = 0 \end{aligned}}$$

⇨ Decoupling of each k component

TDC model - Final system

- Every equation is processed similarly

k = 0 $$\boxed{\mathcal{L}_0(x'_0,\sigma) + \mathcal{L}_1(x'_k,\sigma) = 0}$$

k ≠ 0 $$\boxed{\mathcal{L}_k(x'_k,\sigma) + \mathcal{L}_{k0}(x'_0,\sigma) = 0}$$

TDC model - Final system

- Every equation is processed similarly

k = 0

$$\boxed{\mathcal{L}_0(x'_0,\sigma) + \mathcal{L}_1(x'_k,\sigma) = 0}$$

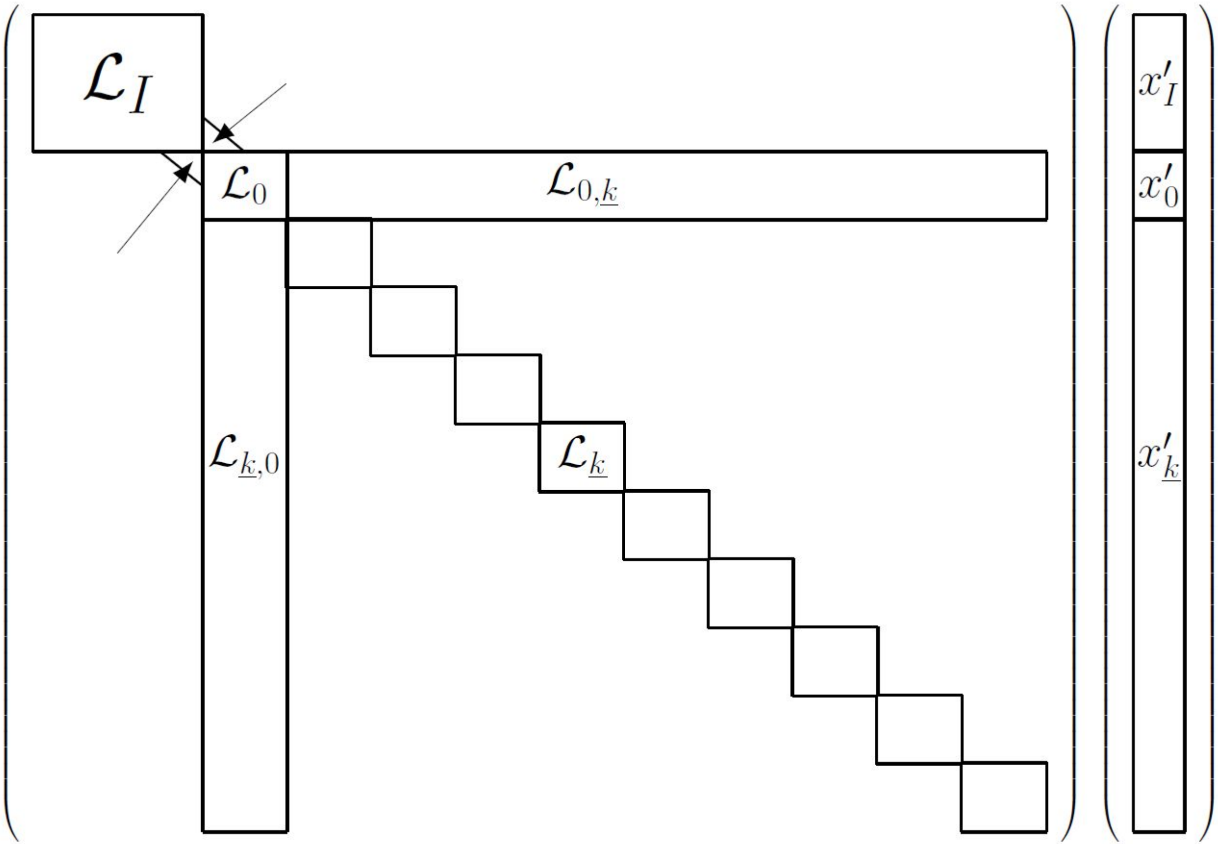
k ≠ 0

$$\boxed{\mathcal{L}_k(x'_k,\sigma) + \mathcal{L}_{k0}(x'_0,\sigma) = 0}$$

- Finite difference
  - Linear operator ⟹ Matrix
  - Eigenvalue problem
- Valid for the entire star
  - Radiative zone
    - k unknowns: $x_k' = 0$ (for $k \neq 0$)

$$\underline{\underline{\mathcal{L}_\sigma}}\, \underline{x'} = 0$$

TDC model - Final system



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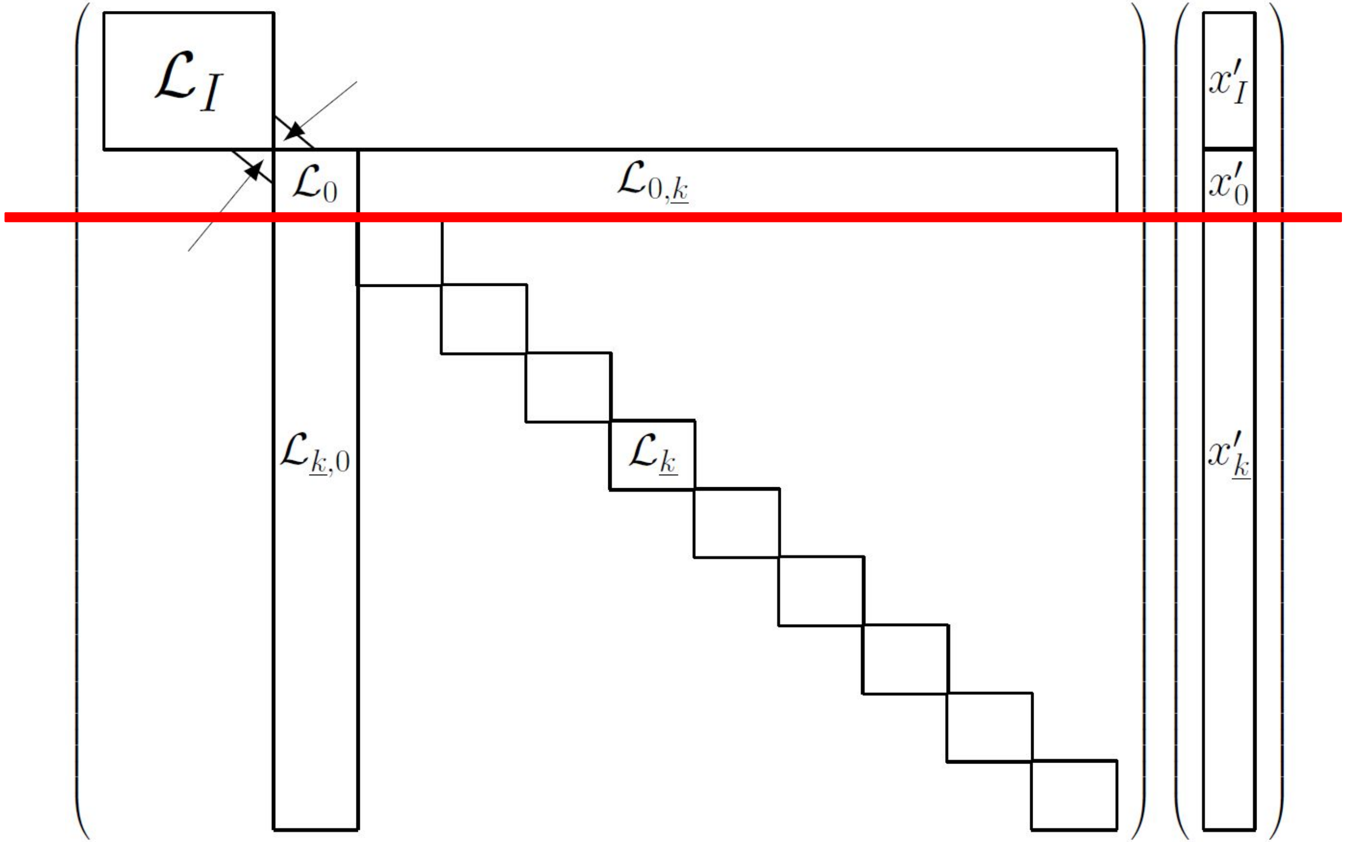

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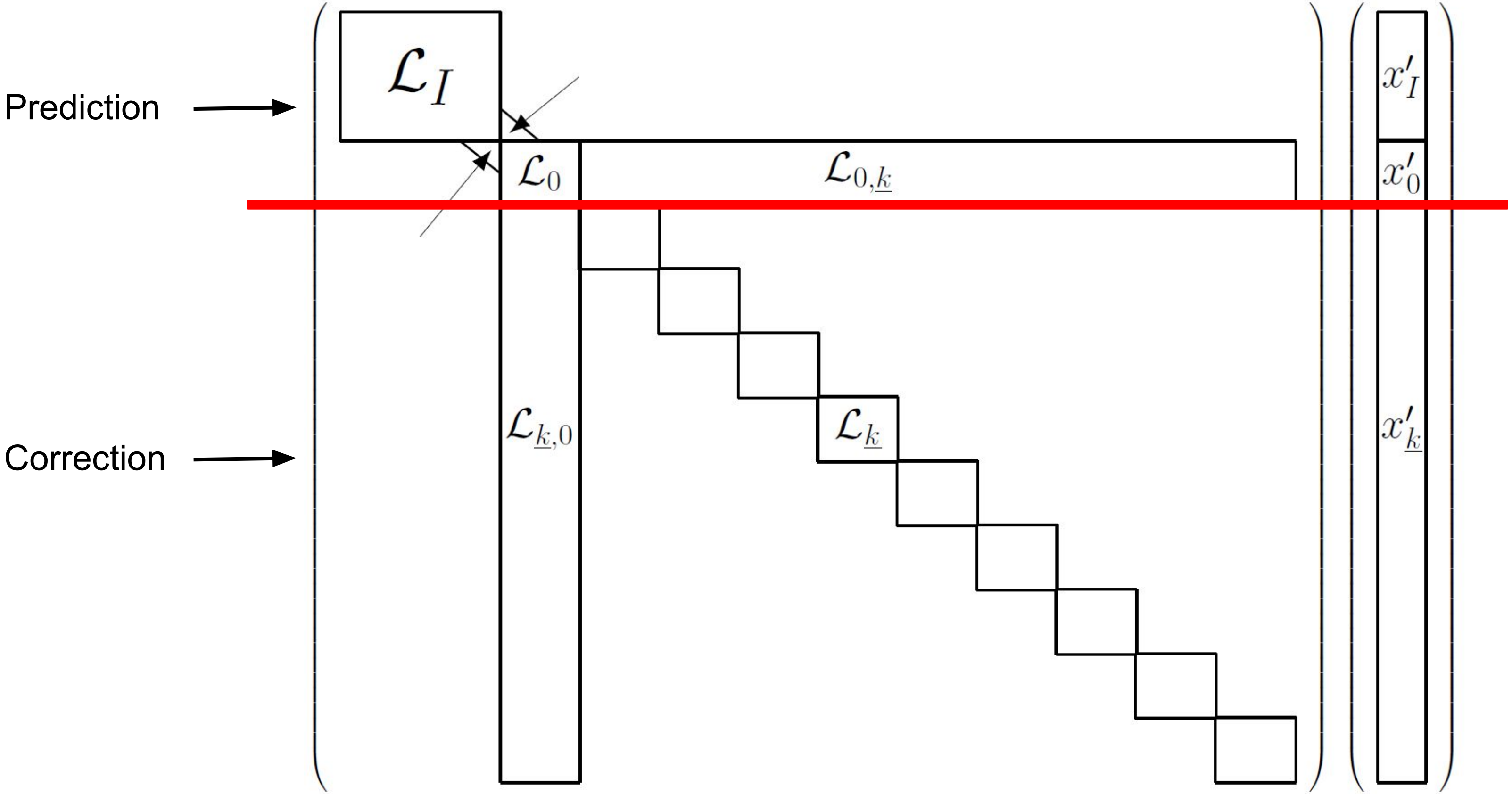


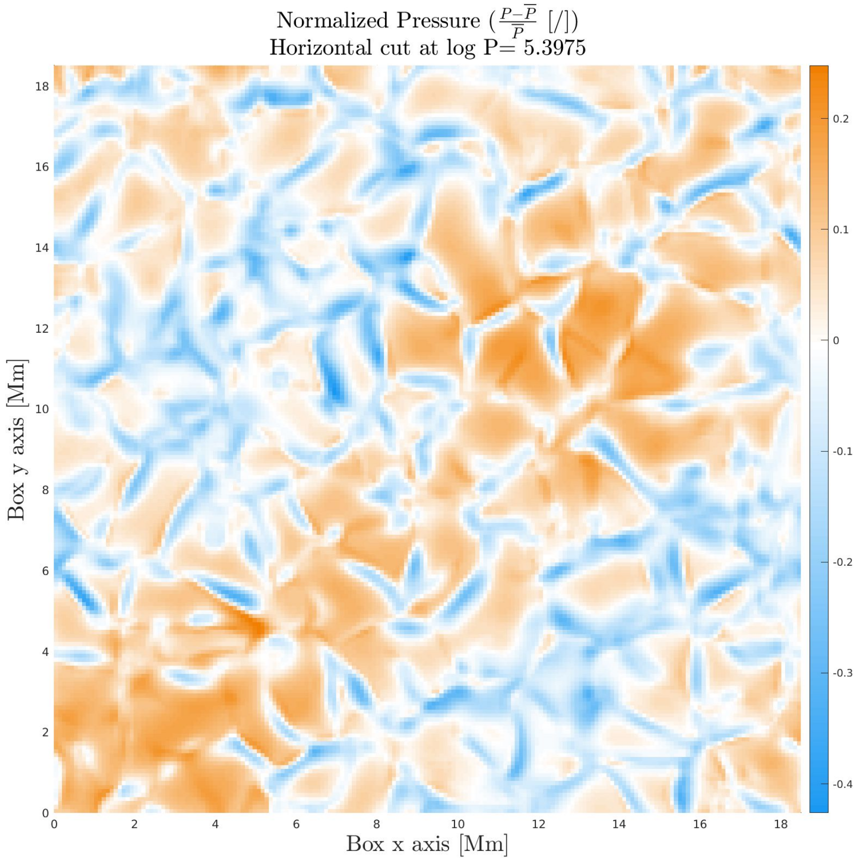
TDC model - Preliminary results

- Unrelaxed solution
  - First iteration only
  - 1D code (MAD) $\Longrightarrow$ x_0' $\Longrightarrow$ x_k'
- Simple boundary conditions ($\forall$ k $\neq$ 0)
  - Stop at the photosphere

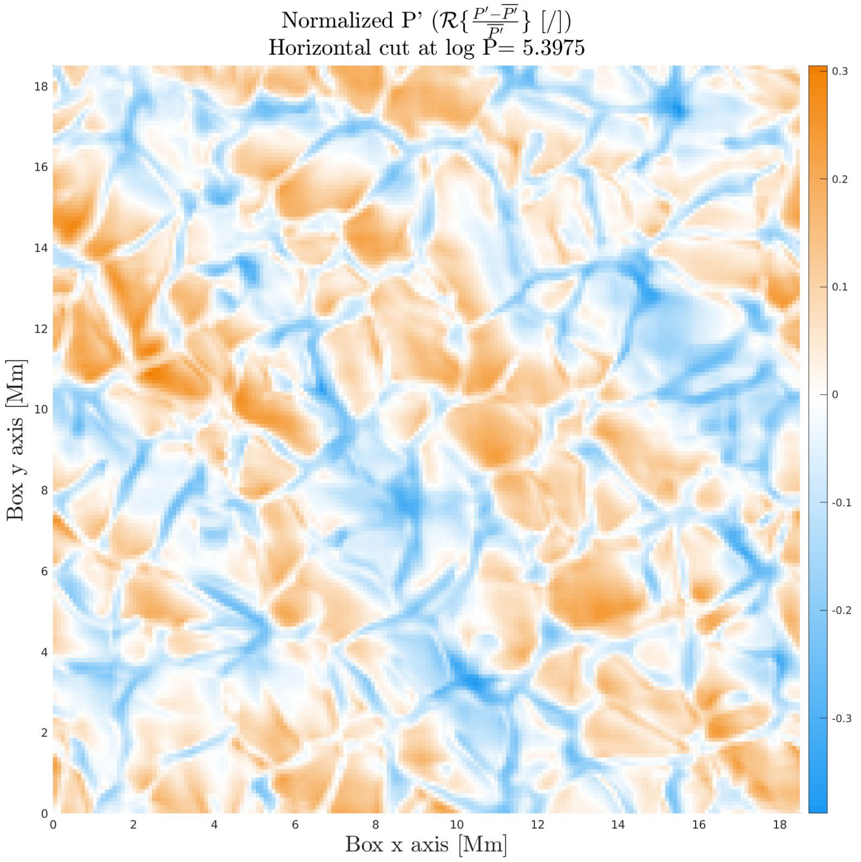
$$P_k' = 0$$
$$\delta(F_z)_k = \sigma_{SB}\,\delta(T^4)_k$$

- Inverse Fourier transform

TDC model - Preliminary results

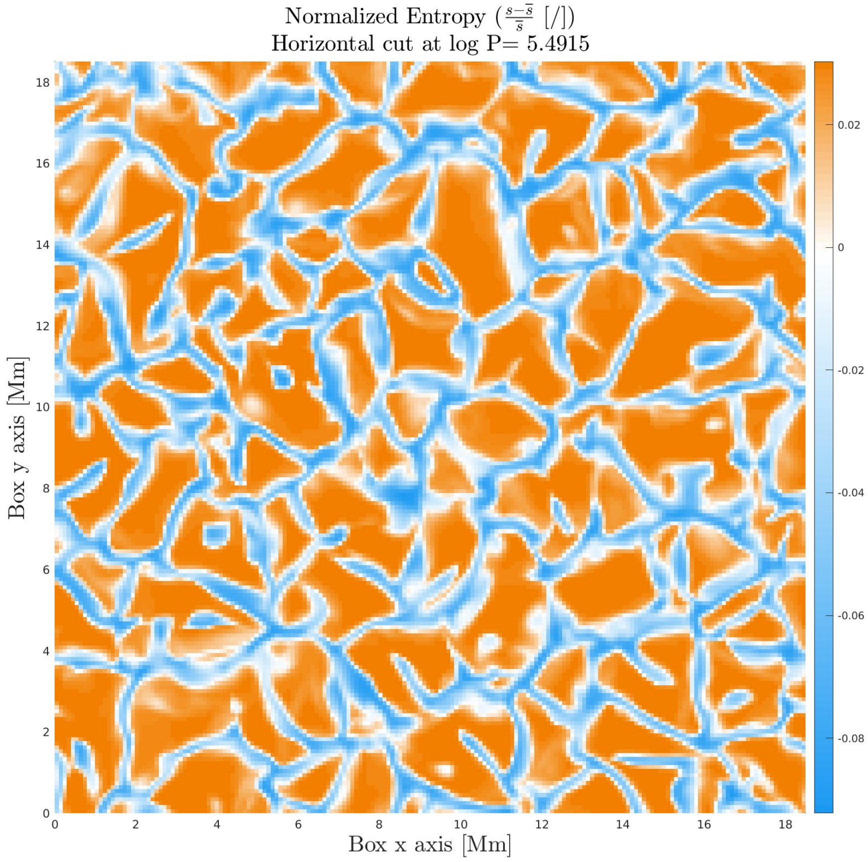


3D simulation

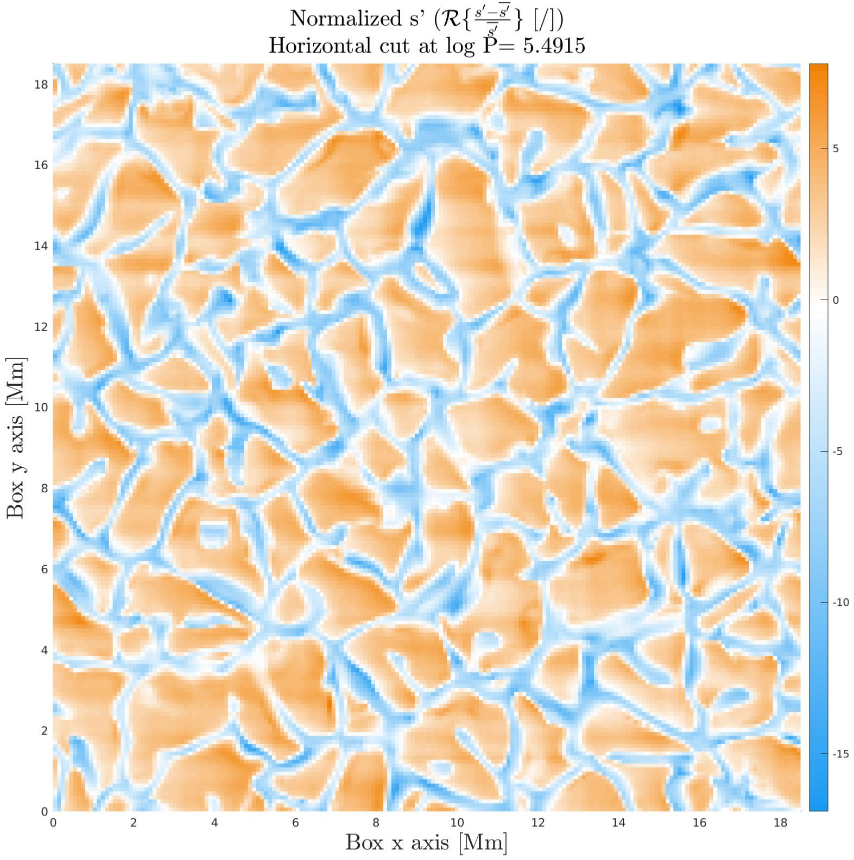


Solution

TDC model - Preliminary results



3D simulation



Solution

TDC model - Preliminary results

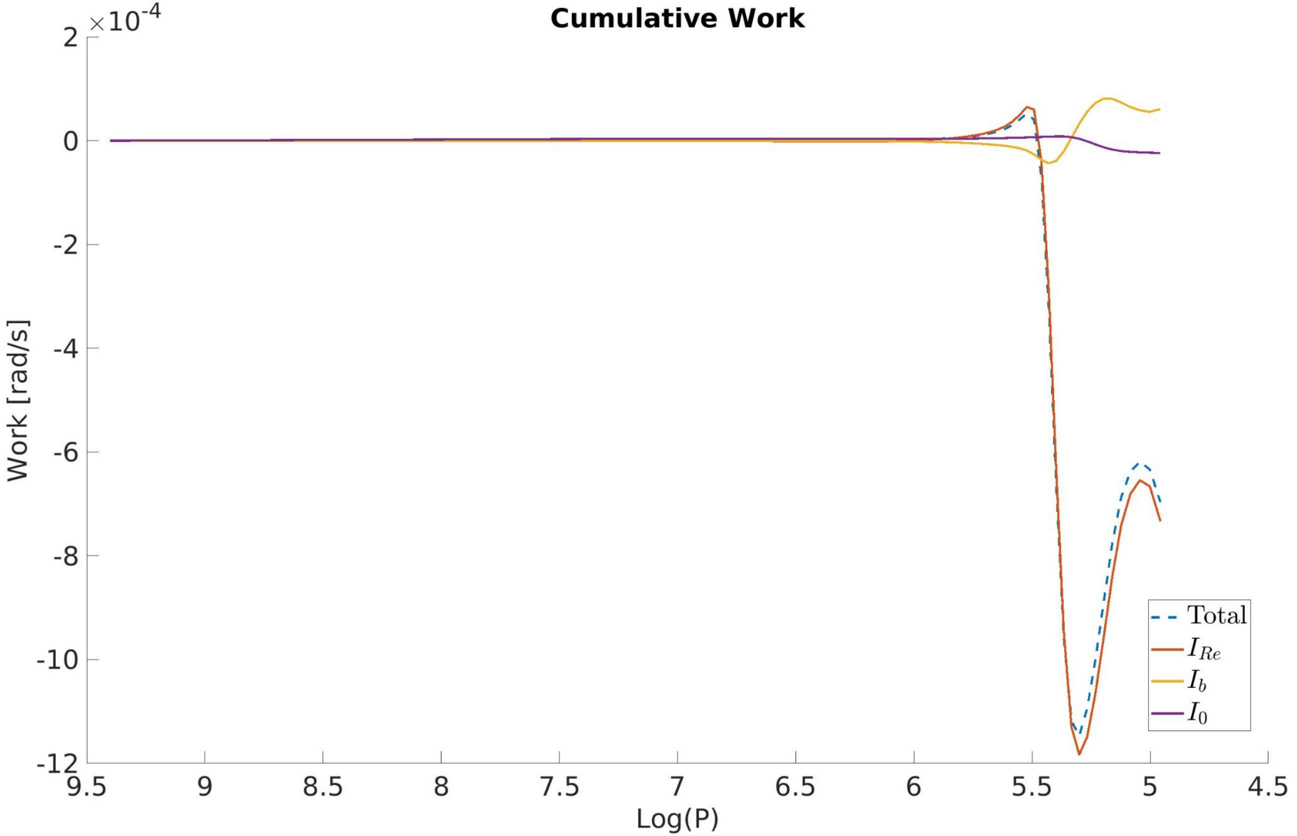
- Work integral

$$\sigma^2 I = I_0(\overline{x'}) + I_{Re}(x' - \overline{x'}) + I_b(x' - \overline{x'})$$

$$\iiint_V R'_{ij}\partial_j v_i'^* \, dV$$

- Extraction of the damping rate
  - Imaginary part of σ^2

TDC model - Preliminary results



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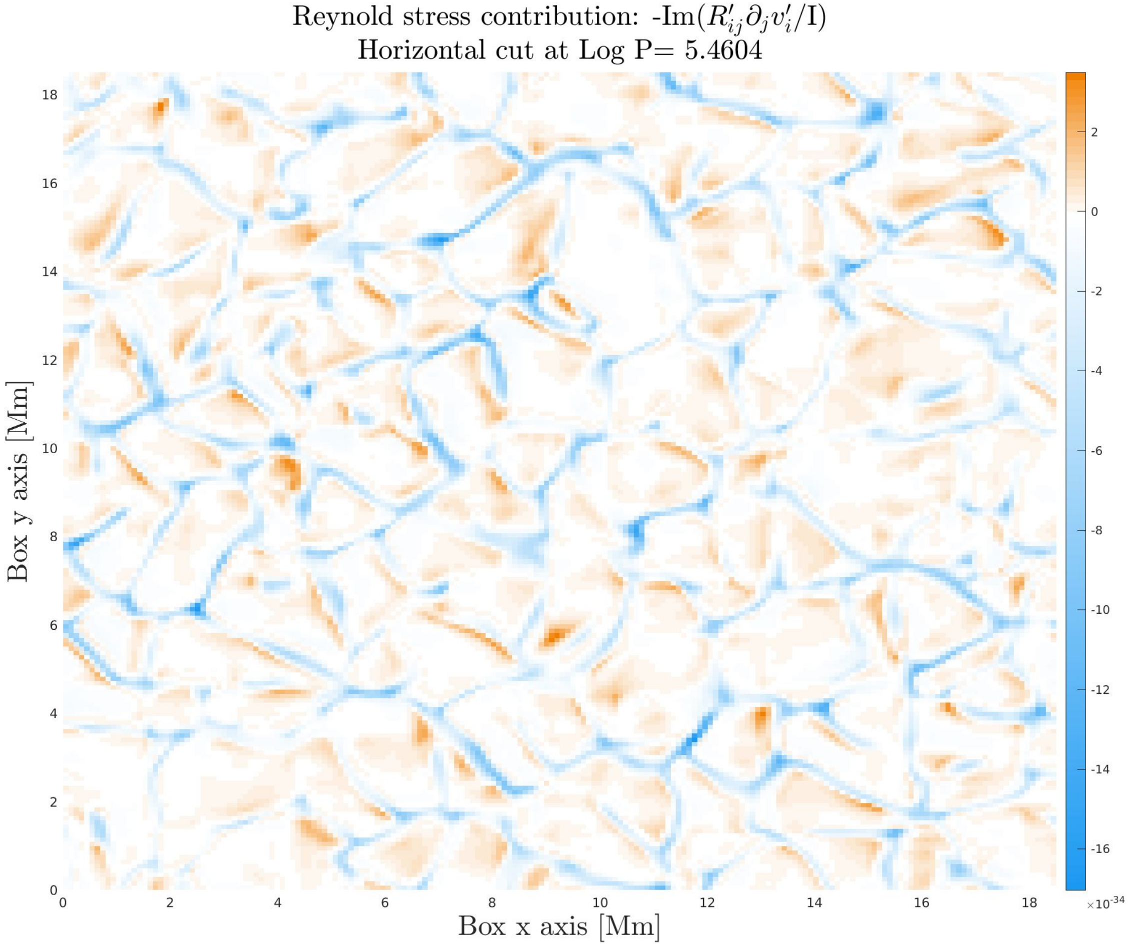

TDC model - Coming soon

- Perspective
  - Improved boundary conditions
  - Include atmosphere
  - High-resolution simulation
- Later on
  - Obtain a relaxed solution
  - Locate region of excitation and damping

TDC model - Coming soon

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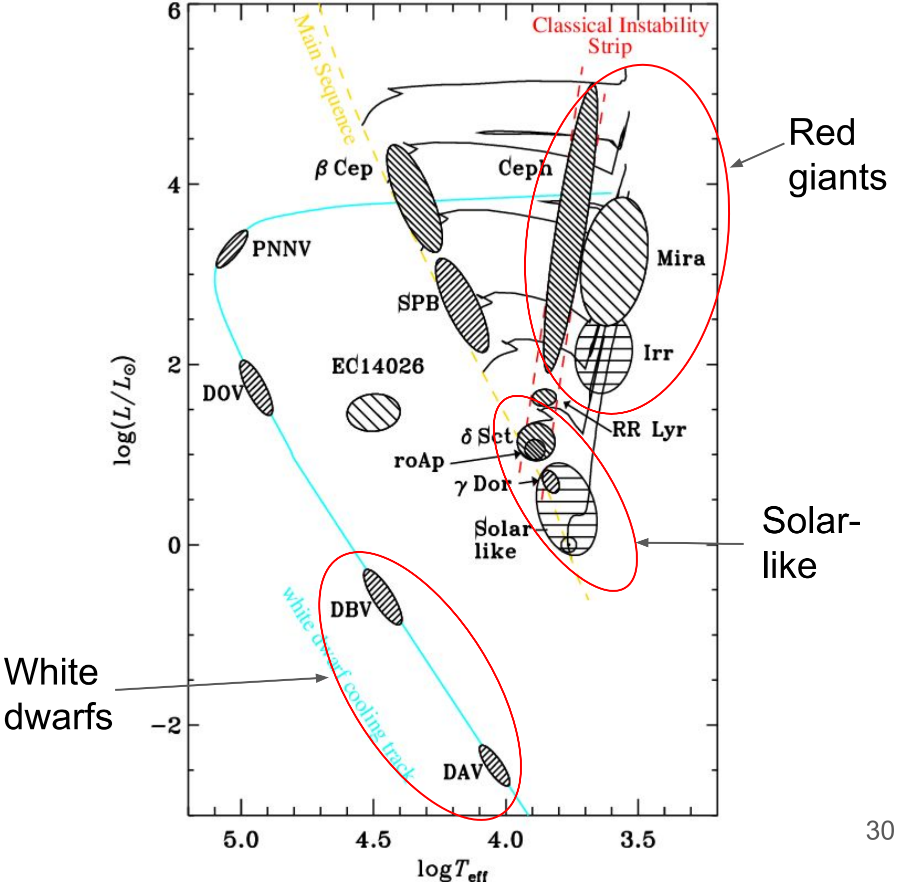
- Later on
  - Obtain a relaxed solution
  - Locate region of excitation and damping
- Other application for the model
  - Solar-like oscillations
  - White Dwarfs
  - γ-Doradus, ...

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