3D static models of close, synchronized binaries in hydrostatic equilibrium with MoBiDICT

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Context

Binary stars

Binary stars: pair of stars gravitationally bounded

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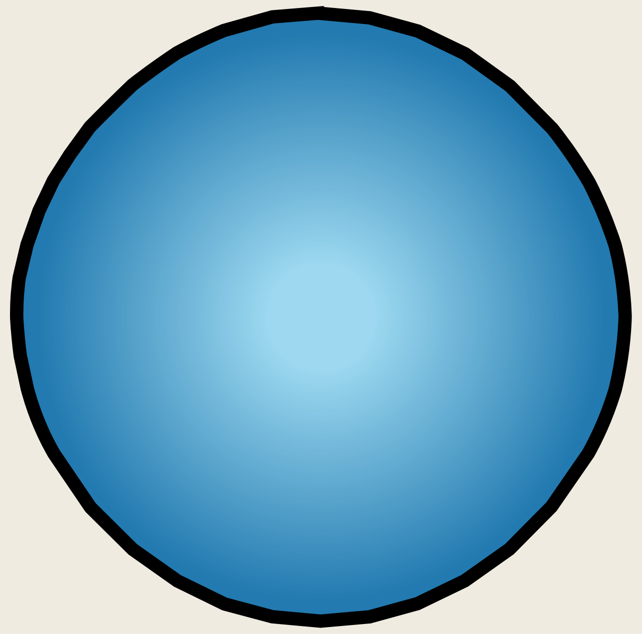

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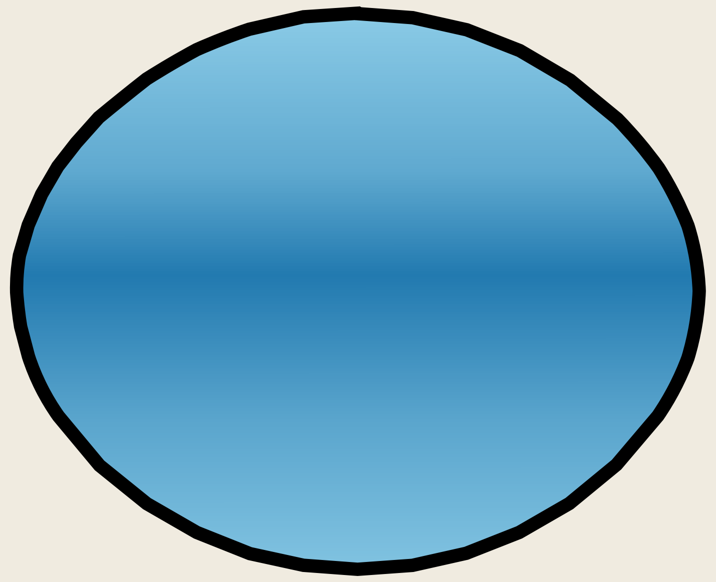

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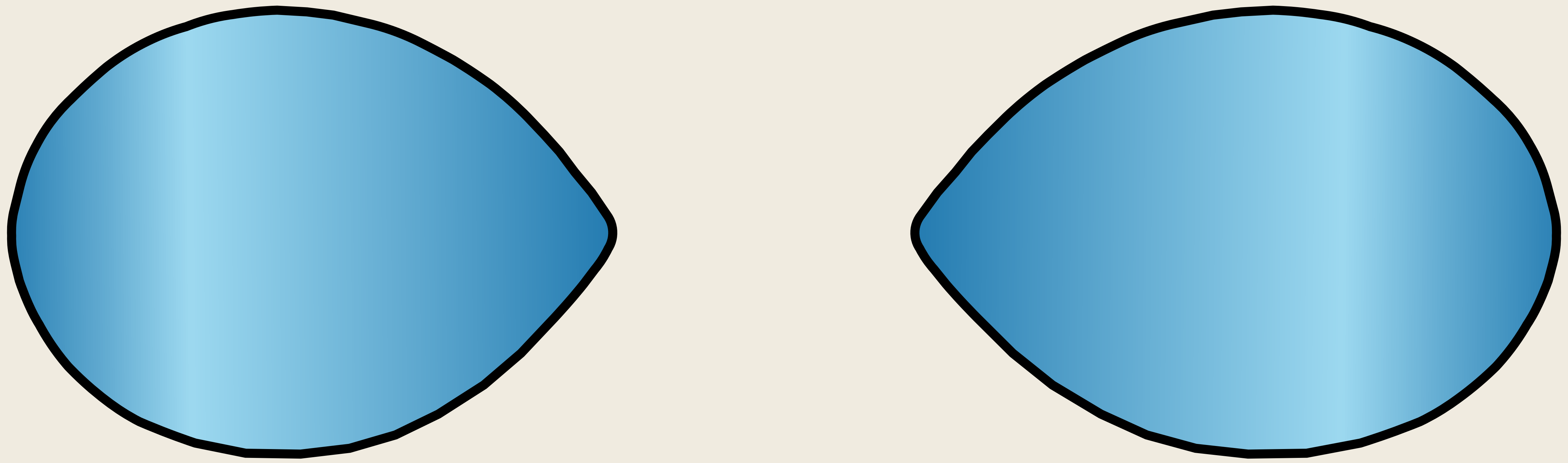

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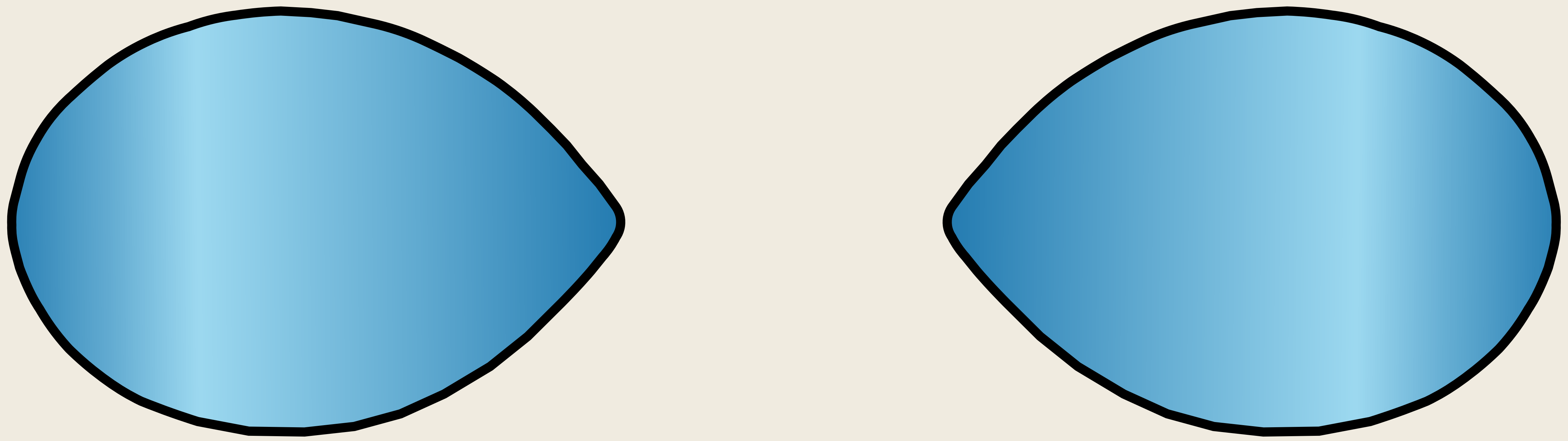
Tidal interaction



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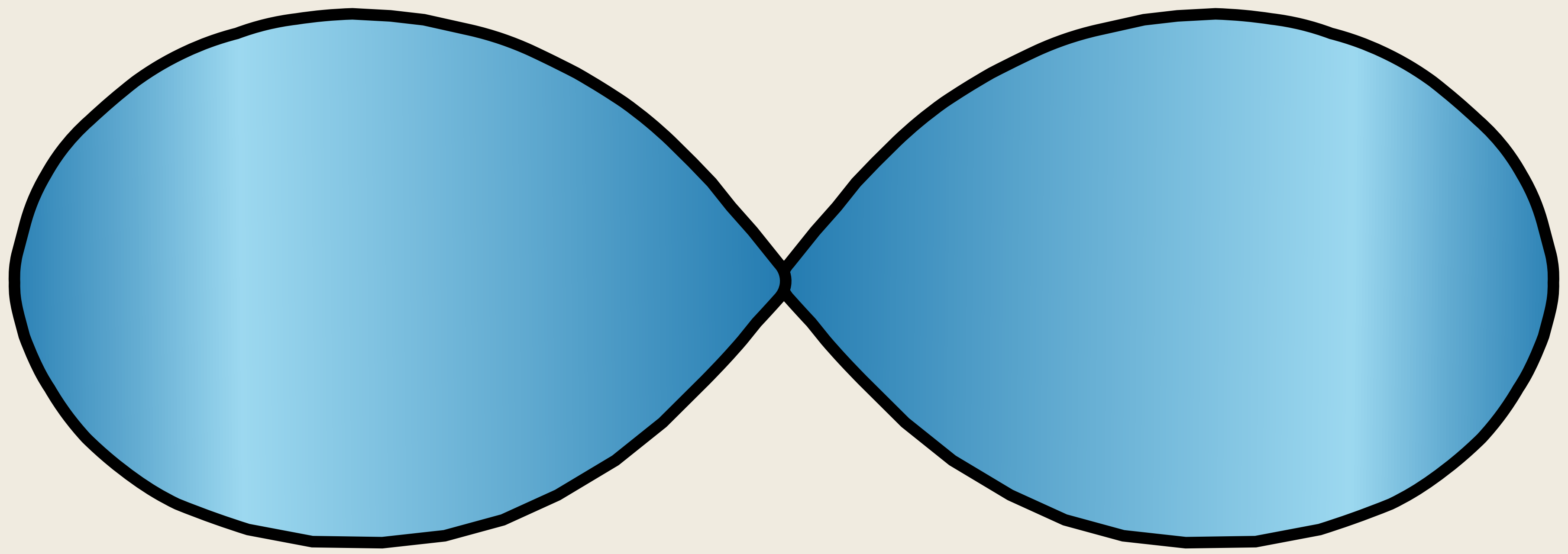
Tidal interaction, common envelope phases or transfert of masse



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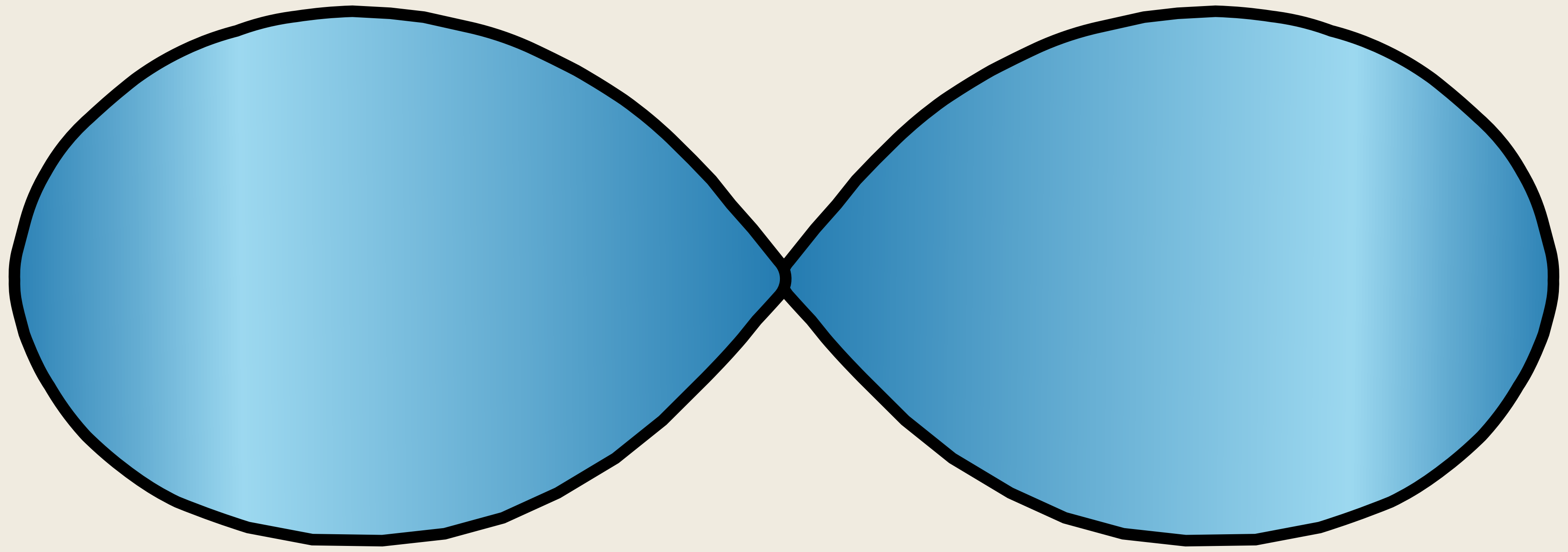

Study of close binaries is tedious due to the breaking of the spherical symmetry of stars.

Presentation of the method

General presentation

Objective: compute the geometrical deformation of binary systems -> computation of 3D static stellar models of stars deformed by tidal and centrifugal forces

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Roche model:

$$\Psi_1(\vec{r}) = -\int_{R_s}^{r} \frac{Gm(r_1)}{r_1^2} dr - \frac{GM_1}{R_s} \quad - \frac{GM_2}{r_2} \quad\quad - \frac{1}{2} s^2 \Omega^2$$

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Our modelling:

$$\Psi_1(\vec{r}) = \Phi_1(r_1, \theta_1, \phi_1) \quad + \Phi_2(r_2, \theta_2, \phi_2) \qquad - \frac{1}{2} s^2 \Omega^2$$

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Gravitational potential, star 1

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Gravitational potential, star 1

Gravitational potential, star 2

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Gravitational potential, star 1

Gravitational potential, star 2

Centrifugal potential

Our modelling:

$$\Psi_1(\vec{r}) = \Phi_1(r_1, \theta_1, \phi_1) \quad + \Phi_2(r_2, \theta_2, \phi_2) \quad -\frac{1}{2}s^2\Omega^2$$

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Gravitational potential, star 1 — Gravitational potential, star 2 — Centrifugal potential

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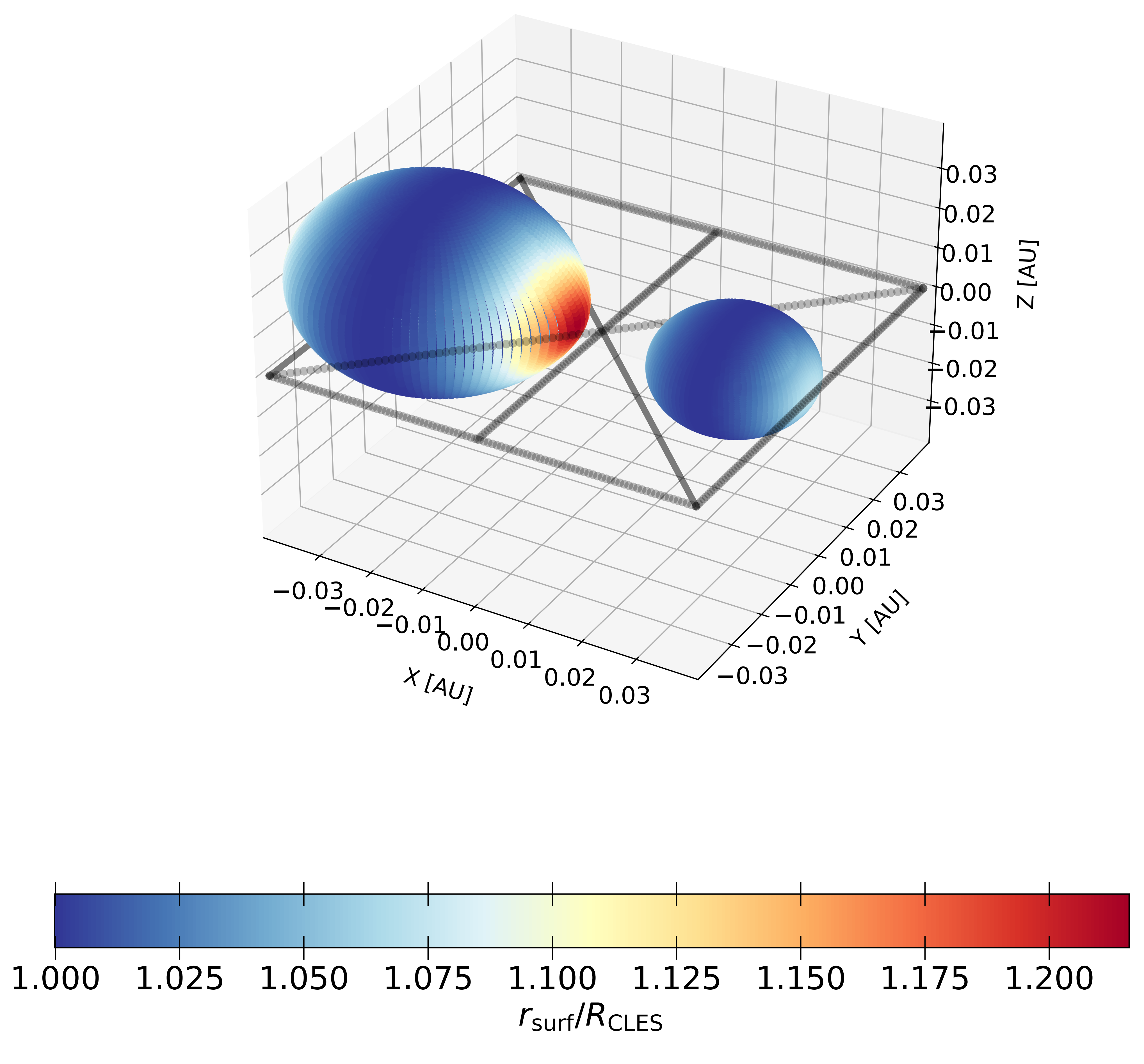
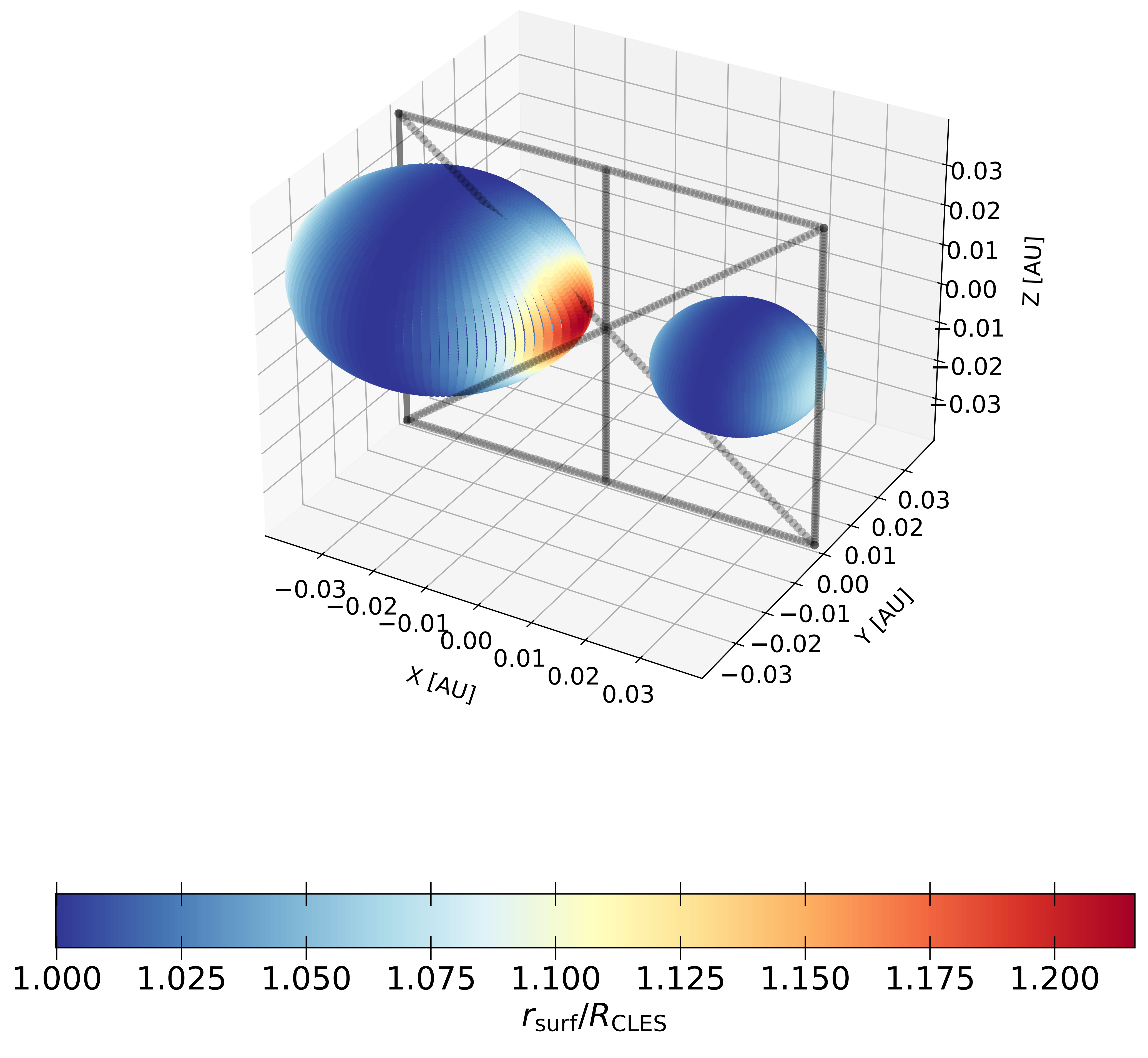
Method: solve Poisson's equation by spectral method until convergence toward the density and potential in 3D (Roxburgh 2004, 2006).

Presentation of the method

Symmetries of the system





Presentation of the method

A conservative problem

In the case of a rigid body rotation all forces are derived from a potential
=> the problem is conservative

=> Pressure and densities are constant on the equipotential

(Can be seen in the equation of hydrostatic equilibrium and its rotational)

=> With a given chemical composition temperature are also constant on the equipotentials

Presentation of the method

Technical details

$\rho_i(r, \mu, \phi) = \rho_{\mathrm{CLES,1D}}(r)$
for each star i

1. as initial parameter we take $\rho_i(r, \mu, \phi) = \rho_{\mathrm{CLES,1D}}(r)$ for each star i composing the system.

Presentation of the method

Technical details

$\rho_i(r,\mu,\phi) = \rho_{\mathrm{CLES,1D}}(r)$
for each star i

$\downarrow$

with $\rho_i(r,\mu,\phi)$ we compute
Ψ_1 and thus Ψ_{tot}

1. as initial parameter we take $\rho_i(r,\mu,\phi) = \rho_{\mathrm{CLES,1D}}(r)$ for each star i composing the system.

2. We project the $\rho_i(r,\mu,\phi)$ on a spherical harmonic basis to have $\rho^m_{i,\ell}$, with the $\rho^m_{i,\ell}(r_i)$ we solve Poisson's equation to obtain $\Psi^m_{i,\ell}(r_i)$ that we project back on the 3D basis to compute the total potential $\Psi_{\mathrm{tot}} = \Psi_1 + \Psi_2 + \Psi_{\mathrm{centri}}$.

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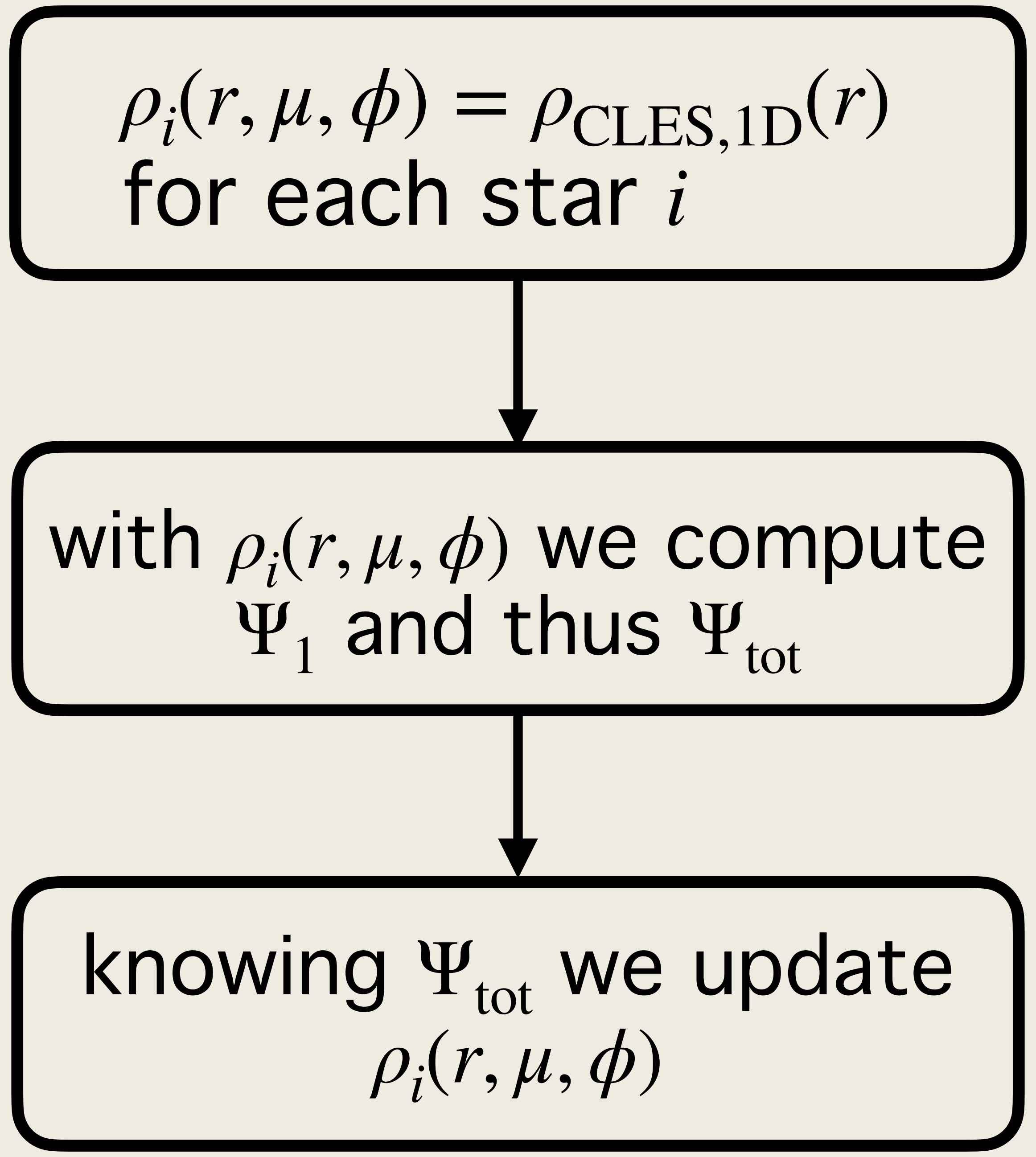

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3. Knowing Ψ_{tot} we interpolate the densities of the entire star assuming that the density of each star along the direction $(\mu_{\mathrm{crit}}, \phi_{\mathrm{crit}})$ is density of the one-dimensional average input models.

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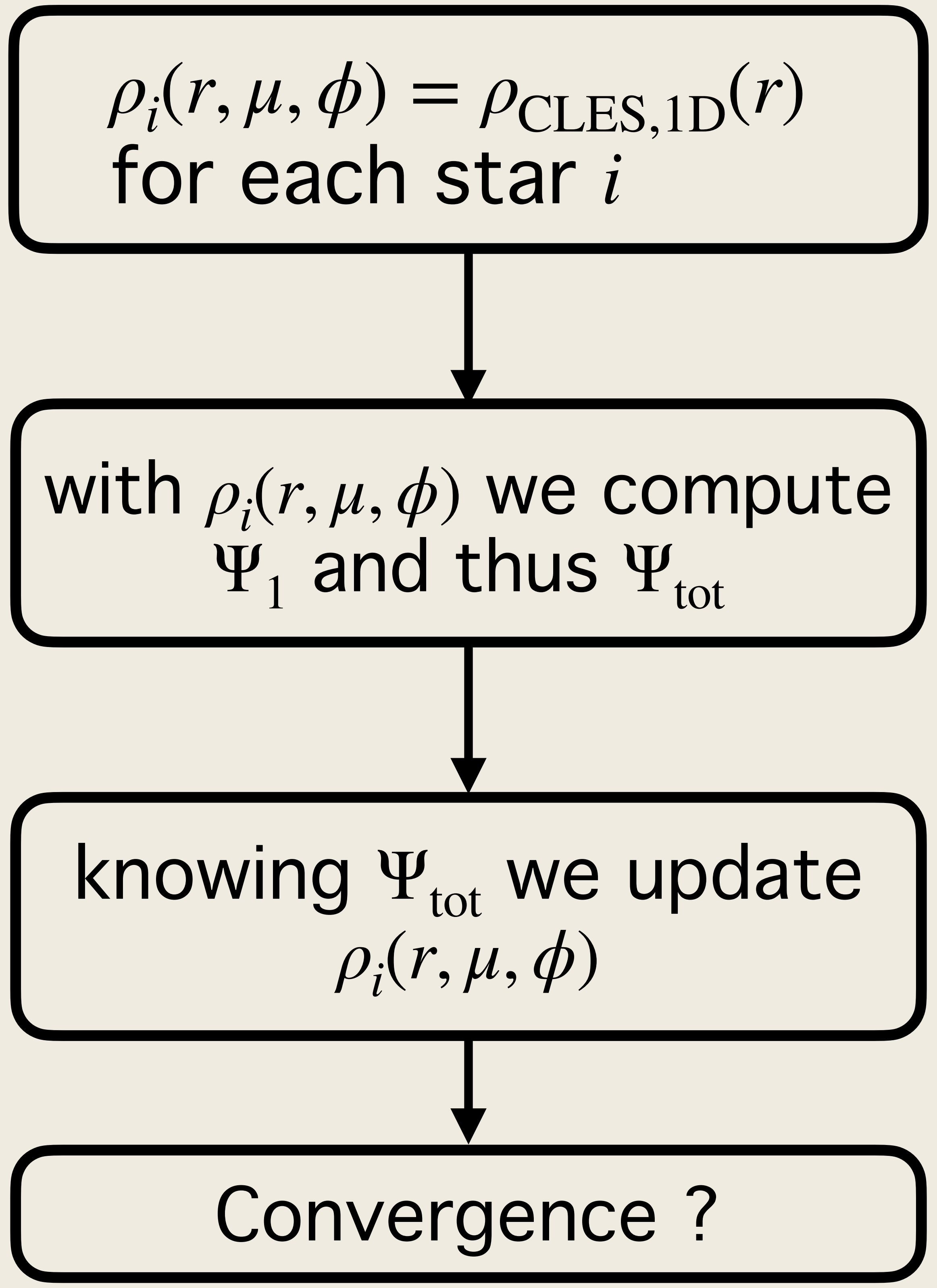

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4. If the model has not converged we go back to step 2.

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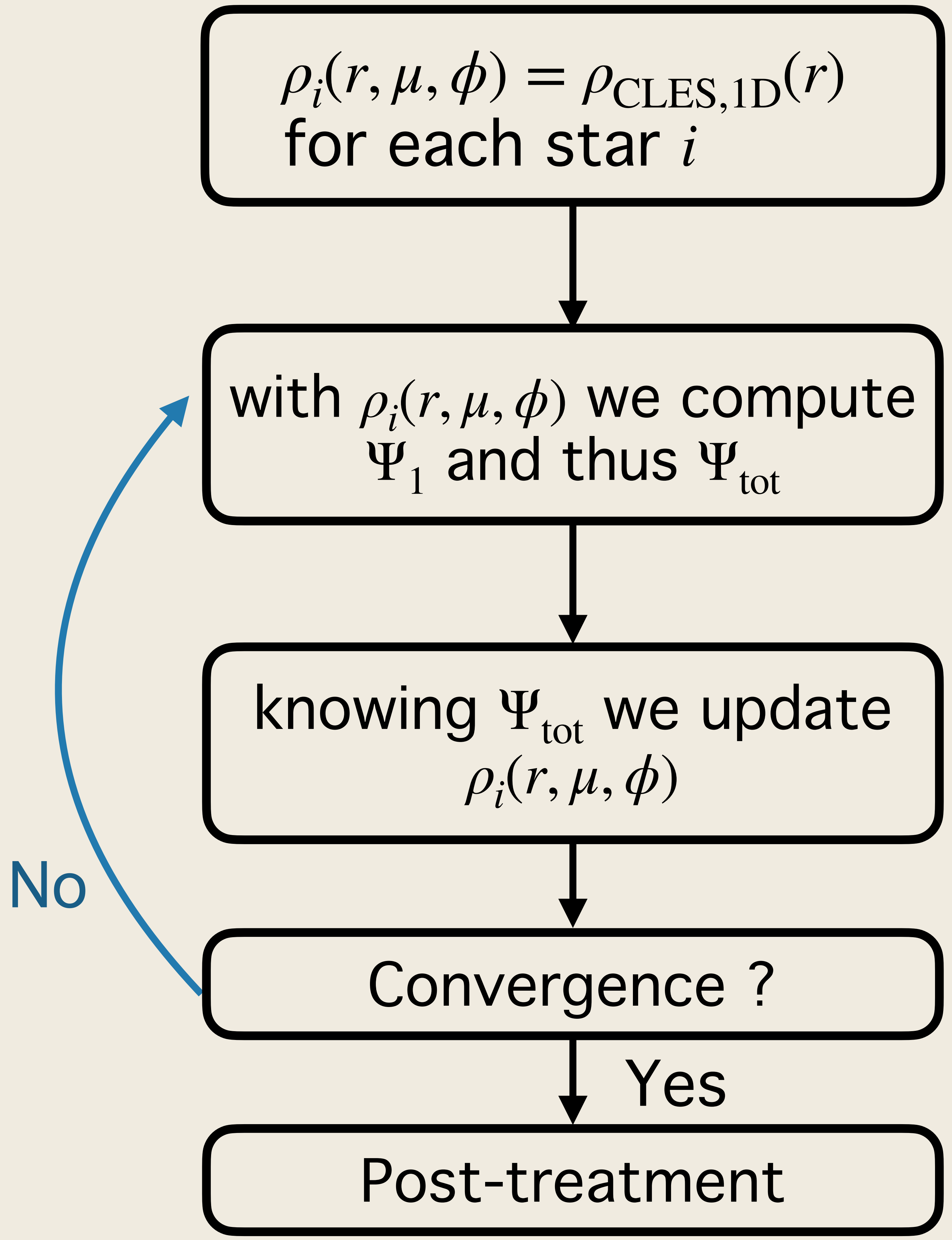

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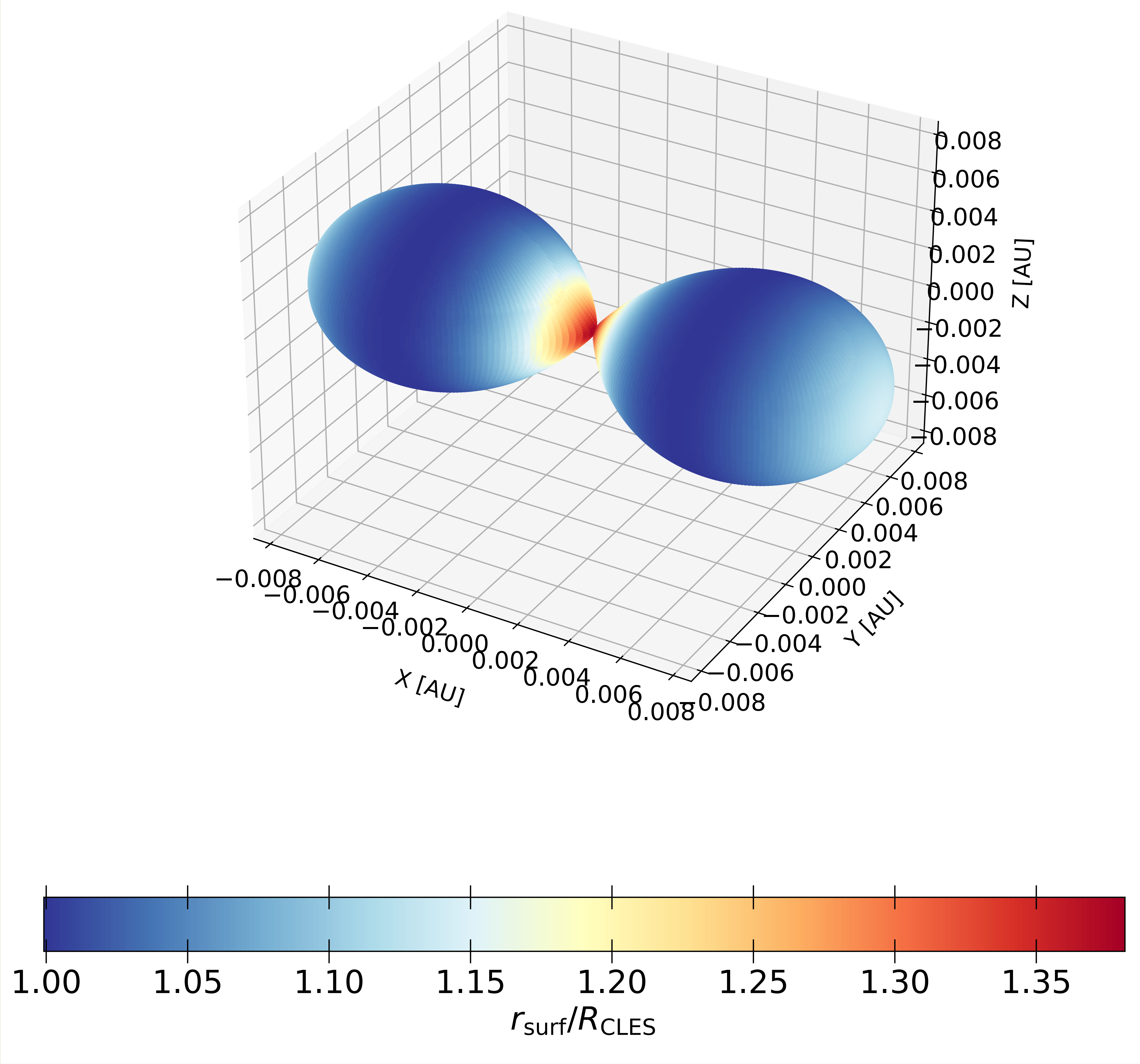
Modelling Results

Binaries of Solar type stars

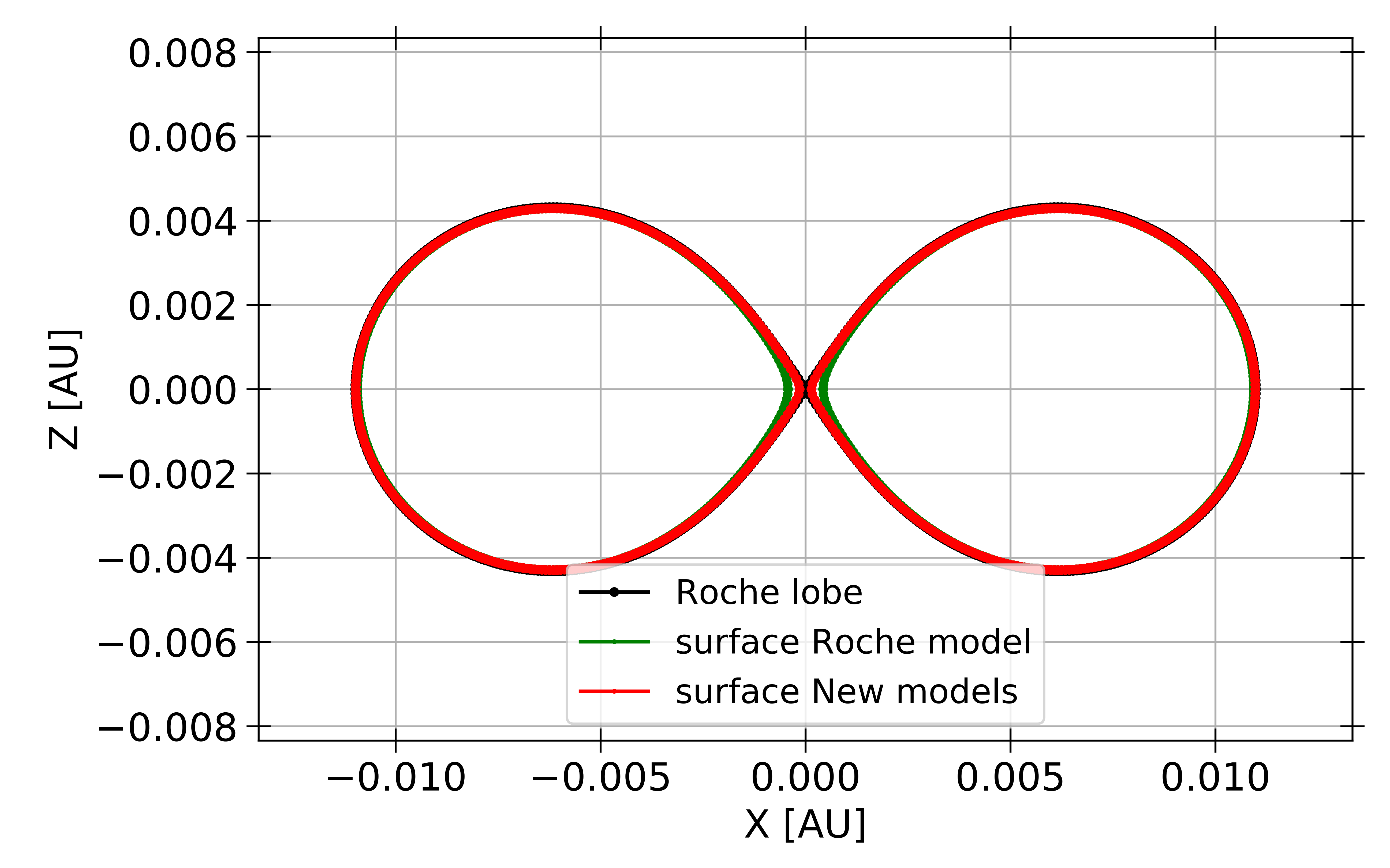


Initial condition of the simulation :

- Separation: 0.01213 AU
- $M_1 = M_2 = 1\ M_{\odot}$ in the MS

Modelling Results

Binaries of Solar type stars - Deformation difference



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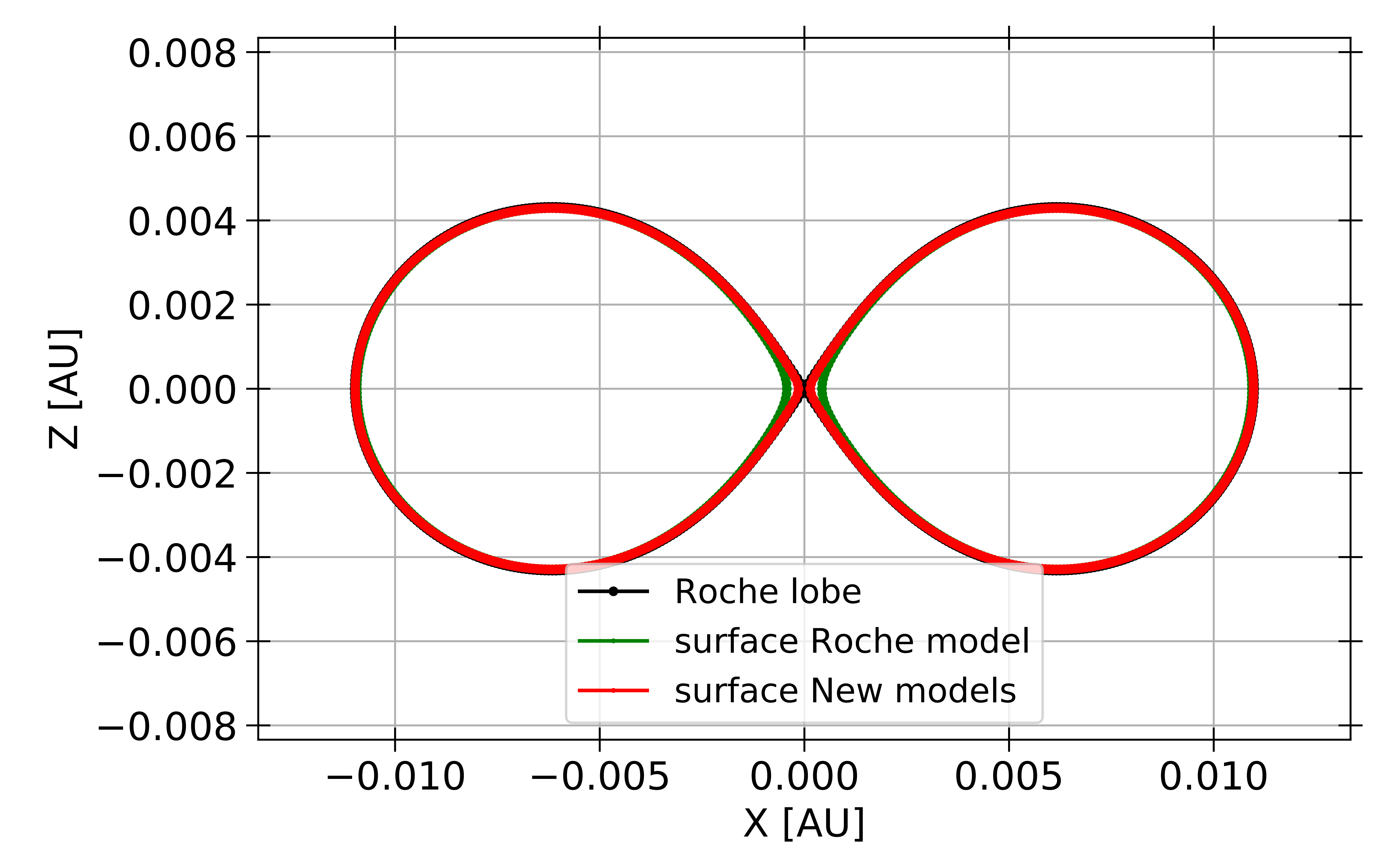

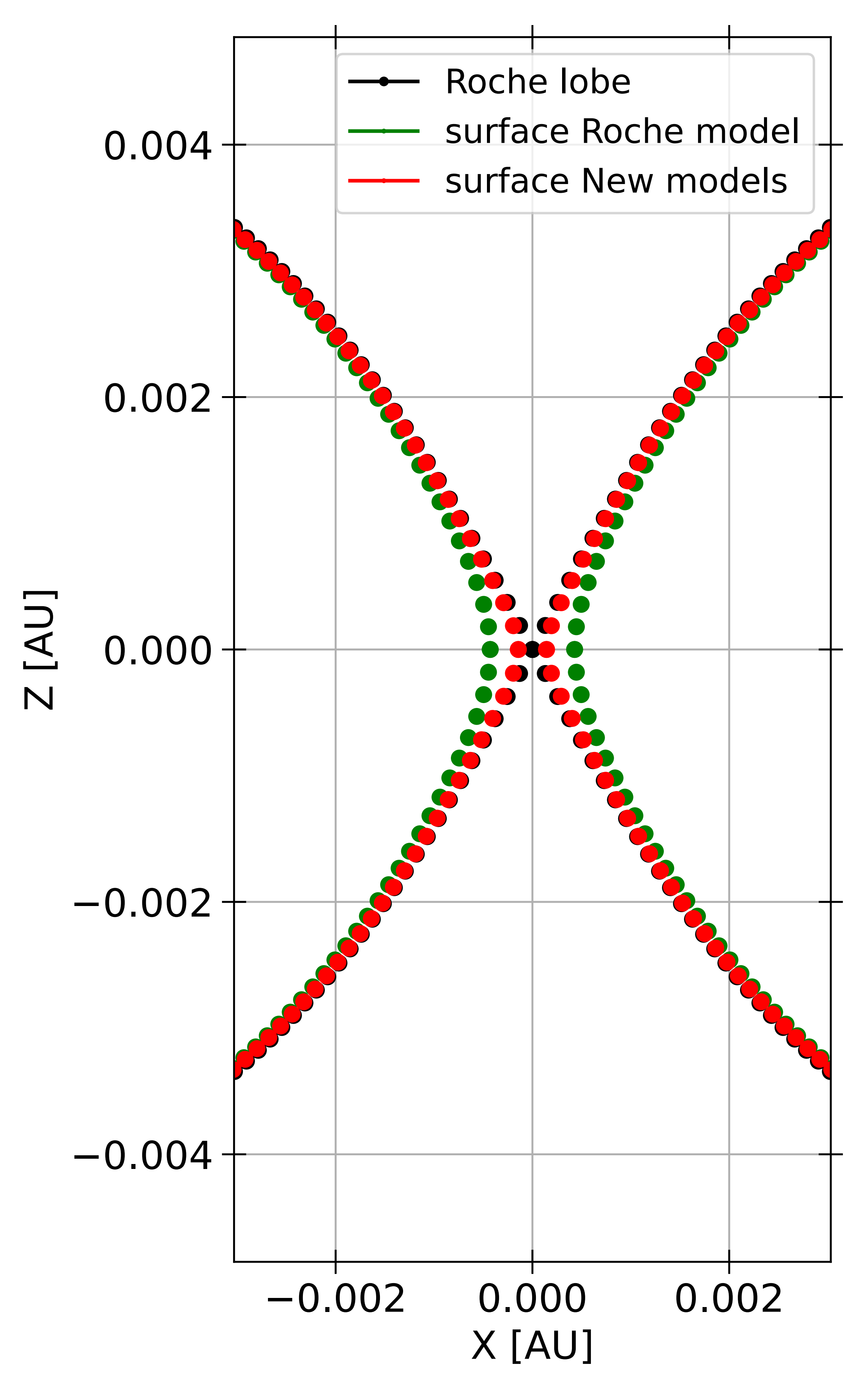

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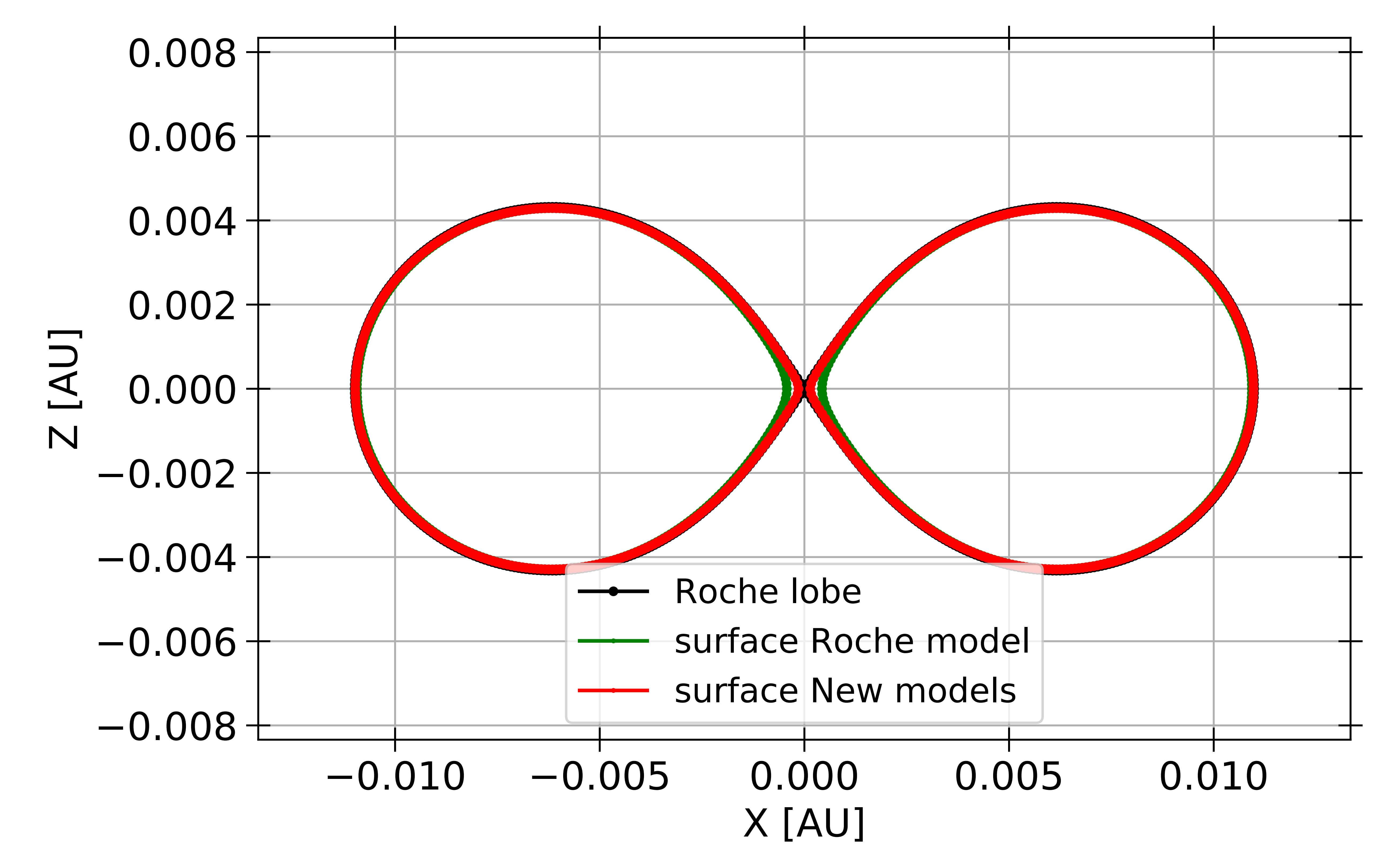
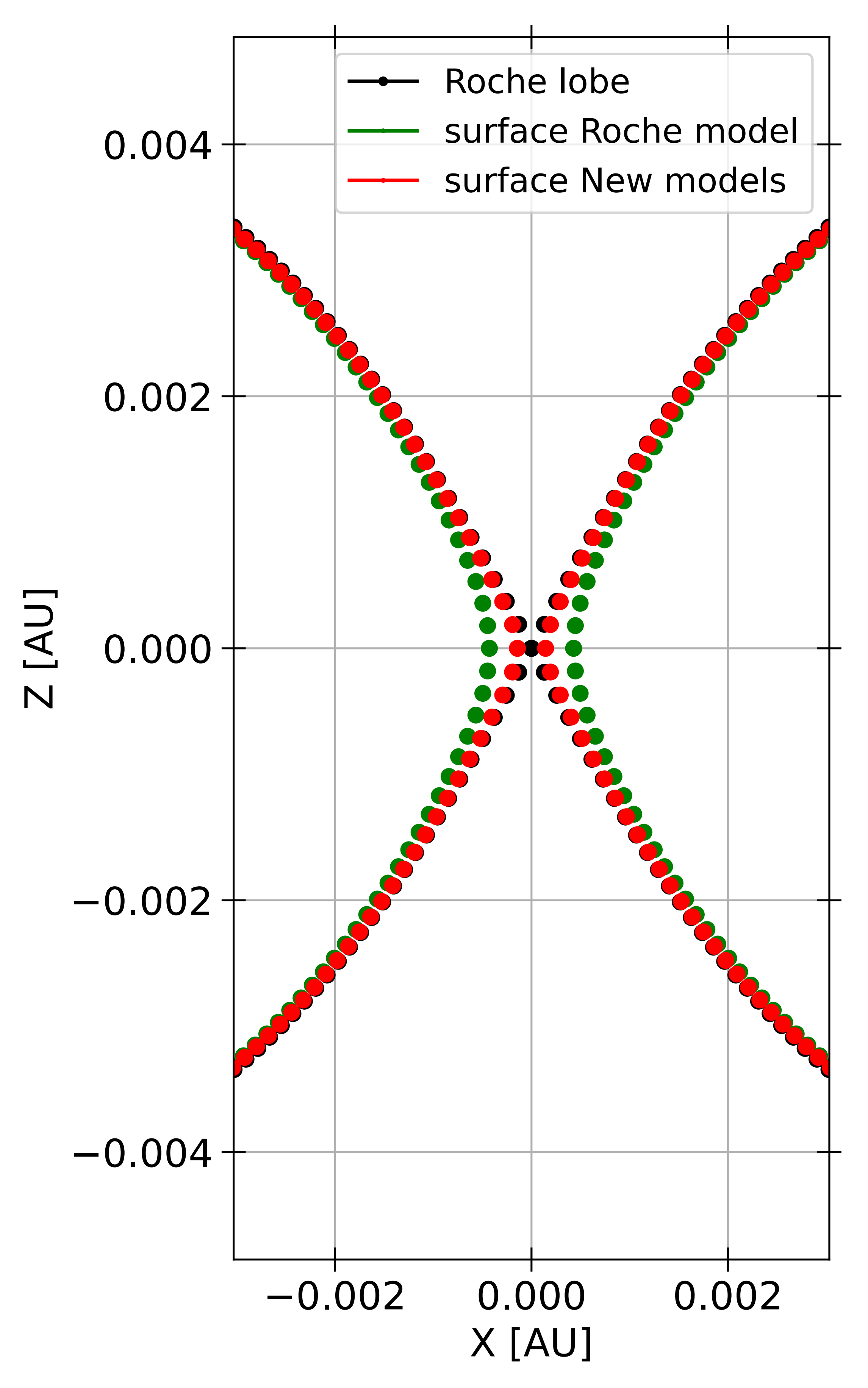
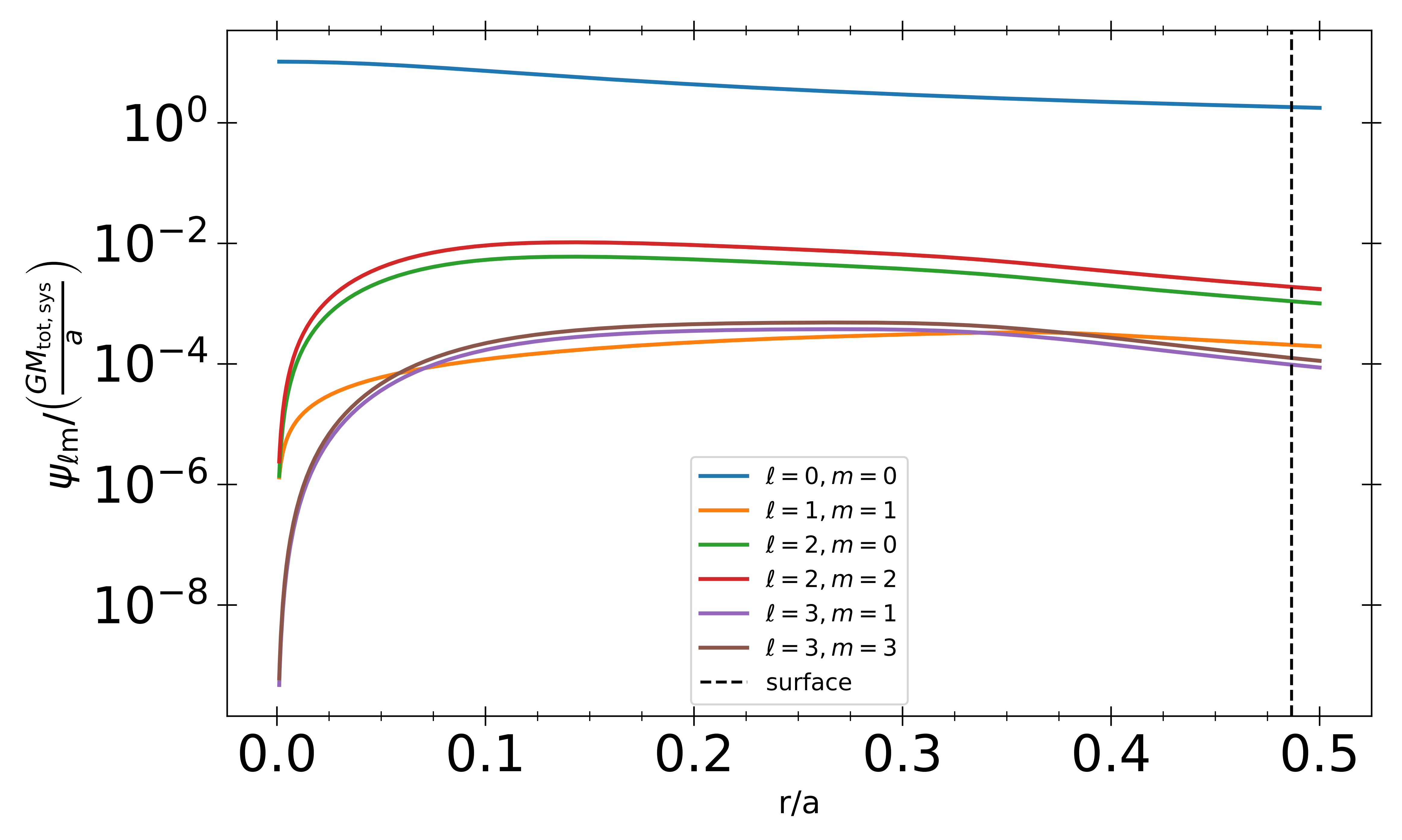


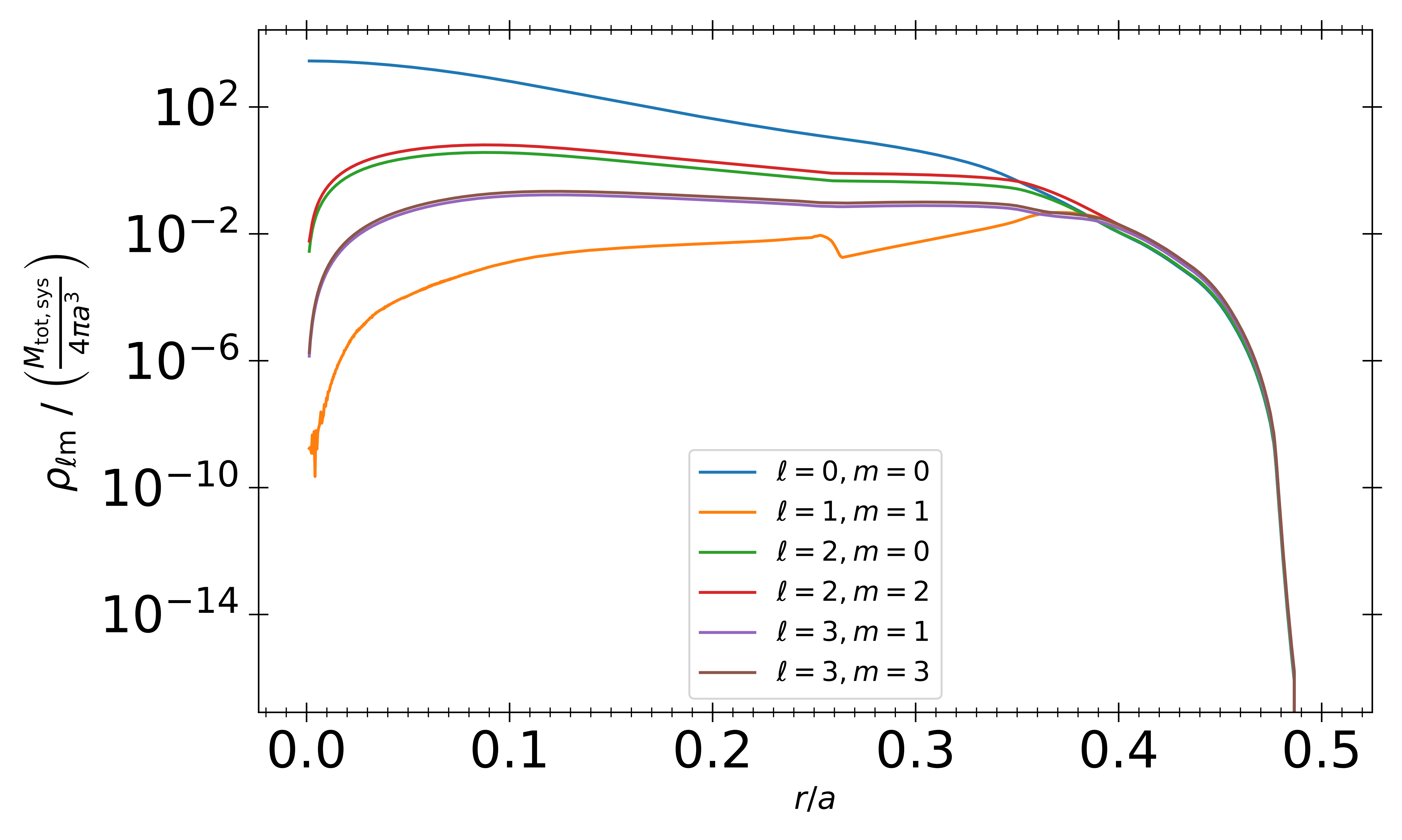


Increase in deformation $\simeq 22\,\%$

Modelling Results

Binaries of Solar type stars - Spectral potential & density





Modelling Results

Dependency on the initial density profile

$$\mathrm{sdB} \sim 0.5\ \mathrm{M}_\odot : \frac{\mathrm{R_{new}} - \mathrm{R_{Roche}}}{\mathrm{R_{Roche}} - \mathrm{R_{CLES}}} = 0.04$$

$$10\ M_\odot : \frac{R_{\mathrm{new}} - R_{\mathrm{Roche}}}{R_{\mathrm{Roche}} - R_{\mathrm{CLES}}} = 0.08$$

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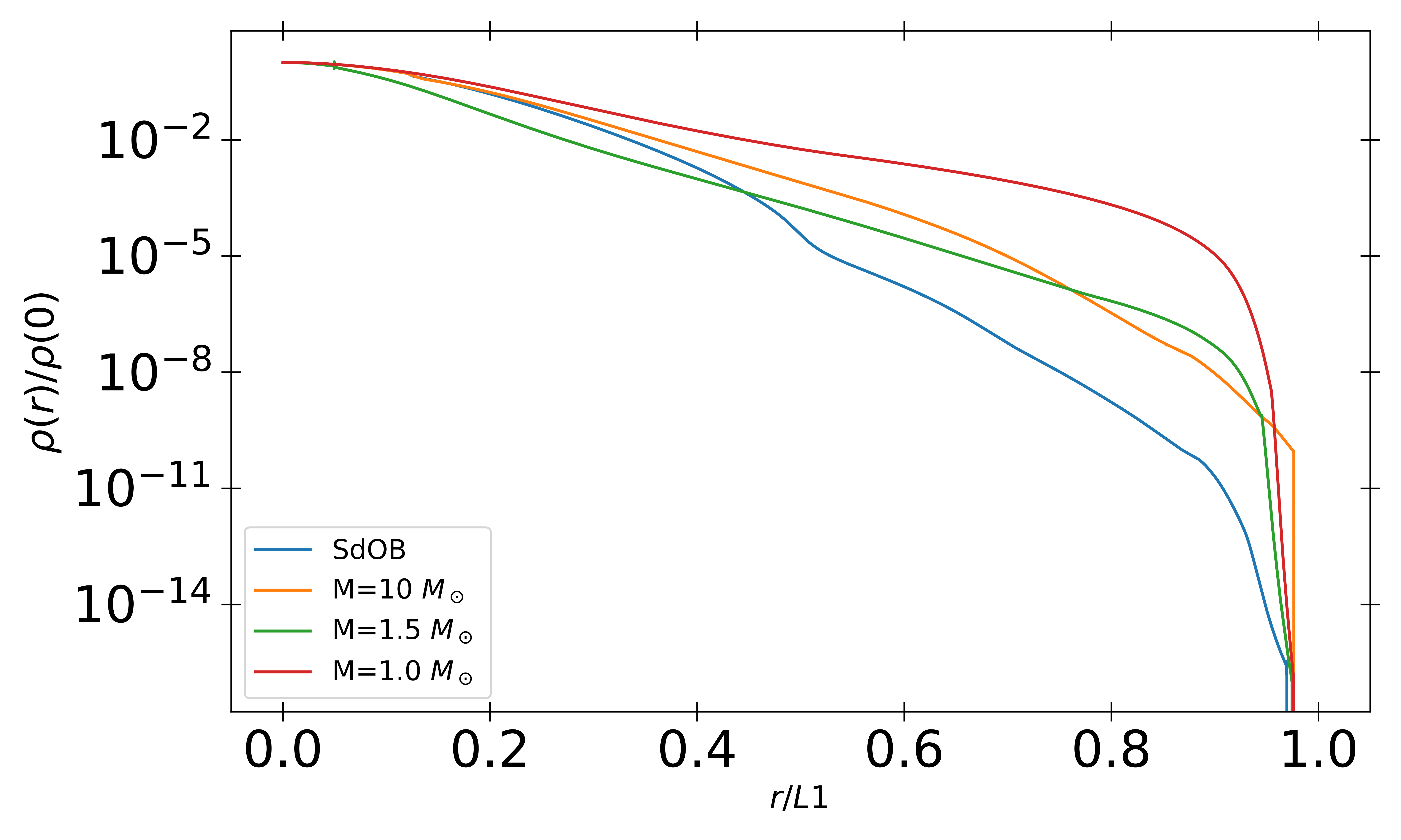


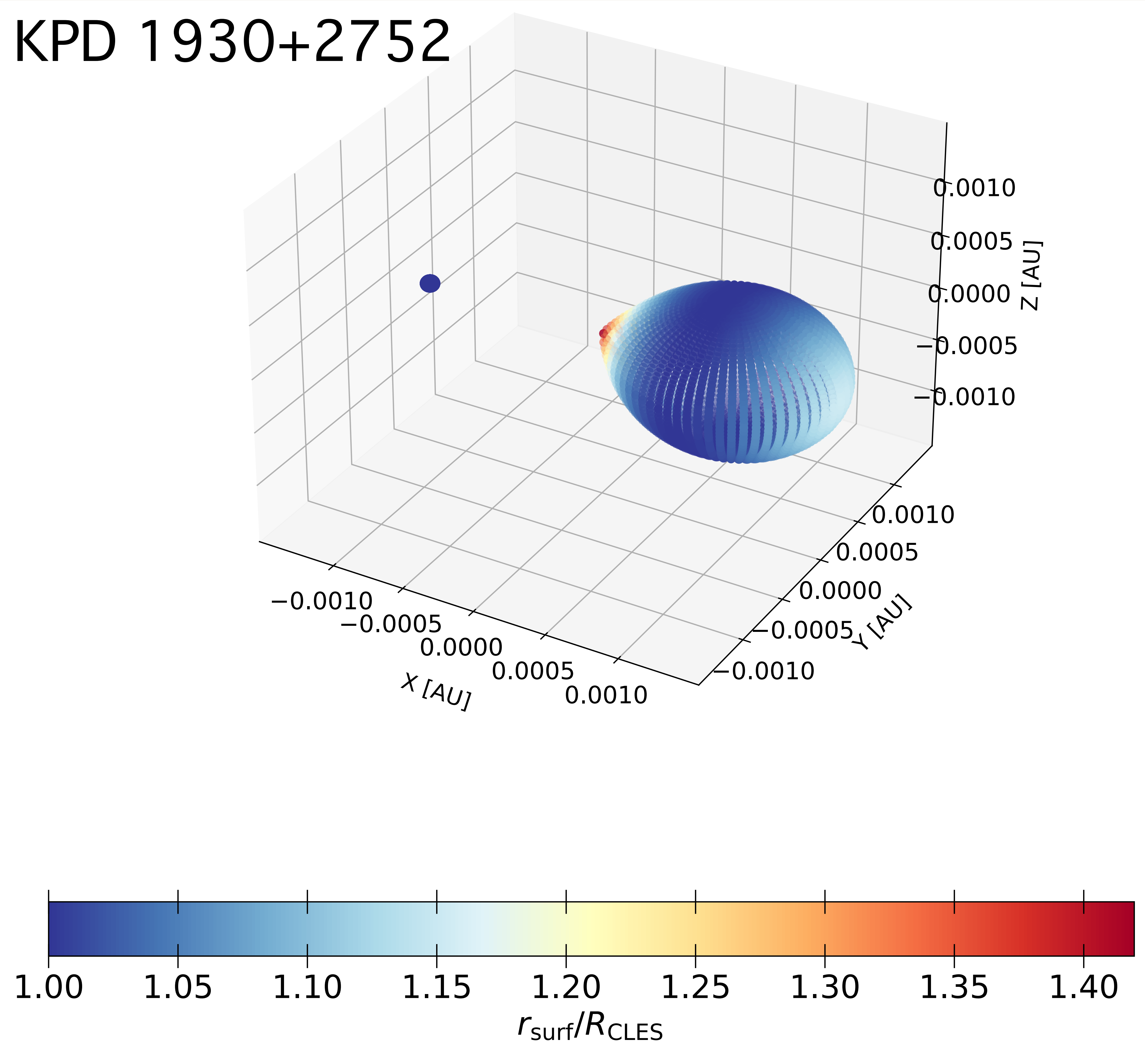
Perspectives

De-synchronization of the system

Two main improvements:

- Study of non synchronized systems with rigid body rotation
- Implementation of non aligned rotation axis

=> breaking of all the symmetries of the system



Perspectives

Coupling with stellar evolution codes

Couple the stellar evolution of Liege to MobiDICT using the method of Kippenhahn et al., 1970 :

$$\frac{dP}{dm_\mathrm{p}} = -f_\mathrm{p}\frac{Gm_\mathrm{p}}{4\pi r_\mathrm{p}^4}, \quad f_\mathrm{p} = \frac{4\pi r_\mathrm{p}^4}{Gm_\mathrm{p}}\frac{1}{\int_\Psi g_\mathrm{eff}^{-1} d\sigma}$$

First step: at the end of each evolution track, compute f_P and f_T and compute again an evolution track

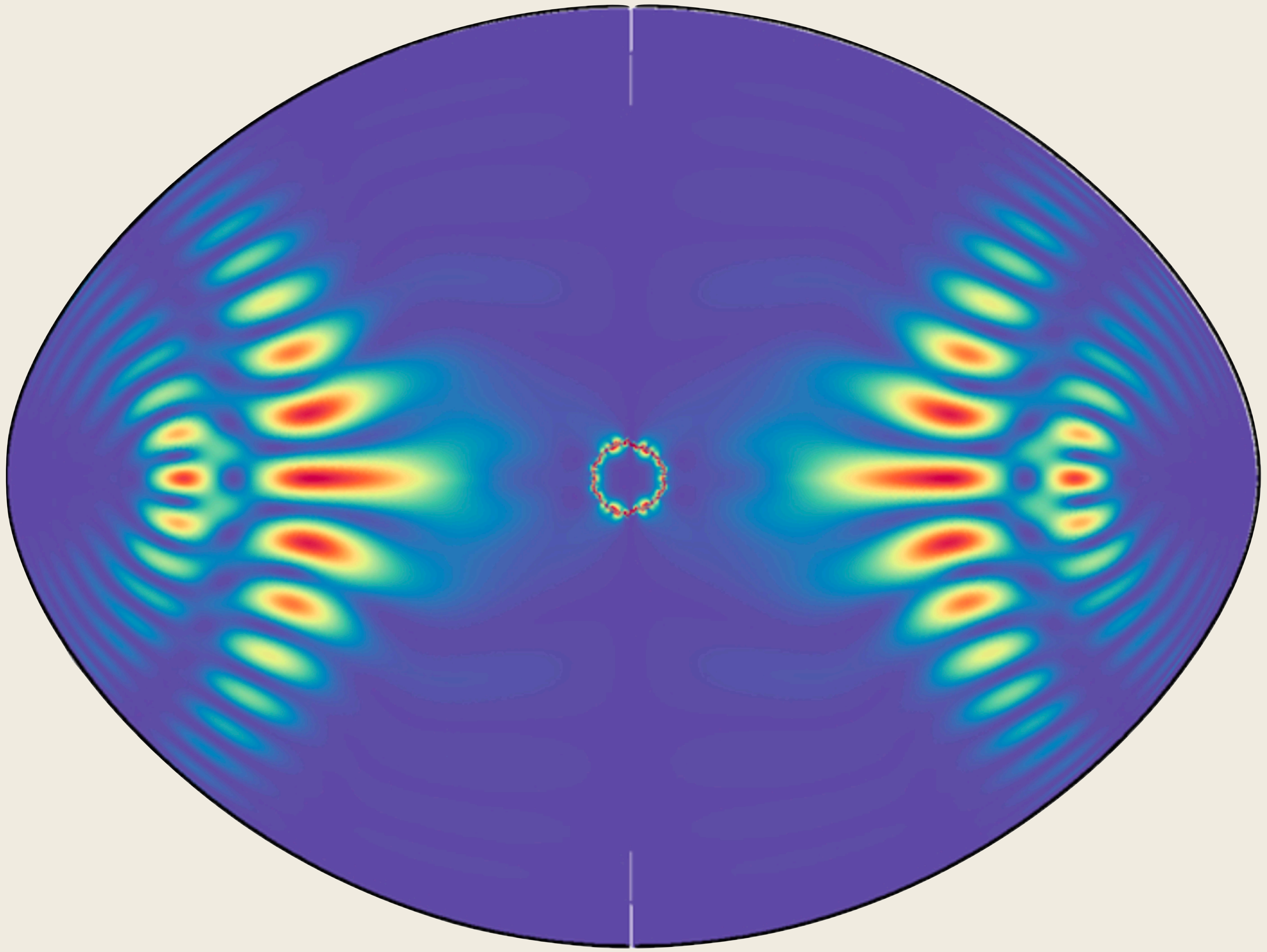
$$\frac{dT}{dm_\mathrm{p}} = -f_\mathrm{T}\frac{3\kappa L_\mathrm{p}}{64\pi^2 r_\mathrm{p}^4 acT^3}, \quad f_\mathrm{T} = \frac{64\pi^2 r_\mathrm{p}^4}{\int_\Psi g_\mathrm{eff}^{-1} d\sigma \int_\Psi g_\mathrm{eff} d\sigma}$$

Perspectives

Stellar 3D oscillation code

Objectives:

- Development of a non-perturbative 3D oscillation code for our new stellar models.
- A new method to identify and classify the oscillation modes in 3D.

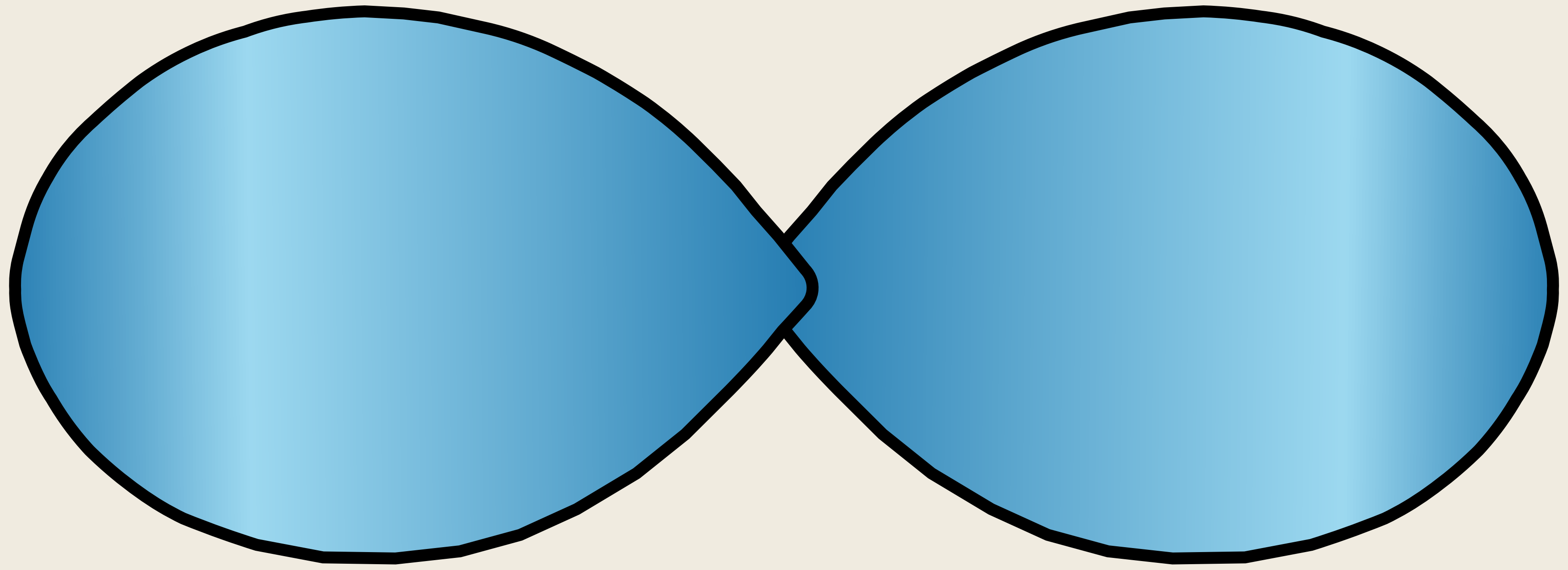


Ouazzani et al., 2015

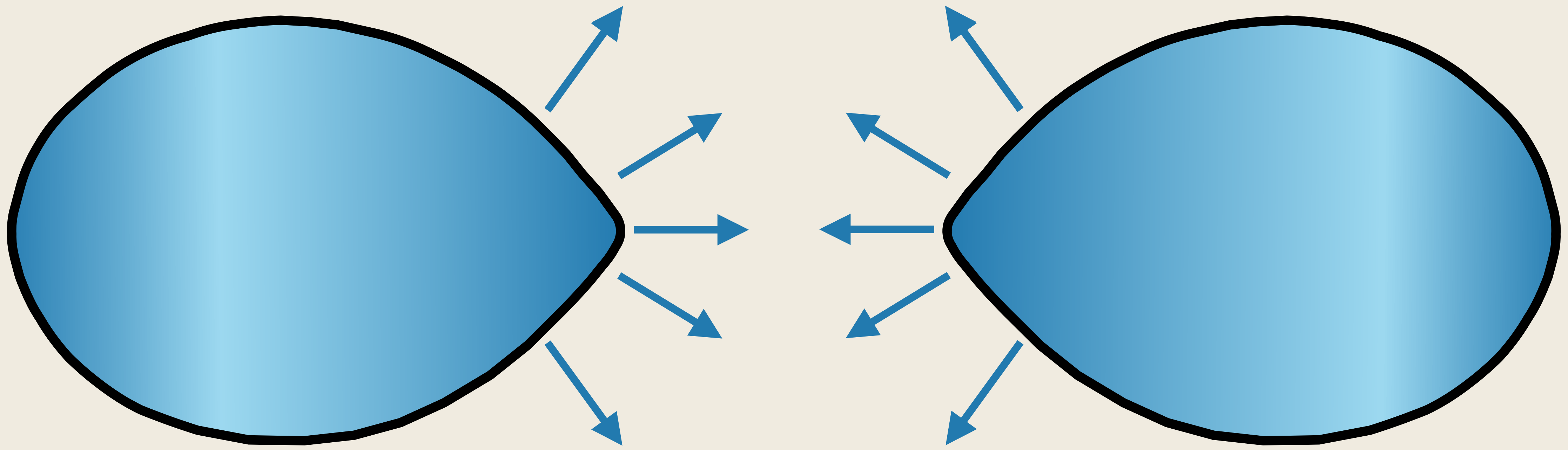
Perspectives

Others

Proper modelling of the phases of common enveloppe



Inclusion of the effects produced by the radiation pressure using an adapted grid of atmospheric models.



Summary

We developed a new method, **MoBiDICT**, to compute the deformations of close, synchronized binaries in 3D.

With **MoBiDICT**, we found a difference in deformation up to 22% compared to the Roche model for solar type stars in the MS.

The differences in deformation with respect to the Roche model are highly dependent on the density profile of the stars studied

In the future we are going to implement the desynchronization of the systems, the coupling of **MoBiDICT** to stellar evolution codes and develop a 3D oscillation code associated with our 3D stellar models.

Thank you for your attention !

Presentation of the method

Technical details

1. as initial parameter we take $\rho_i(r,\mu,\phi) = \rho_i(r,\mu_{\text{crit}},\phi_{\text{crit}}) = \rho_{\text{CLES,1D}}(r)$ for each star i composing the system.

2. solve Poisson's equation knowing $\rho_i(r,\mu,\phi)$ and we compute $\Psi_{\text{tot}} = \Psi_1 + \Psi_2 + \Psi_{\text{centri}}$ for the grid of each star.

$$\frac{1}{r_i^2}\frac{d}{dr_i}\left(r_i^2\frac{d\Psi^m_{i,\ell}}{dr_i}\right) - \frac{\ell(\ell+1)}{r_i^2}\Psi^m_{i,\ell} = 4\pi G\rho^m_{i,\ell}$$

Adimensioning of the procedure :

$$x_i = \frac{r_i}{a}; \quad \Lambda^m_{i,\ell} = \rho^m_{i,\ell}\left(\frac{M_{\text{tot,sys}}}{4\pi a^3}\right)^{-1}; \quad \Upsilon^m_{i,\ell} = \Psi^m_{i,\ell}\left(\frac{GM_{\text{tot,sys}}}{a}\right)^{-1}; \quad \Omega^2 = \omega^2\left(\frac{GM_{\text{tot,sys}}}{4\pi^2 a^3}\right)^{-1}$$

Presentation of the method

Technical details

3. assuming that the density of each star along the direction $(\mu_{\mathrm{crit}}, \phi_{\mathrm{crit}})$ is density of the one-dimensional average input models, we can interpolate the density one each grid point taking advantage that the density should be constant on a given equipotential.

4. estimation of the differences $\delta\rho_i(r, \mu, \phi)$ and $\delta\Psi_i(r, \mu, \phi)$ that we are using as convergence indicator.

5. we start back to step 2 if the model has not converged.

Annexe

Méthode : Pression de radiation

- Effet de surface -> modélisation à postériori.
- Couplage des modèles stellaires aux atmosphères (lois T(τ,T_{eff},$\log g_{\mathrm{eff}}$), atmosphères type CMFGEN)

1. Calculer le flux totale venant de l'étoile 2 sur un point de l'étoile 1.
2. L'équation d'équilibre hydrostatique devient $\frac{1}{\rho}\frac{dP}{dr} = g_{\mathrm{eff}} + \kappa\frac{F_{21}}{c}$ qu'on intègre grâce aux équations d'états et T(τ).
3. On colle l'atmosphère en profondeur ($\tau = 100$) en s'assurant la continuité de P.