

# Entropy-calibrated modelling with CESTAM

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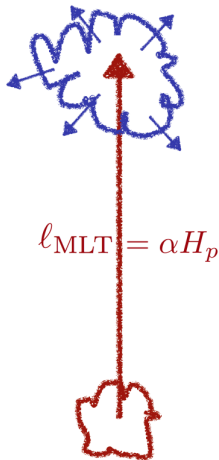


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How do we model convection in 1D stellar evolution codes?

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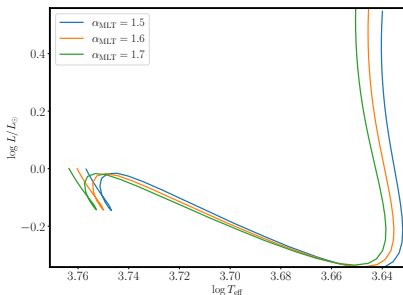


- Convection : extremely complex  $\Rightarrow$  *ad hoc* theories : MLT, CGM, ...  $\rightsquigarrow$  free parameter.
- MLT : Hot gas parcel rises to a height  $\ell \propto \alpha_{\text{MLT}} H_p$ .  $\alpha_{\text{MLT}}$  controls the convective flux.
- How do we choose  $\alpha$ ?
  - ▶ From calibration;
  - ▶ Compute grid of models with different  $\alpha$
  - ▶ Set to solar value;

Is there a more physical way to choose it?  
Should  $\alpha$  stays fixed along evolution?

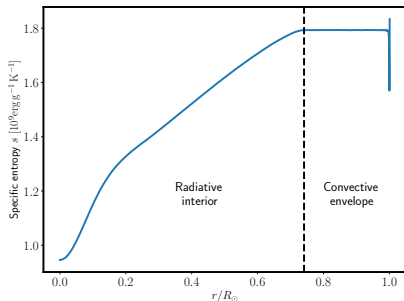
## Can $\alpha$ be linked to other quantities?

$\alpha$  controls the stellar radius  $R$ .



$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

But  $R$  is also controlled by  $s_{\text{ad}}$ , the entropy of the adiabat.



E.g., in a polytropic, completely convective model, (e.g. Ireland & Browning, 2018)

$$R \propto \exp\left(\frac{\gamma - 1}{3\gamma - 4} \frac{\mu s_{\text{ad}}}{N_A k_B}\right)$$

**Then  $\alpha$  and  $s_{\text{ad}}$  are linked.**

## How do we know what $s_{\text{ad}}$ a star should have ?

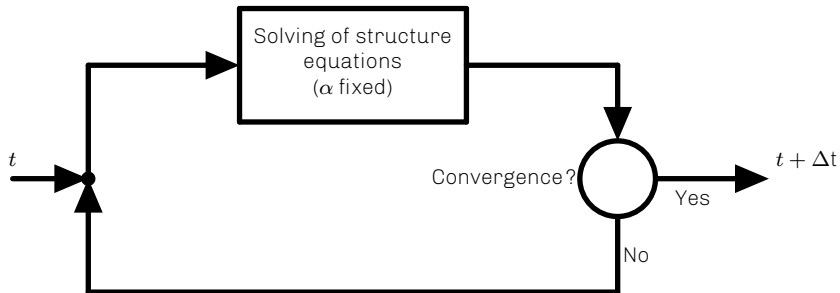
- From precise 3D modelling of convective upper layers (stagger, CO5BOLD, ...).  $s_{\text{ad}}$  **is an input of the models.**
- Using grids of 3D atmospheres : prescription for  $s_{\text{ad}}$  as a function of  $T_{\text{eff}}$ ,  $\log g$ ,  $Z$ .
- So far, three prescriptions :
  - ▶ **Ludwig+99** : Based on 2D atm. models at fixed metallicity and with a chemical composition close to GS98 (proto-Sun).
  - ▶ **Magic+13** : Based on 3D atm. models (STAGGER grid).  $[\text{Fe}/\text{H}] \in [-4.0; +0.5]$ . Chem. composition :  $\simeq$  AGS09 (pres. Sun).
  - ▶ **Tanner+16** : Same as Magic+13 but with different mathematical form.

$\Rightarrow$  we can determine  $s_{\text{ad}}$  knowing  $T_{\text{eff}}$ ,  $\log g$  and  $Z$ .  
How do we relate  $\alpha$  to  $s_{\text{ad}}$  ?

## Entropy-calibration

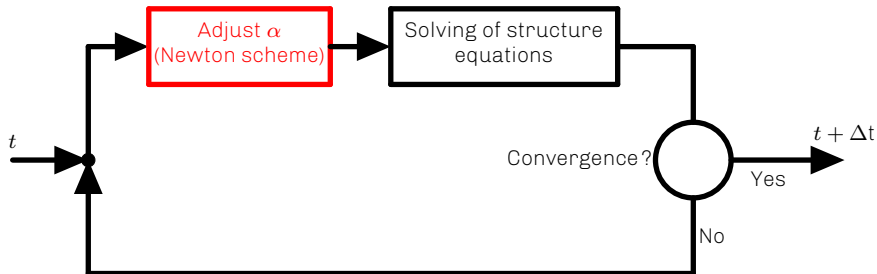
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**In a traditional stellar evolution code** (e.g CESTAM; Morel+95, Lebreton+08, Marques+13) :



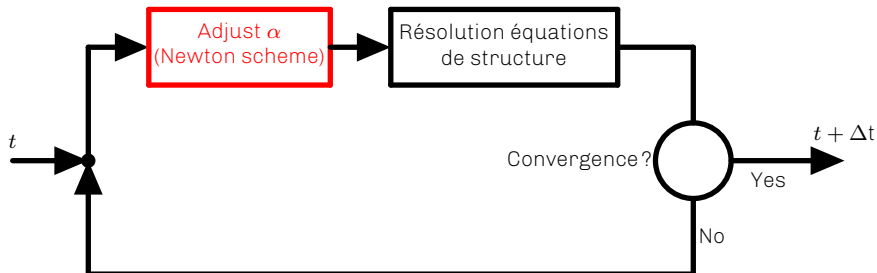
## Entropy-calibration, general idea (Spada+2018,2019,2021):

The goal is to adjust  $\alpha$  along evolution so that  $s_{\text{ad}}$  in 1D models matches  $s_{\text{MHD}}$  obtained from prescriptions.



## Entropy-calibration, general idea (Spada+2018,2019,2021):

The goal is to adjust  $\alpha$  along evolution so that  $s_{\text{ad}}$  in 1D models matches  $s_{\text{MHD}}$  obtained from prescriptions.



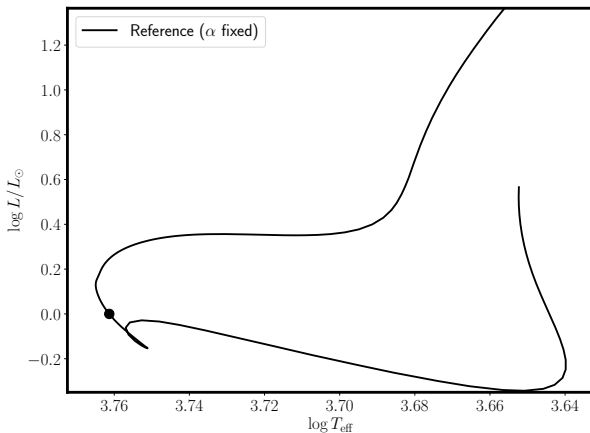
New implementation in CESTAM.

Why redo the work of Spada+?

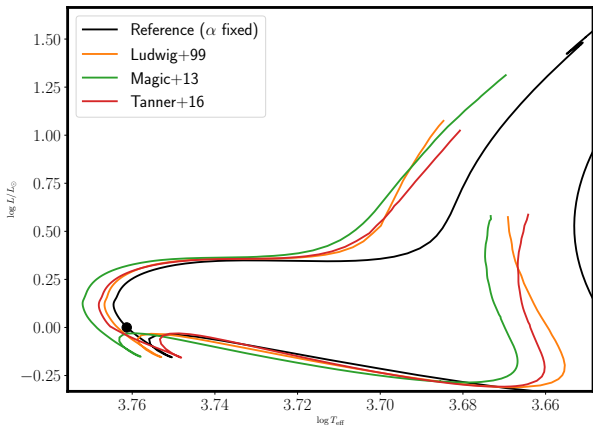
- Different code (YREC (Demarque+08)  $\rightarrow$  CESTAM). Different way of computing convective envelope;
- Lot of care should be taken when using entropy prescriptions.

Gravitational settling, GS98.

Tuning of  $Y_0$  and  $\alpha_{\text{MLT}} (\simeq 1.81)$  to obtain  $T_{\text{eff},\odot}$ ,  $L_{\odot}$ .







$\Rightarrow$  Large discrepancies ( $\Delta T_{\text{eff}} > 100\text{K}$ ).

It's different. Is it better?

## Prescriptions should be corrected

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- Entropy is defined up to a constant. EoS tables used in 2D and 3D MHD models and 1D evolution models are not the same.  
⇒ Addition of an **offset  $ds$** , computed using a reference model (Spada+2018,2019).
- The entropy varies with the chemical composition :

$$s \propto \frac{1}{\mu} \ln(\dots) \quad (1)$$

The mean molecular weight  $\mu$  is different in MHD models and in your 1D model.

⇒ **Multiplicative factor  $f_\mu = \mu_{\text{RHD}}/\mu_{\text{1D}}$**  (Spada+2021).

Final corrected form :

$$s_{\text{MHD}} \rightsquigarrow s_{\text{MHD}} f_\mu + ds \quad (2)$$

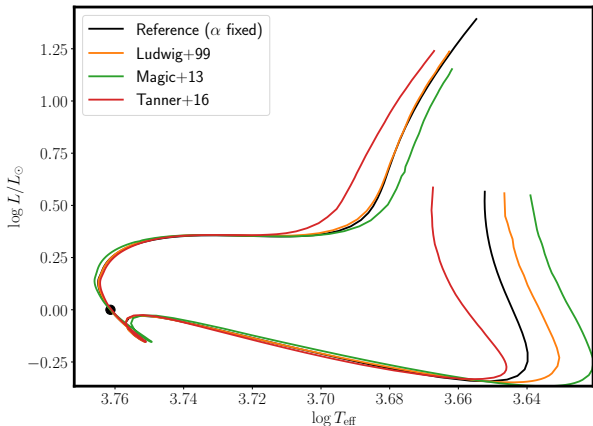
## Prescriptions should be corrected

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But also, prescription's coefficients from original paper should be used.

- Ludwig+99 Based on 2D models → less accurate adiabatic entropies
- Magic+13 & Tanner+16 : original paper used entropies at the bottom of the simulate box instead of  $s_{ad}$

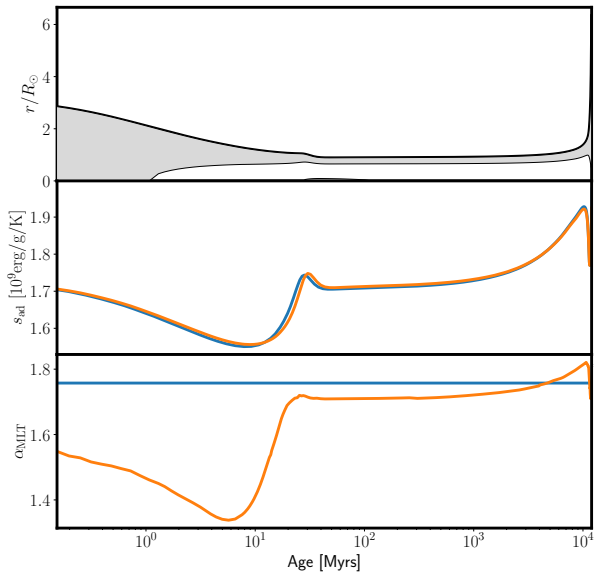
Using data from the CIFIST grid (computed with C05BOLD : Ludwig+09; Freytag+12), we recalibrated all the parameters for the different prescriptions.



What is the cause of discrepancies during PMS and RGB?

## $\alpha_{\text{MLT}}$ evolution

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$$s \propto \ln T^{3/2} / \rho$$

$$\text{Virial th. : } T \propto R^{-1},$$

$$\rho \propto R^{-3}$$

• PMS : contraction phase.

$$\Rightarrow s \searrow$$

• RGB : expansion phase

$$R \nearrow \Rightarrow s \nearrow$$

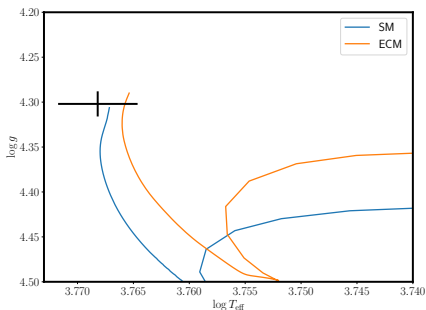
## 16Cyg A

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- Observables (16 CygA; Karovicova+2021) :

$\log g$	$T_{\text{eff}}$	$L/L_{\odot}$	[Fe/H]
$4.302 \pm 0.014$	$5864 \pm 48$	$1.511 \pm 0.0043$	$0.15 \pm 0.05$

- Calibration through Levenberg-Marquardt algorithm (OSM; R. Samadi).
- Physics : AGS09, MLT, gravitational settling.
- Standard model (SM). Adjustable parameters : Age,  $M$ ,  $\alpha_{\text{MLT}}$  (fixed),  $Y_0$ .  
Targets :  $\log g$ ,  $T_{\text{eff}}$ ,  $L/L_{\odot}$  and [Fe/H].
- Entropy-calibrated model (ECM). Adjustable parameters : Age,  $M$ ,  $Y_0$ . Targets :  $\log g$ ,  $T_{\text{eff}}$  and [Fe/H].



	$\log g$	$T_{\text{eff}}$	$L/L_{\odot}$	$[\text{Fe}/\text{H}]$	
Obs.	$4.302 \pm 0.014$	$5864 \pm 48$	$1.511 \pm 0.0043$	$0.15 \pm 0.05$	
SM	4.306	5850	1.511	0.19	
ECM	4.290	5826	1.600	0.18	
	Age	$M$	$\alpha_{\text{MLT}}$	$Y_0$	$R/R_{\odot}$
SM	7617	$1.06 \pm 0.01$	$2.02 \pm 0.01$	$0.273 \pm 0.004$	1.20
ECM	$6293 \pm 939$	$1.10 \pm 0.03$	Varying	$0.267 \pm .002$	1.24

PLATO expected accuracies. Age :10%; Mass : 15%, Radius : 2%.

## Conclusions

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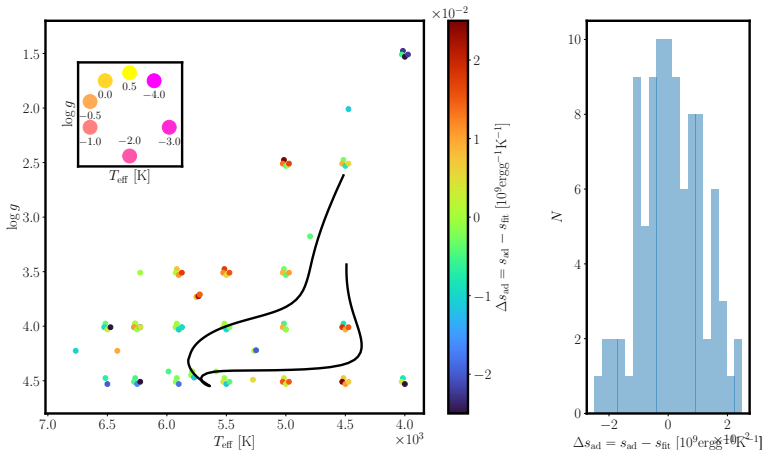
- ▶ Numerical scheme is robust and we recover results obtained by F. Spada with YREC.
- ▶ Sorted out the different prescriptions and improved them through corrections (see Manchon+, in prep for more details).
- ▶ Large impact for PLATO accuracy of model-dependent parameters.
- ▶ Changes PMS and RGB location of Solar type stars.
- ▶ Calibration independent of physics (contrary to prescription of  $\alpha$ ).

Now :

- ▶ More detailed tests on benchmark stars (seismic,...).
- ▶ What impact it has on depth of CZ? Could have an impact on transport processes.



Error between Magic+13 prescription, with coefficients calibrated on the CIFIST grid



The CIFIST grid includes models with very low  $[\text{Fe}/\text{H}]$ . What happens when we remove them?

Error between Magic+13 prescription, with coefficients calibrated on the CIFIST grid reduced to  $-1.0 \geq [\text{Fe}/\text{H}] \geq +0.5$ :

