Transport of angular momentum in 2D

L. Manchon, J. P. C. Marques, R.-M. Ouazzani

IAS Institut d'Astrophysique Spatiale Orsay

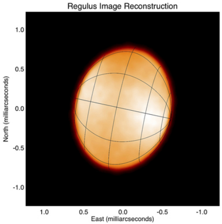
l'Observatoire de Paris — LESIA
Laboratoire d'Études Spatiales et d'Instrumentation en Astrophysique

MAX PLANCK INSTITUTE FOR SOLAR SYSTEM RESEARCH

Why study rotation ?

In standard stellar evolution models, rotation is neglected.
However,

- **Generates a dynamo** that sustains magnetic field $\rightarrow$ impact on planetary atm. ;
- **Induces additional transport of chemicals** ;
- **Breaks spherical symmetry** $\rightarrow \boldsymbol{\nabla} p/\rho \nparallel \mathbf{g}$;



Regulus

Current modelling of the transport of angular momentum (TAM)

- Convective zone : chemically homogeneous $\rightarrow$ no impact on the age.

$\rightarrow$ we focus on radiative zone : rotation $\leftrightarrow$ turbulence, diffusion, magnetic fields, overshoot...

3 simplifying ideas (Zahn 92, Spiegel & Zahn 92) :

- Turbulence-induced viscosity diffuses $\mathbf{\Omega}$.
- Meridional circulation advects $\mathcal{J}$.
- Shellular rotation approximation ($\nu_{\mathrm{h}} \gg \nu_{\mathrm{v}}$) : $\mathbf{\Omega}$ independent of latitude.

$\Rightarrow$ We can keep our 1D approach : $r \rightarrow p$.

Shear-induced turbulence

$\Rightarrow$smooths horizontal gradients ($\nu_{\mathrm{h}} \gg \nu_{\mathrm{v}}$).

$\Rightarrow$ **Shear-induced turbulence diffuses Ω.**

Current modelling of the transport of angular momentum (TAM)

- Convective zone : chemically homogeneous $\rightarrow$ no impact on the age.

$\rightarrow$ we focus on radiative zone : rotation $\leftrightarrow$ turbulence, diffusion, magnetic fields, overshoot...

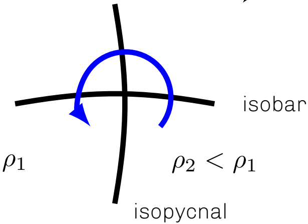
3 simplifying ideas (Zahn 92, Spiegel & Zahn 92) :

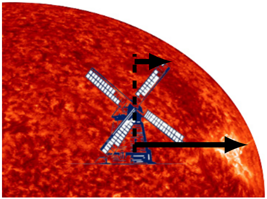
- Turbulence-induced viscosity diffuses $\boldsymbol{\Omega}$.
- Meridional circulation advects $\boldsymbol{\mathcal{J}}$.
- Shellular rotation approximation ($\nu_\mathrm{h} \gg \nu_\mathrm{v}$) :
  $\boldsymbol{\Omega}$ independent of latitude.

$\Rightarrow$ We can keep our 1D approach : $r \rightarrow p$.

Star is in baroclinic equilibrium :

$$\frac{\boldsymbol{\nabla} p \times \boldsymbol{\nabla} \rho}{\rho^2} = r \sin\theta \frac{\partial \Omega^2}{\partial z} \mathbf{e}_\varphi$$





$\Rightarrow$ Variations of Ω along the rotation axis are determined by variations of ρ along isobares **alone**.

$\Rightarrow$ Only way to fulfil the baroclinic equilibrium : meridional circulation.

Current modelling of the transport of angular momentum (TAM)

- Convective zone : chemically homogeneous $\rightarrow$ no impact on the age.

$\rightarrow$ we focus on radiative zone : rotation $\leftrightarrow$ turbulence, diffusion, magnetic fields, overshoot...

3 simplifying ideas (Zahn 92, Spiegel & Zahn 92) :

- Turbulence-induced viscosity diffuses $\boldsymbol{\Omega}$.
- Meridional circulation advects $\boldsymbol{\mathcal{J}}$.
- **Shellular rotation approximation** $(\nu_\mathrm{h} \gg \nu_\mathrm{v})$:
  $\boldsymbol{\Omega}$ independent of latitude.

$\Rightarrow$ We can keep our 1D approach : $r \rightarrow p$.

Does it reproduce observations ?

No !

	Observations	Models
Sun	CZ : $\partial\Omega/\partial\theta \neq 0$	$\partial\Omega/\partial\theta = 0$
	RZ : $\partial\Omega/\partial r \simeq 0$	$\partial\Omega/\partial r \neq 0$
Red Giants	$\Omega_{\rm core} > \Omega_{\rm surf}$ but ...	...$\Omega_{\rm core,model}$ strongly overestimated

$\Rightarrow$ Motivates the need for additional transport mechanisms... :

- Waves : Mixed modes (Belkacem et al., 2015ab), IGW (Kumar et al. 1999 ; Pinçon et al. 2016) ;
- Hydro- or MHD-instabilities : GSF, ABCD, Rayleigh-Taylor ; MRI, Taylor-Spruit

$\Rightarrow$ ... But also require to modify standard stellar model.

We need to break spherical symmetry and go to 2D

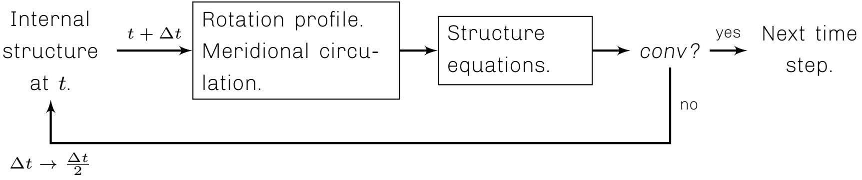
- Rotating stars are not spherical.
- Transition RZ/CZ : Boundary condition on Ω depends on θ (obs.).
- Additional transport mechanisms :
  - Instability criteria : depend on θ.
  - AM transported by waves : depends on $\Omega(\theta)$.
- Shellular rotation may not be valid near the pole (because rotation velocity vanishes : $\nu_{\rm h} \gg \nu_{\rm v}$ not verified).

In CESTAM

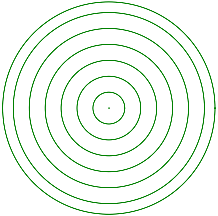
CESTAM : Code d'Évolution Stellaire, avec Transport, Adaptatif et Modulaire. (Morel 97, Morel & Lebreton 08, Marques et al. 13)

$$\begin{aligned}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi\rho r^2}, & \frac{\partial T}{\partial m} &= \frac{\partial p}{\partial m}\frac{T}{p}\nabla, \\
\frac{\partial p}{\partial m} &= -\frac{\mathcal{G}m}{4\pi r^4}, & \frac{\partial l}{\partial m} &= \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - T\frac{\partial s}{\partial t}
\end{aligned}$$

$$m(r) \rightarrow \text{1D} \rightarrow \text{spherical}$$

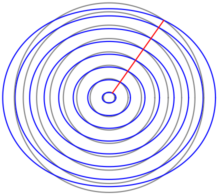


Modified structure equations



$$
\begin{aligned}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi\rho r^2}, & \frac{\partial T}{\partial m} &= \frac{\partial p}{\partial m}\frac{T}{p}\nabla, \\
\frac{\partial p}{\partial m} &= -\frac{\mathcal{G}m}{4\pi r^4}, & \frac{\partial l}{\partial m} &= \varepsilon_{\mathrm{nuc}} - \varepsilon_\nu - T\frac{\partial s}{\partial t}
\end{aligned}
$$

Average on isobar

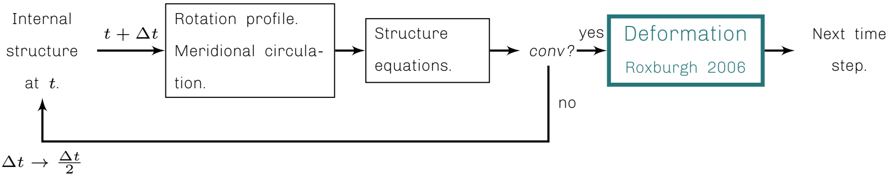


$$
\frac{\partial r_p}{\partial m_p} = \frac{1}{4\pi\breve{\rho} r_p^2}, \quad \frac{\partial T}{\partial m_p} = \frac{\mathcal{G}m_p}{4\pi r_p^4}\boldsymbol{f_p}\min\left[\nabla_{\mathrm{conv}}, \nabla_{\mathrm{rad}}\frac{\boldsymbol{f_T}}{\boldsymbol{f_p}}\right],
$$

$$
\frac{\mathrm{d}p}{\mathrm{d}m_p} = \frac{\mathcal{G}m_p}{4\pi r_p^4}\boldsymbol{f_p}, \quad \frac{\partial L_p}{\partial m_p} = \frac{\left\langle \left(\varepsilon_{\mathrm{nucl}} - T\frac{\partial s}{\partial t} - \varepsilon_\nu\right)\boldsymbol{g}_{\mathrm{eff}}^{-1}\right\rangle}{\left\langle \boldsymbol{g}_{\mathrm{eff}}^{-1}\right\rangle}
$$

Deformation – Roxburgh's method (2006)

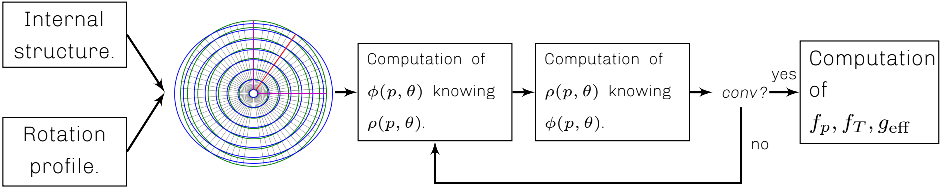
$f_p, f_T, g_{\text{eff}} \Rightarrow$ need the coordinates of isobars, ϕ, ρ on the isobars, etc...



Deformation – Roxburgh's method (2006)

Legendre polynomial decomposition :

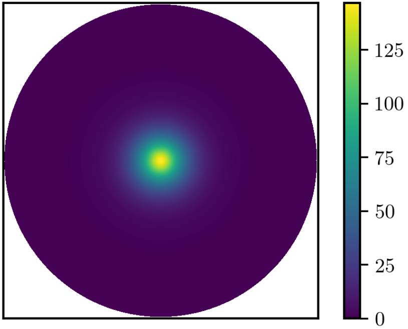
$$\rho = \overline{\rho} + \sum_{\ell > 0} \widetilde{\rho}_\ell P_\ell(\cos\theta)$$



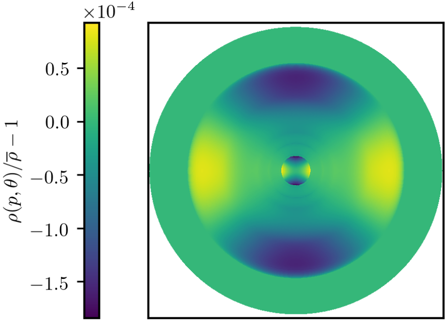
Deformation – Roxburgh's method (2006)

$1 M_\odot$

No Rotation



Initial rotation period : 3 days

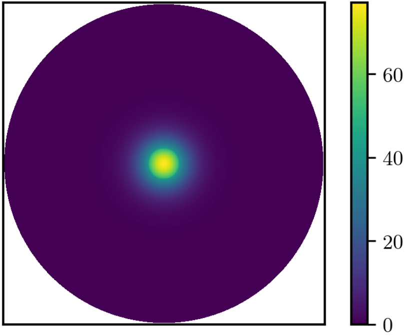


$20\% \Omega_{\mathrm{K}}$

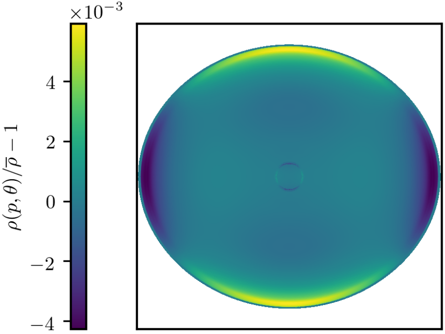
Manchon et al. (in prep.)

$2M_\odot$

No Rotation



Initial rotation period : 3 days

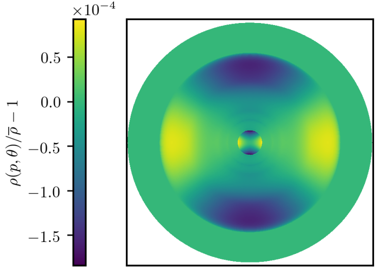


Manchon et al. (in prep.)

$57\%\Omega_{\mathrm{K}}$

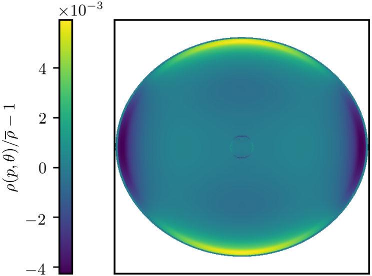
$\Rightarrow$ We can now follow the 2D structure along evolution.

$M = 1M_\odot$ and $P_{\rm disk} = 3$ days



$20\%\Omega_{\rm K}$

$M = 2M_\odot$ and $P_{\rm disk} = 3$ days



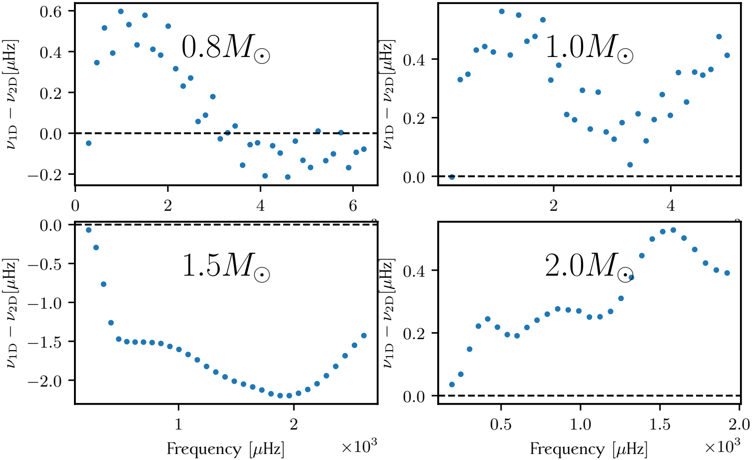
$57\%\Omega_{\rm K}$

$$\frac{\mathrm{d}\rho}{\mathrm{d}\theta} = -\rho \frac{\frac{\partial}{\partial r}\left(\Omega^2 r^2 \sin\theta\cos\theta\right) - \frac{\partial}{\partial \theta}\left(\Omega^2 r \sin^2\theta\right)}{\frac{\partial \phi}{\partial r} - \Omega^2 r \sin^2\theta}. \tag{1}$$

If $\Omega \searrow$ with $r \nearrow$: $(\dots) < 0$, $(\dots) > 0$ (shell. rot.), $(\dots) > 0$ (local break-up criterion) $\Rightarrow \mathrm{d}\rho/\mathrm{d}\theta > 0$ and ρ increases from pole to equator.

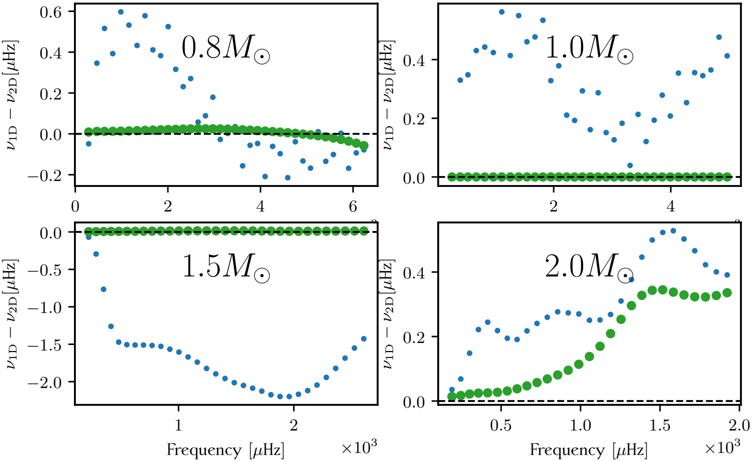
Does the deformation module introduce substantial numerical noise?

- Fréquences : ACOR (Ouazzani+2015).
- 1D (`.osc`) vs 2D (`.osc2d`) (bleu).



Seismic validation (non-rotating case)

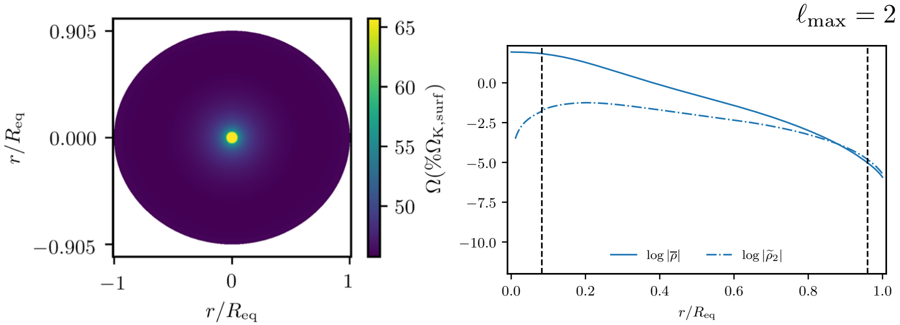
- Fréquences : ACOR (Ouazzani+2015).
- 1D vs 2D (bleu) et (1D + dérivée 2D) vs 2D (vert).



Remaining gaps : glitches in the derivative of ρ at transition RZ/CZ.

Deformation with rotation – maximum order

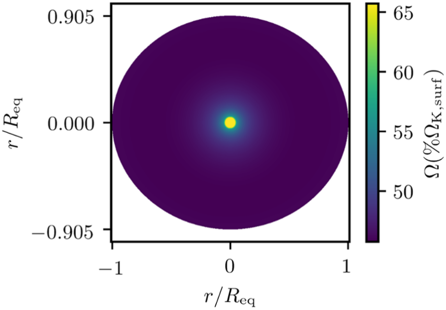
$$\rho = \overline{\rho} + \sum_{\ell>0}^{\ell_{\max}} \widetilde{\rho}_\ell P_\ell(\cos\theta)$$



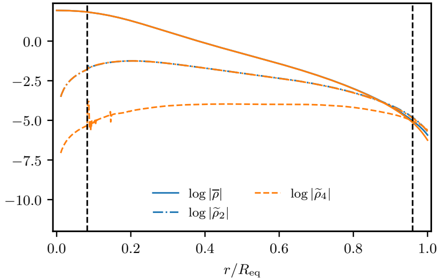
Manchon et al. (in prep.)

Deformation with rotation – maximum order

$$\rho = \overline{\rho} + \sum_{\ell>0}^{\ell_{\max}} \widetilde{\rho}_\ell P_\ell(\cos\theta)$$



$\ell_{\max} = 4$

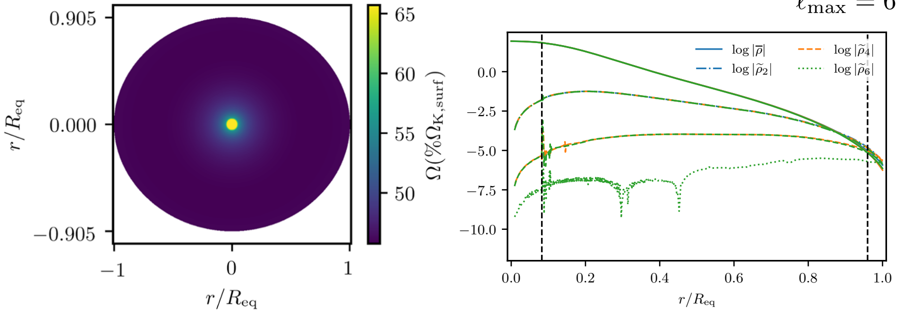


Manchon et al. (in prep.)

Deformation with rotation – maximum order

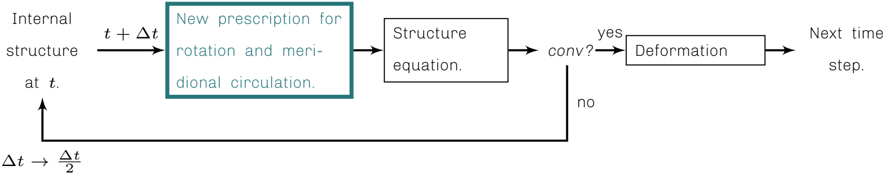
$$\rho = \overline{\rho} + \sum_{\ell>0}^{\ell_{\max}} \widetilde{\rho}_\ell P_\ell(\cos\theta)$$

$\ell_{\max} = 6$



Manchon et al. (in prep.)

Transport of angular momentum



Transport of angular momentum

$$\boxed{\frac{\partial}{\partial t}\left(\rho r^2 \sin^2\theta\Omega\right)} + \boxed{\boldsymbol{\nabla}\cdot\left(\rho r^2 \sin^2\theta\Omega\boldsymbol{\mathcal{U}}\right)} = \boxed{\frac{\sin^2\theta}{r^2}\frac{\partial}{\partial r}\left(\rho\nu_{\mathrm{v}} r^4 \frac{\partial\Omega}{\partial r}\right)} + \boxed{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\rho\nu_{\mathrm{h}}\sin^3\theta\frac{\partial\Omega}{\partial\theta}\right)}$$

Variation of angular momentum in a layer, per u. time.

Advection of angular momentum by the meridional circulation.

Vertical diffusion of Ω by the turbulent viscosity.

Horizontal diffusion of Ω by the turbulent viscosity.

Old vs. new prescription

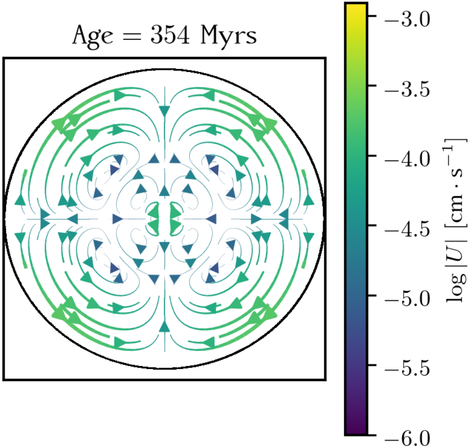
Zahn 1992, Talon et al. 1997, Maeder et al. 1998

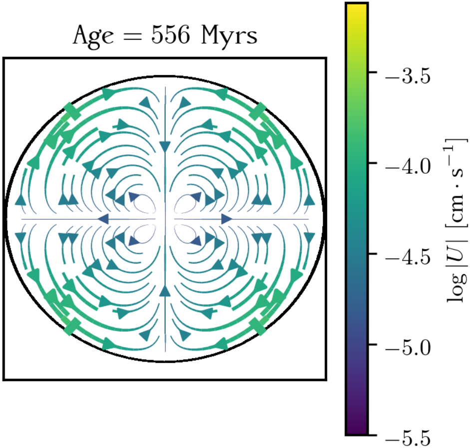
- Limited to $\ell = 2$.
- Approximation of $\mathbf{g}_{\text{eff}}$.
- Depends on $\frac{\partial\widetilde{\rho}_\ell/\bar{\rho}}{\partial r}$, $\frac{\partial\widetilde{\mu}_\ell/\bar{\mu}}{\partial r}$.

Mathis & Zahn 2004 + deformation :

- Possible at any ℓ.
- $\mathbf{g}_{\text{eff}}$ (MZ04 or Roxburgh 2006).
- Depends on $\frac{\partial\widetilde{T}_\ell/\bar{T}}{\partial r}$ $\rightarrow$ Better handling of steep gradients.

$M = 2.5 M_\odot$, $P_{\rm disk} = 4$ days.

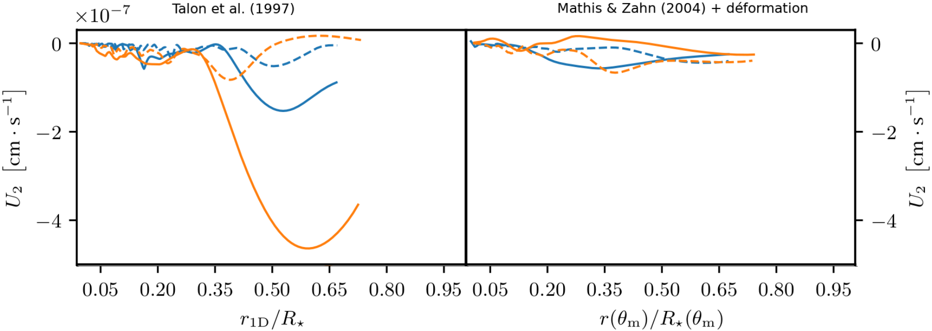




$\Omega \simeq 37\% \Omega_{\rm K}$

$\Omega \simeq 49\% \Omega_{\rm K}$

Manchon et al. (in prep.)



Manchon et al. (in prep.)

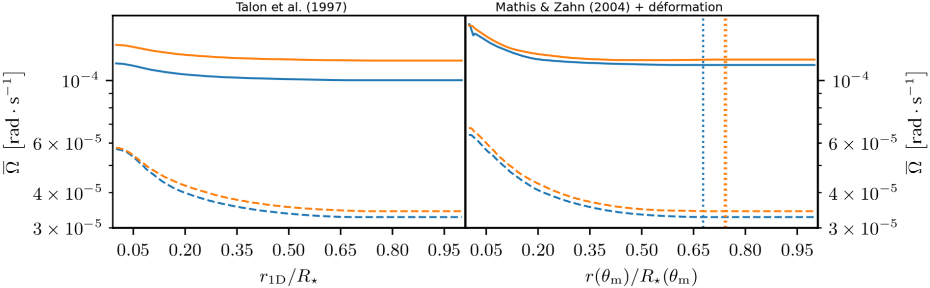
- orange : $0.8 M_\odot$
- blue : $1.0 M_\odot$
- dashed lines : slow rotators
- solid lines : fast rotators

Conclusion

- First stellar evolution code with 2D treatment. Deformation module almost costless in terms of computation time.
- New angular momentum transport prescription : includes 2D effects (possibility to go beyond $\ell = 2$); + stable and fater than code purely 1D

Perspectives :

- Framework to test additional transport mechanisms.
- Allow to improve seismic diagnostics for rotating stars.
- A code of 2^{nd} generation for PLATO.



Manchon et al. (in prep.)

- orange : $0.8M_\odot$
- blue : $1.0M_\odot$
- dashed lines : slow rotators
- solid lines : fast rotators