Atelier Codes en Physique Stellaire AIPS - PNPS, Meudon, June 29th, 2022.

Modelling waves in stellar interior with the MUSIC code

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IGWs & artificial boosting in solar-like stars

Convective penetration and waves excitation

Acoustic waves

Impact of radial geometry

Intermediate-mass stars

3D

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Solar-like reference model

➡ Based on 1D model

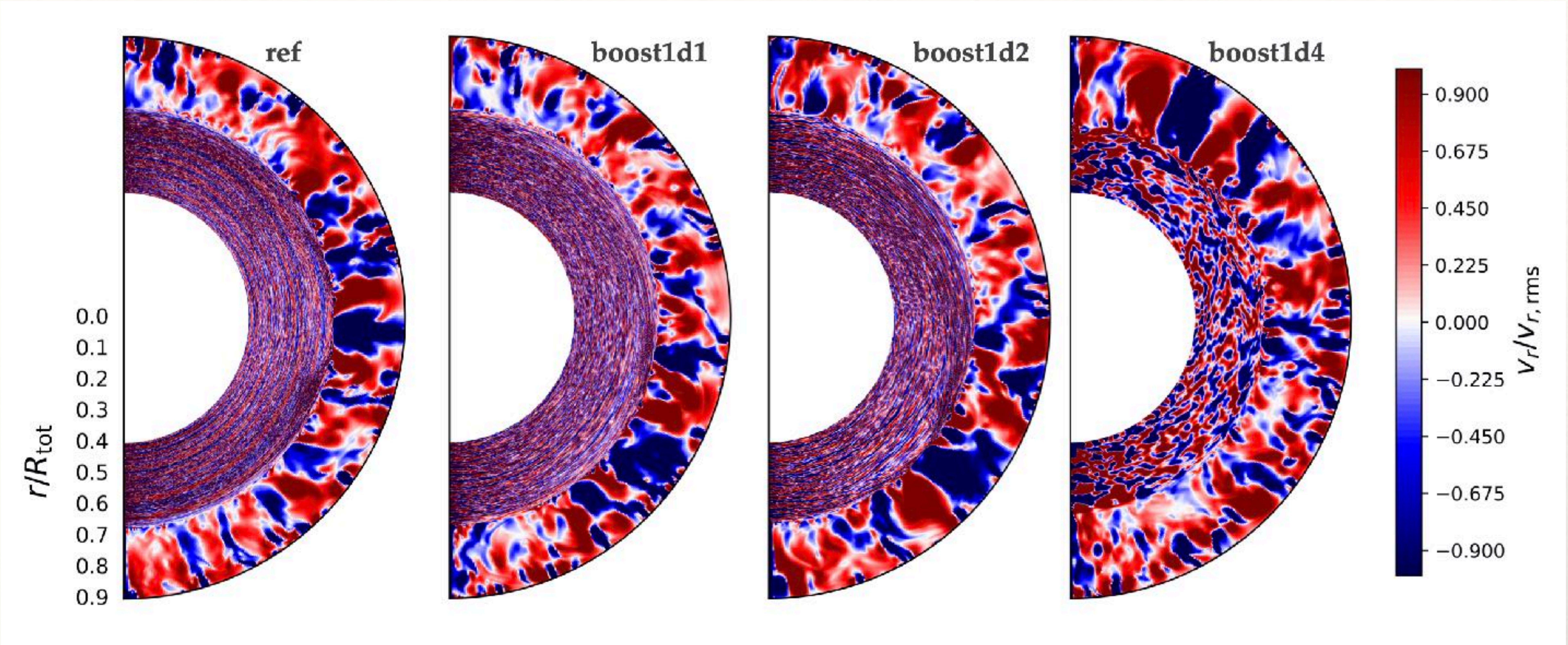
Radial range $r_{in} = 0.4\ R_{star}$

$r_{out} = 0.9\ R_{star}$

Co-latitudinal range $\theta = [0; \pi]$

Models	L/L_{star}
• **ref**	1
• **boost1d1**	10
• **boost1d2**	10^2
• **boost1d4**	10^4

Radial velocity



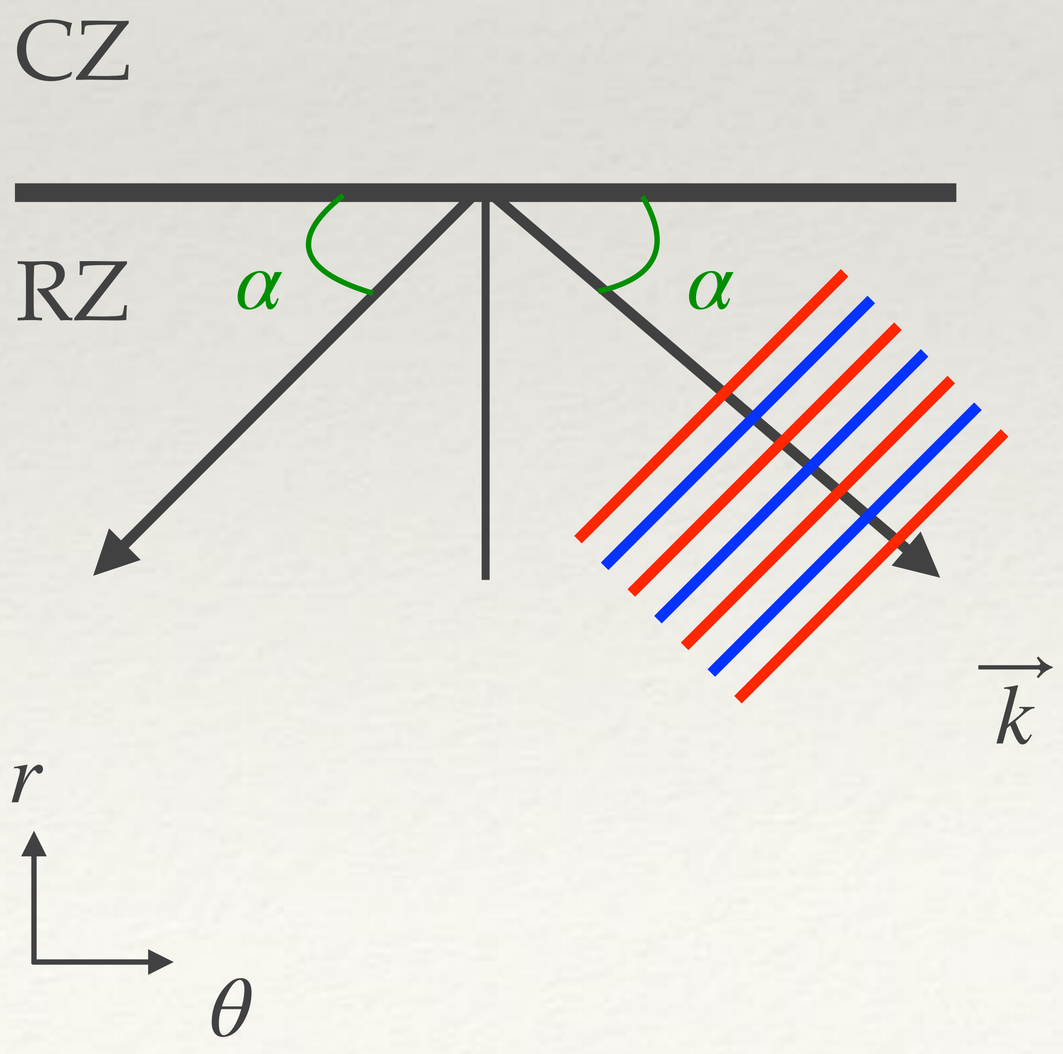
Thin concentric “circles” -> wavefronts of IGW

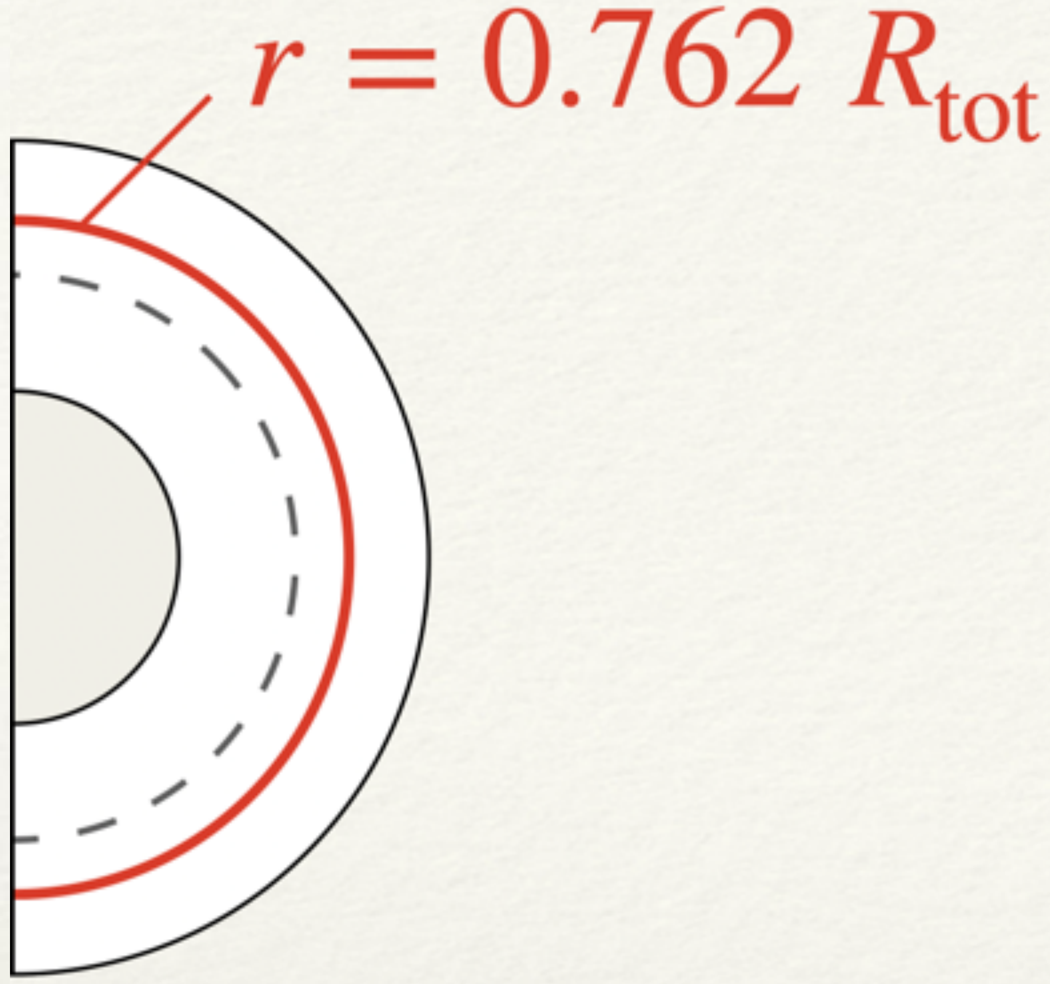
↳ More inclined = higher frequencies ⟶

Dispersion relation for IGW

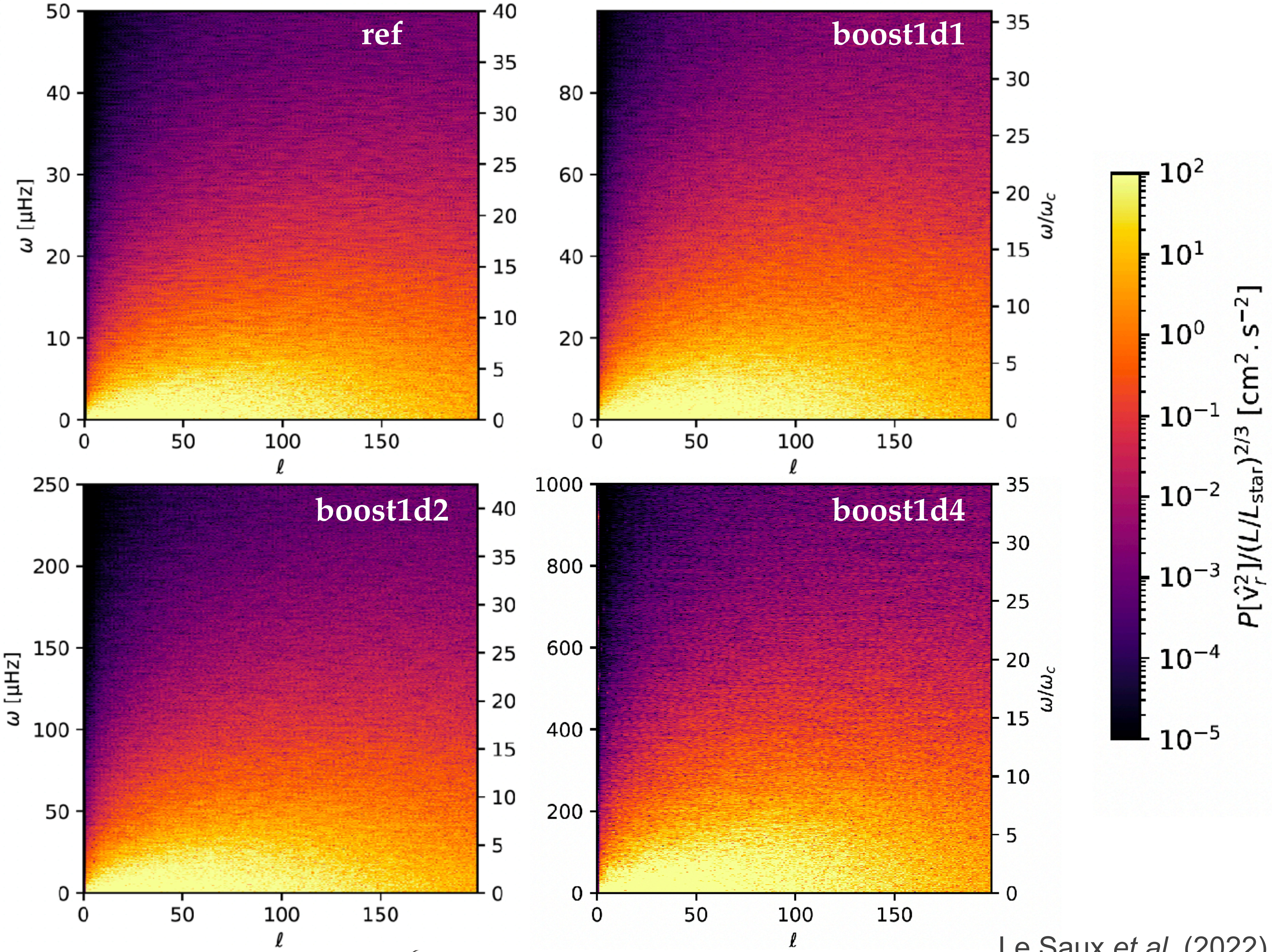
$$\frac{\omega}{N} = \pm \frac{k_h}{k} = \pm \cos(\alpha)$$

(e.g. Vallis, 2017)

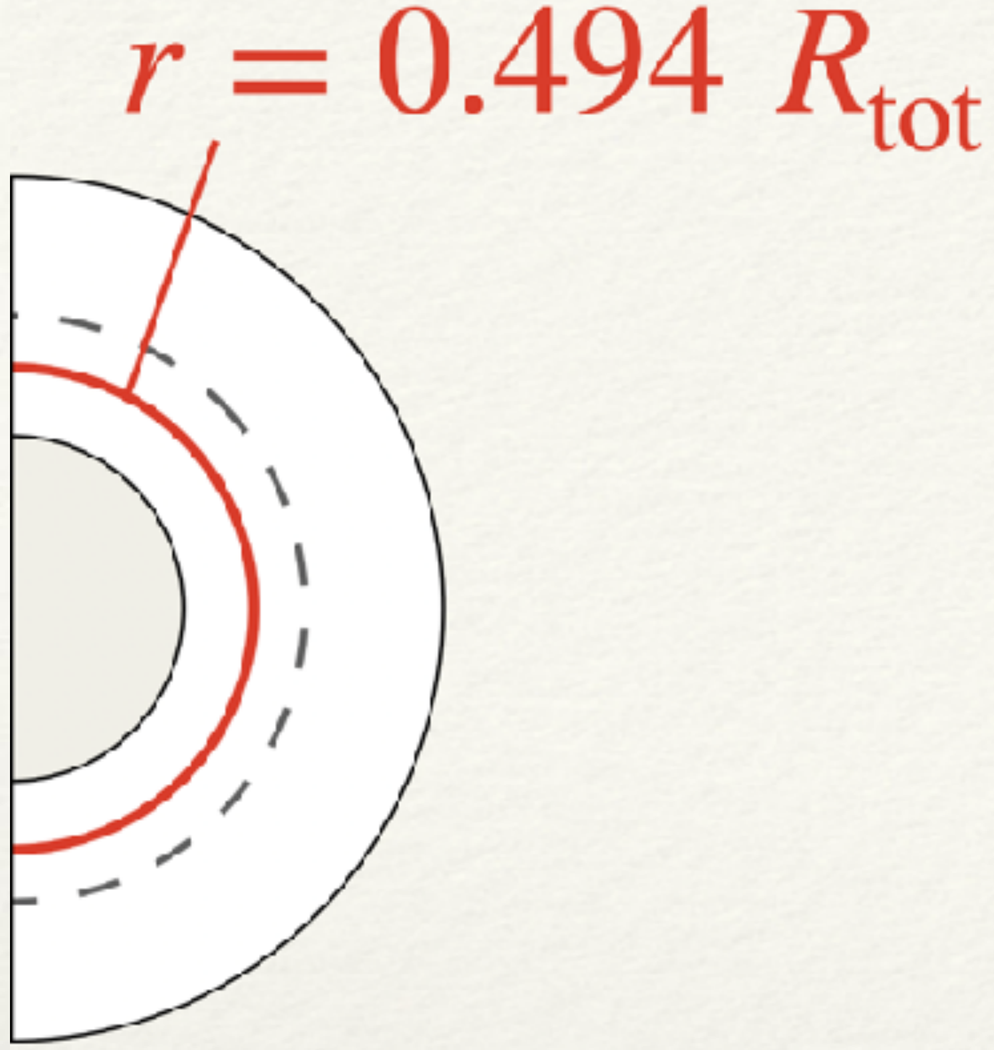


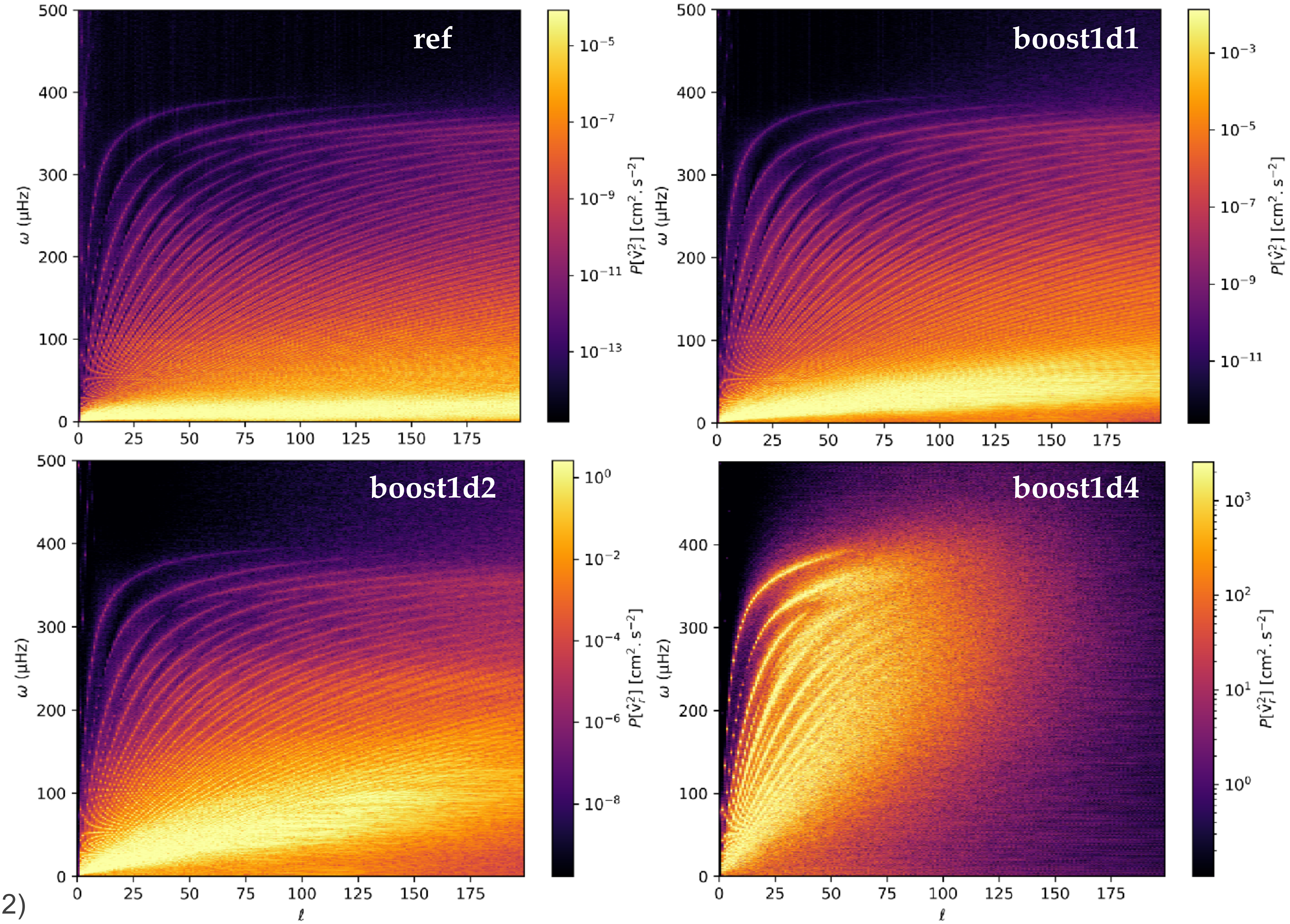


Models	ω_{conv} (μHz)
ref	1.25
boost1d1	2.78
boost1d2	5.88
boost1d4	28.57



Le Saux *et al.* (2022)

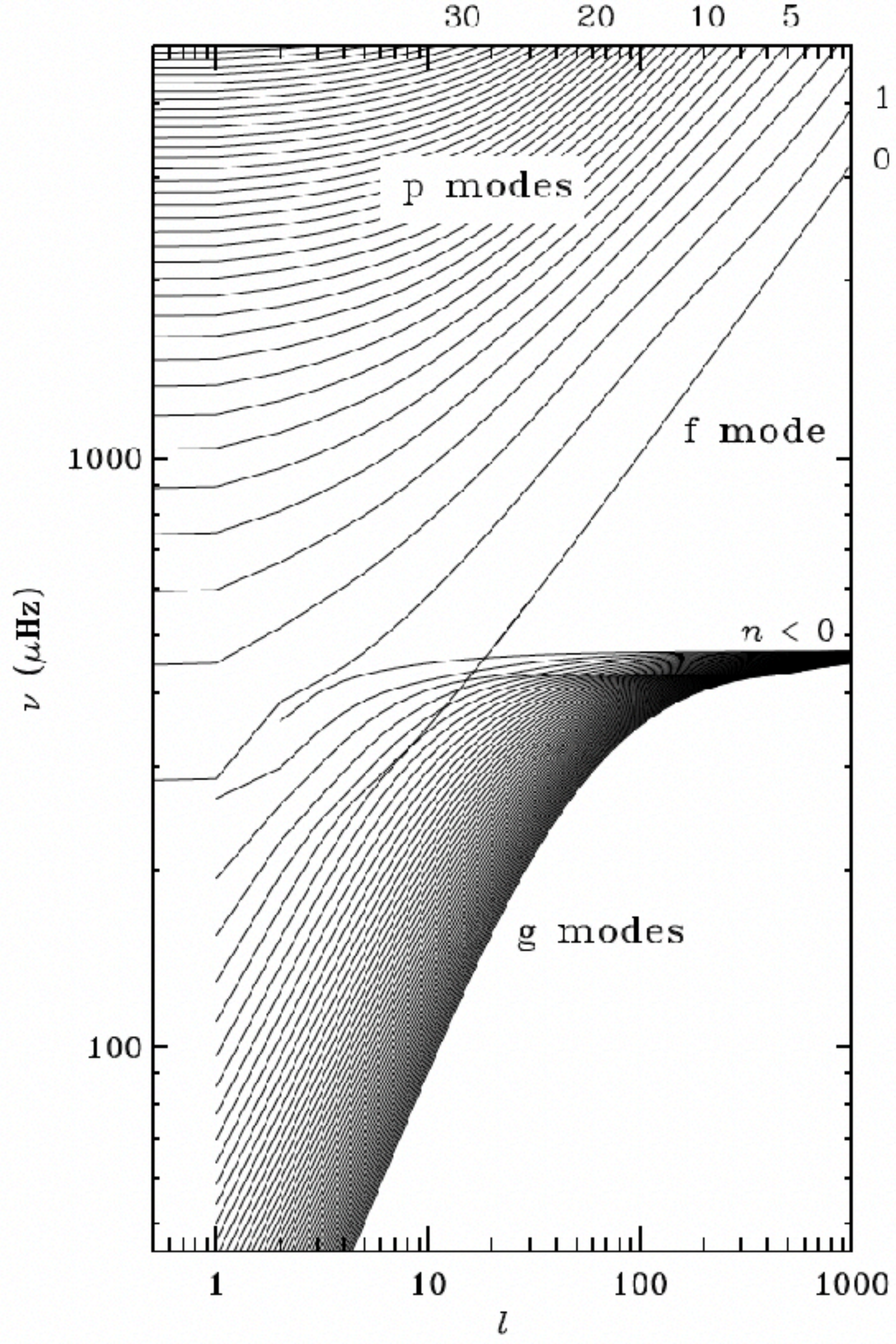




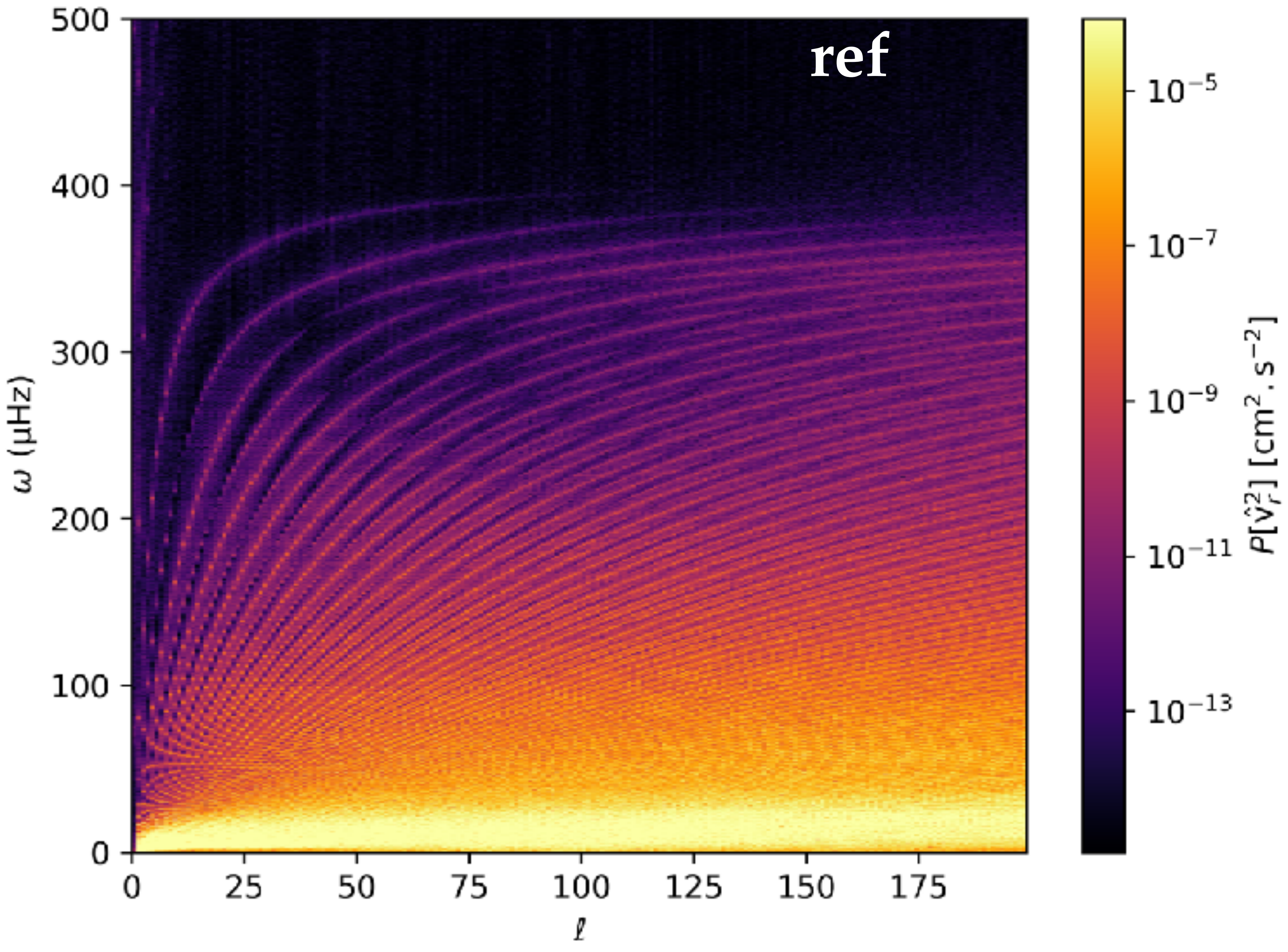
No scaling relation in the radiative zone!

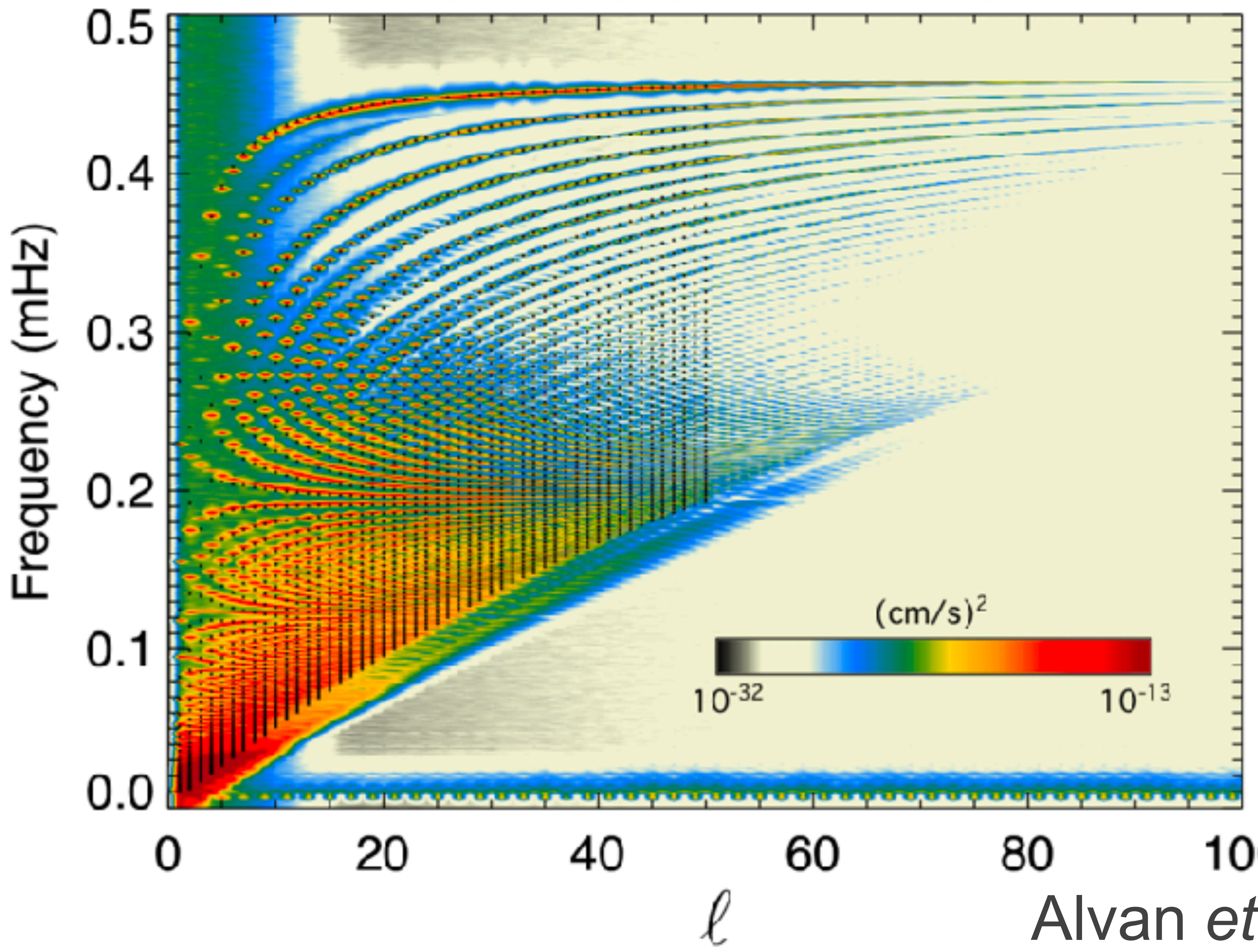
Le Saux *et al*. (2022)

Comparison



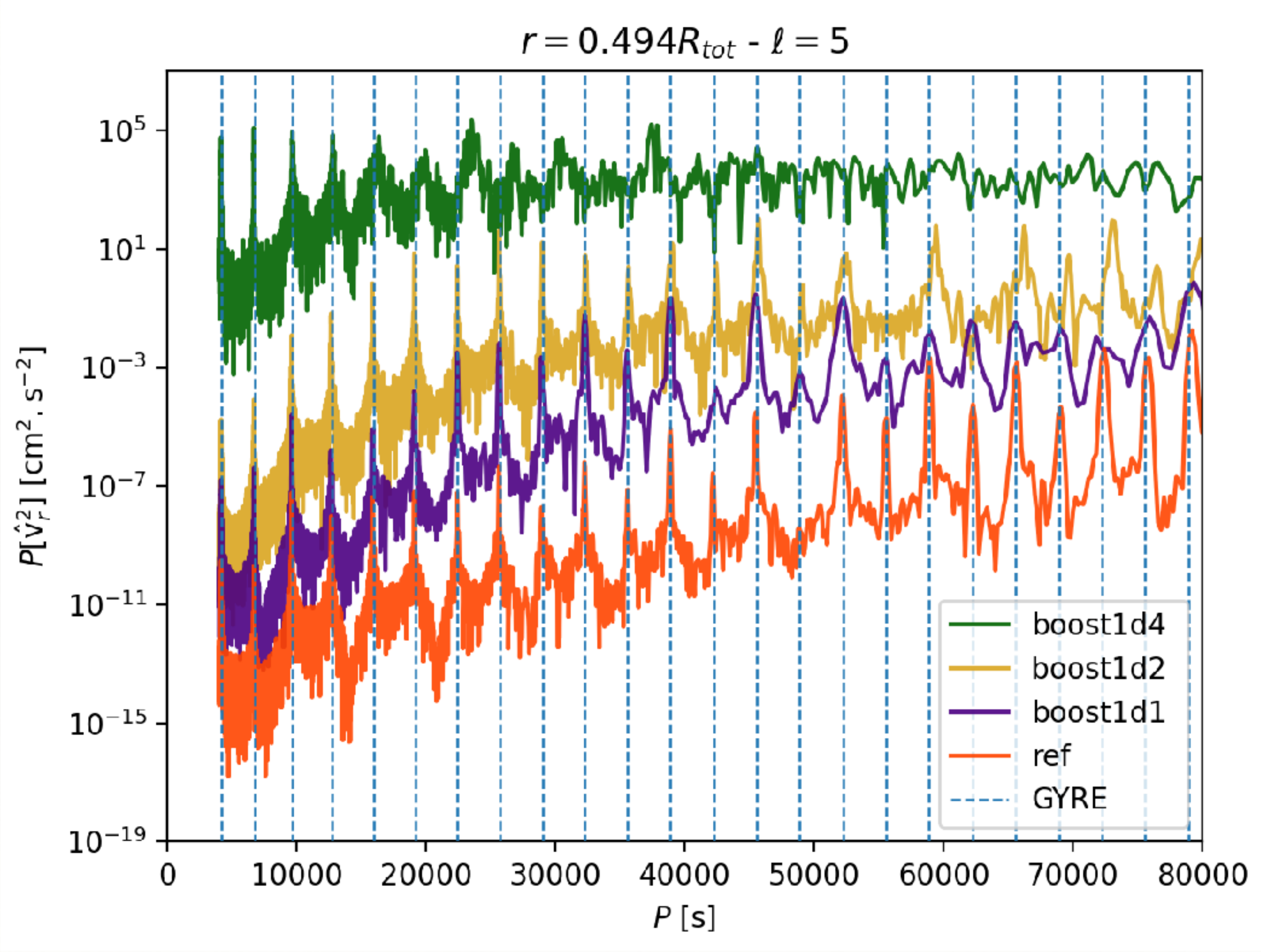
Christensen-Dalsgaard (2014)





Alvan *et al.* (2014)

Peaks = g-modes



$$\ell = 5$$

Comparison with GYRE (version 6.0)

(Townsend *et al.*, 2013, 2018)

$$P = \frac{1}{\omega}$$

Radial wave flux → Depends on the excitation mechanism

Reynolds stress excitation

Stein (1967),
Press (1981),
Goldreich & Kumar (1990),
Garcia-Lopez & Spruit (1991),
Kumar *et al.* (1999),
Lecoanet & Quataert (2013)

Plumes excitation

Townsend (1966),
Rieutord & Zahn (1995),
Montalbán & Schatzman (2000),
Pinçon *et al.* (2016)

Radial wave flux → Depends on the excitation mechanism

Lecoanet & Quataert (2013)

Discontinuous

$$\frac{\mathrm{d}F^{\mathrm{D}}}{\mathrm{d}\ln\omega\,\mathrm{d}\ln k_{\mathrm{h}}} \propto k_h^4\omega^{-13/2}$$

Linear

$$\frac{\mathrm{d}F^{\mathrm{L}}}{\mathrm{d}\ln\omega\,\mathrm{d}\ln k_{\mathrm{h}}} \propto k_h^{13/3}\omega^{-41/6}d^{1/3}$$

tanh

$$\frac{\mathrm{d}F^{\mathrm{T}}}{\mathrm{d}\ln\omega\,\mathrm{d}\ln k_{\mathrm{h}}} \propto k_h^5\omega^{-15/2}d$$

Pinçon *et al.* (2016)

$$\frac{\mathrm{d}F^{\mathrm{P}}}{\mathrm{d}\ln\omega\;\mathrm{d}\ln k_{\mathrm{h}}} \propto \mathrm{e}^{-\omega^2/4\nu_{\mathrm{p}}^2}$$

In MUSIC

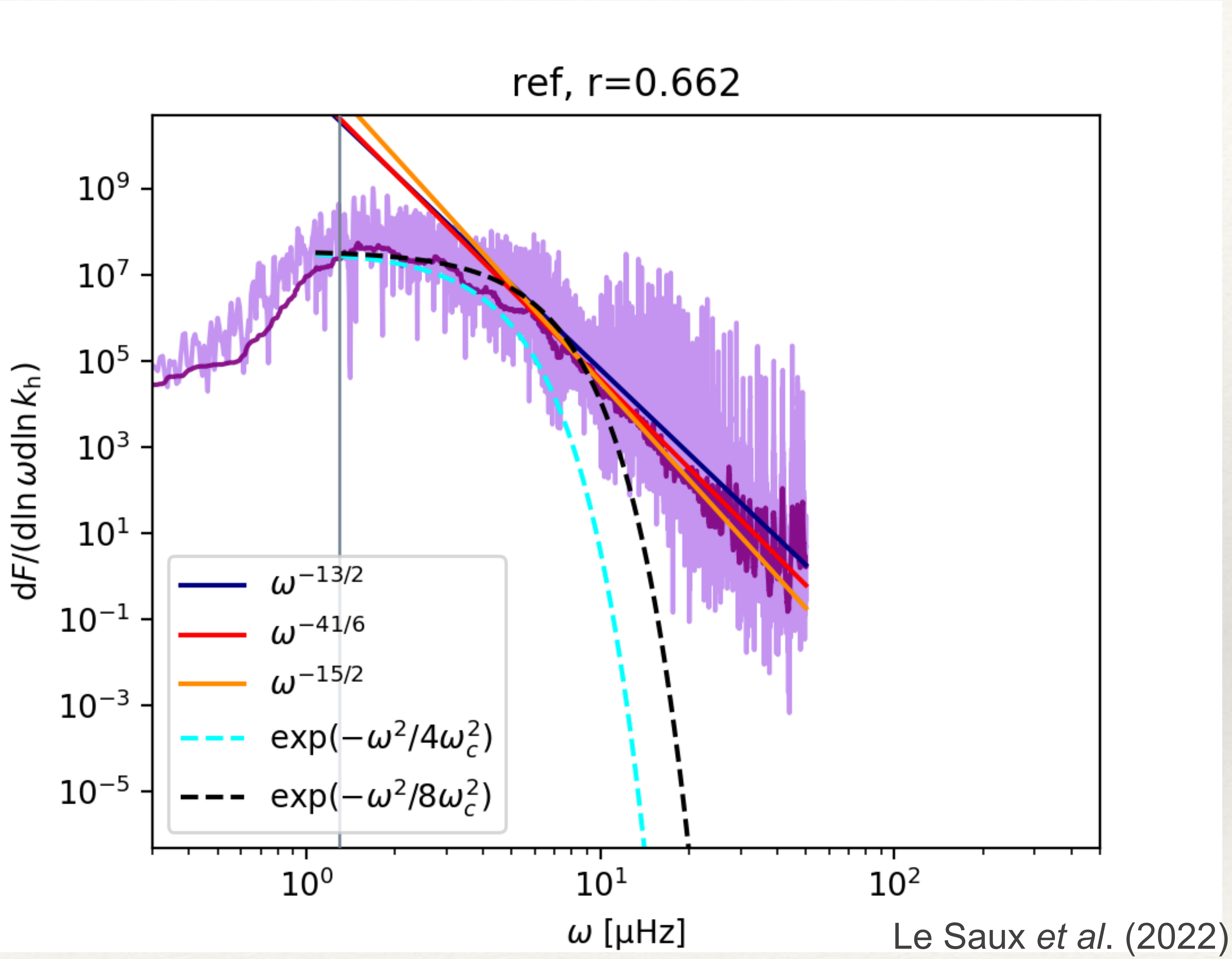
$$\frac{\mathrm{d}F}{\mathrm{d}\ln\omega\;\mathrm{d}\ln k_{\mathrm{h}}} \sim \frac{1}{2}\rho T_S N\omega r P[\hat{\mathrm{v}}_r^2](r,\omega,\ell)$$

Radial wave flux

$$\ell = 10$$

$$r = r_{\rm conv} - l_{\rm max}$$

For definition of $l_{\rm max}$ see Baraffe et al. (2021)

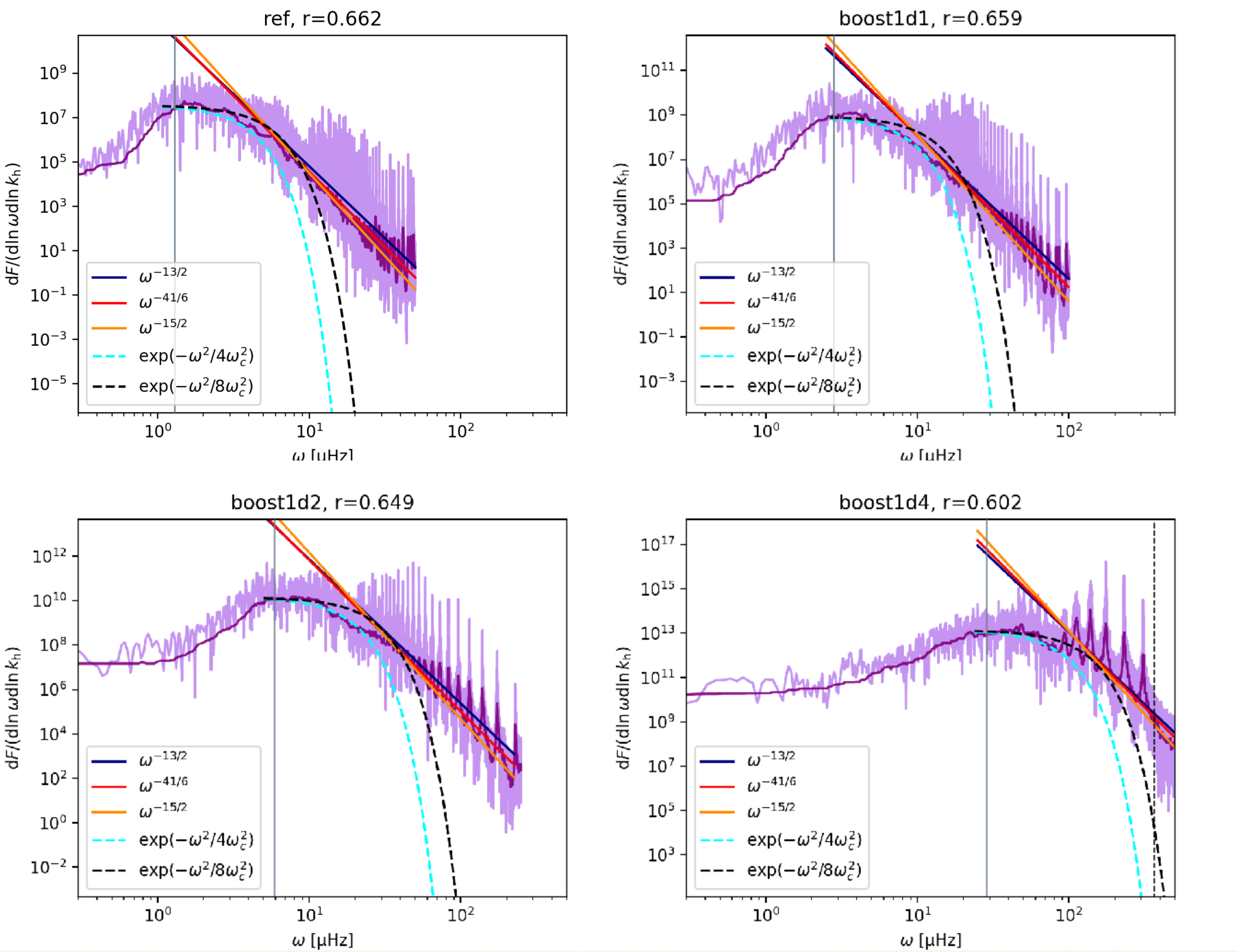


Le Saux *et al*. (2022)

Radial wave flux

$$\ell = 10$$

$$r = r_{\text{conv}} - l_{\max}$$



Wave damping

IGW can transport angular momentum, through 3 processes:

- Radiative damping (Press, 1981; Schatzman, 1993; Zahn, 1997)
- Critical layers (Alvan *et al.*, 2013)
- Non-linear wave breaking (e.g. Gervais *et al.*, 2018)

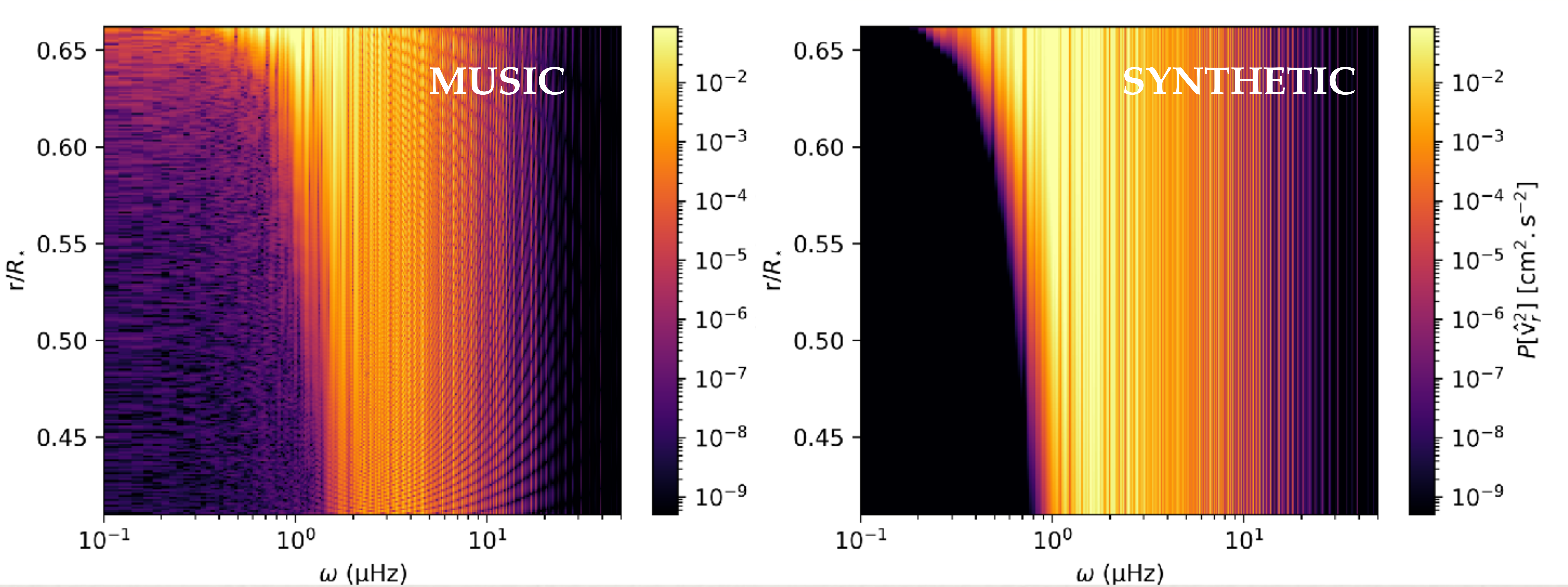
From Press (1981), wave amplitude = $\mathrm{v}_r \propto \rho^{-1/2} \times (\text{geometric term}) \times \mathrm{e}^{-\tau/2}$

$$\rightarrow \quad \tau(r, \ell, \omega) = [\ell(\ell+1)]^{3/2} \int_r^{r_\mathrm{e}} \kappa_\mathrm{rad} \frac{N^3}{\omega^4} \frac{\mathrm{d}r}{r^3}$$

In MUSIC $P_\mathrm{theory}[\hat{\mathrm{v}}_r^2](r, \ell_0, \omega) = P[\hat{\mathrm{v}}_r^2](r_\mathrm{conv} - l_\mathrm{max}, \ell_0, \omega) \times \mathrm{e}^{-\tau(r,\ell_0,\omega)}$

Wave damping

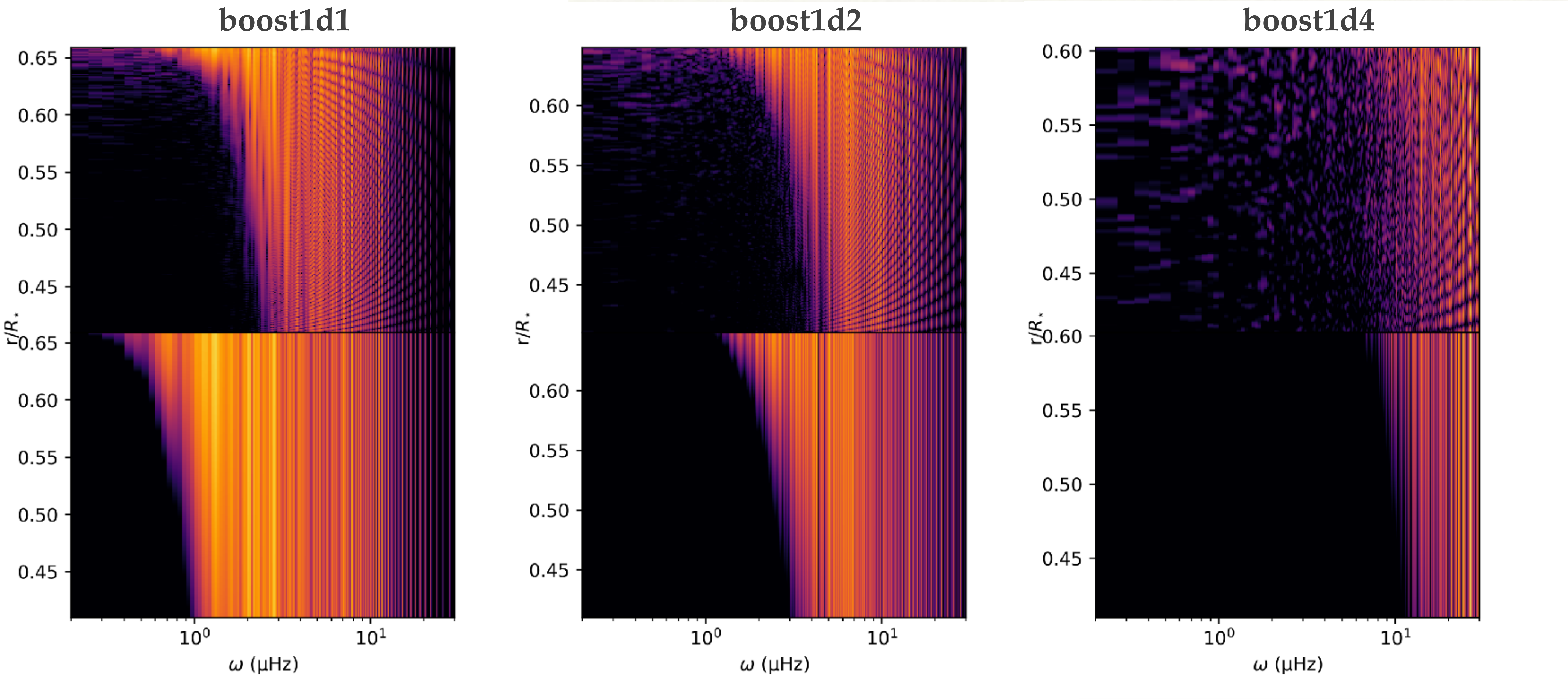
$\ell = 5$



Angular momentum transport (thus evolution of rotation profile) depend on damping rate

$$\rightarrow \quad \tau(r,\ell,\omega) = [\ell(\ell+1)]^{3/2} \int_r^{r_e} \kappa_{\text{rad}} \frac{N^3}{\omega^4} \frac{\mathrm{d}r}{r^3}$$

Wave damping



Very important to increase the radiative diffusivity by the same coefficient as the luminosity!

IGWs & artificial boosting in solar-like stars

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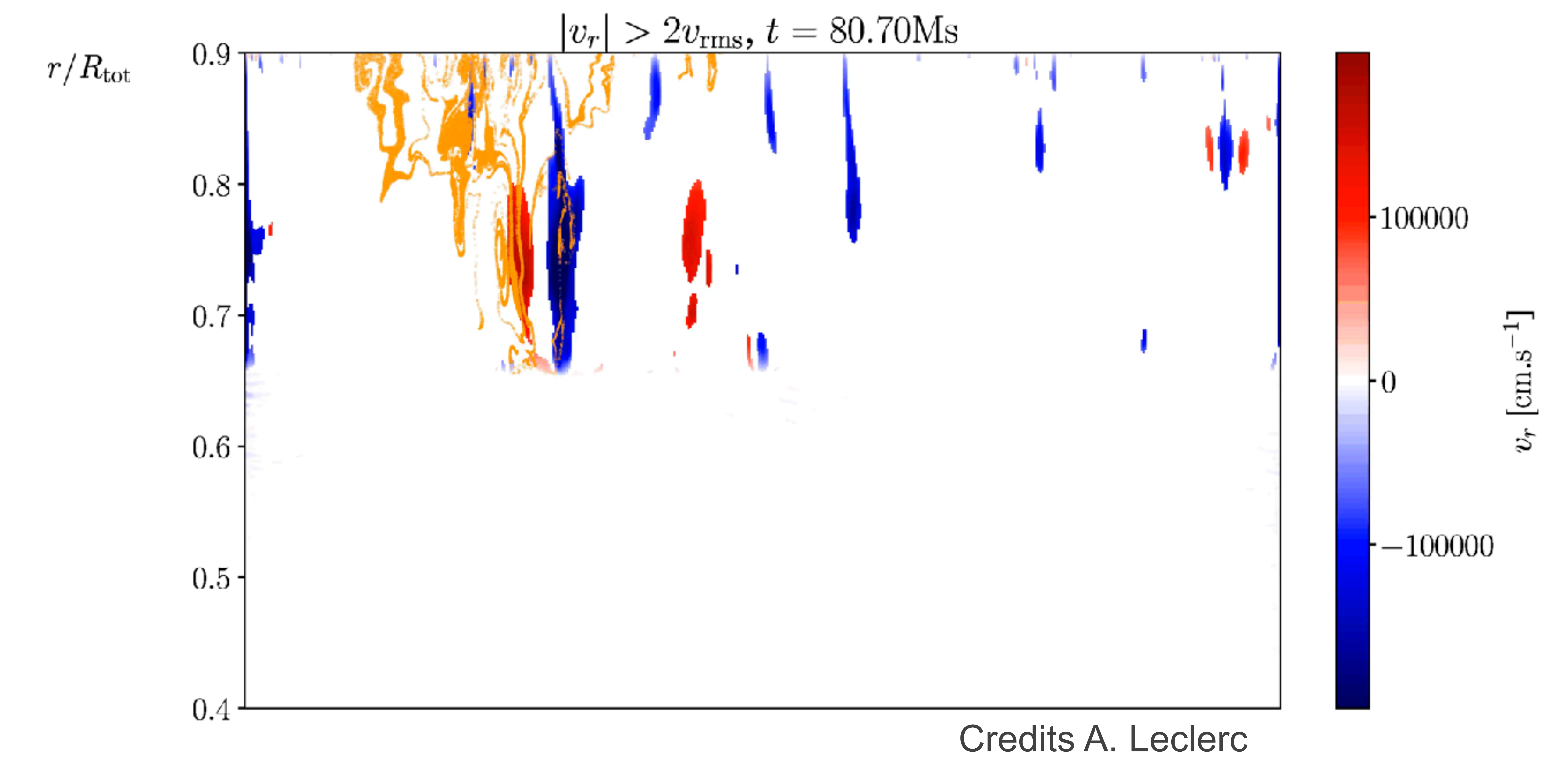
Plume characterisation

Identification of convective plumes using Lagragian particule tracers



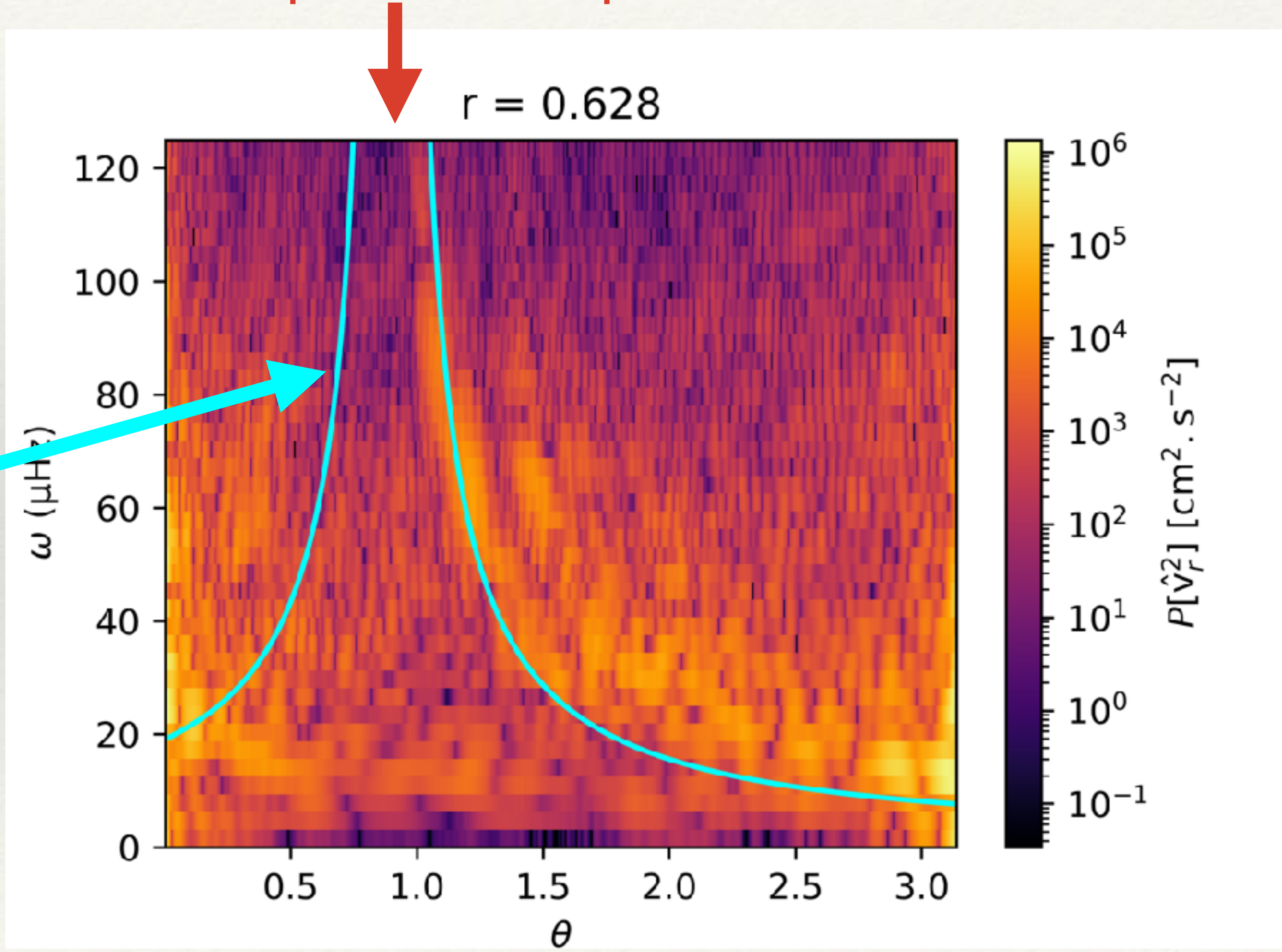
Credits A. Leclerc

IGW excitation by convective plumes

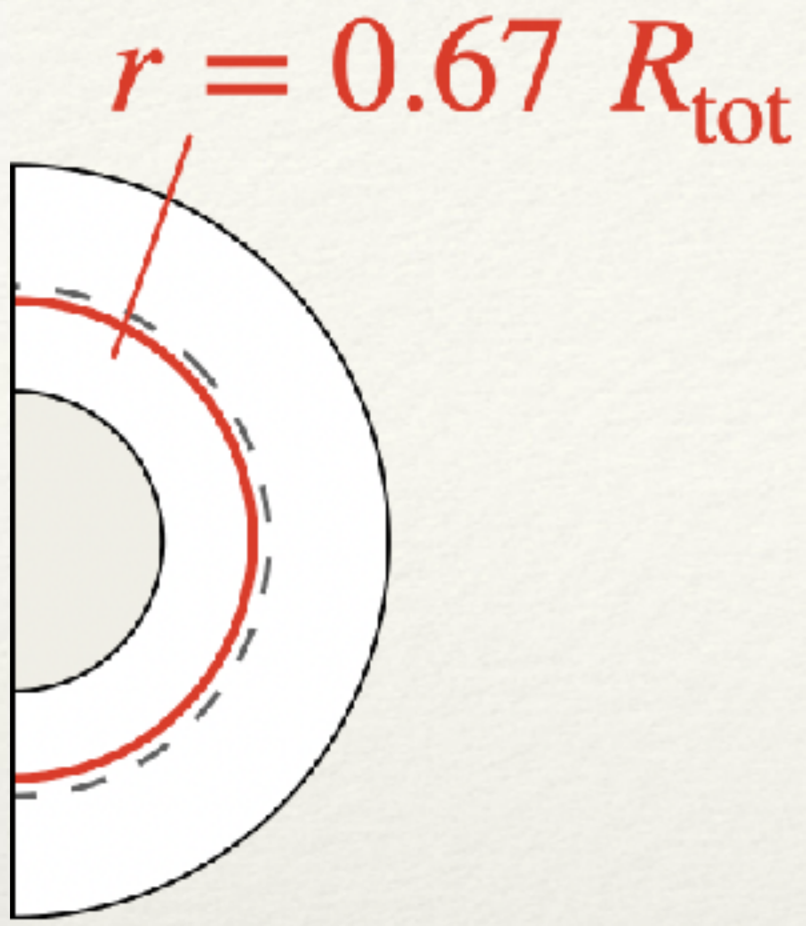
Comparison with theoretical dispersion relation for IGW

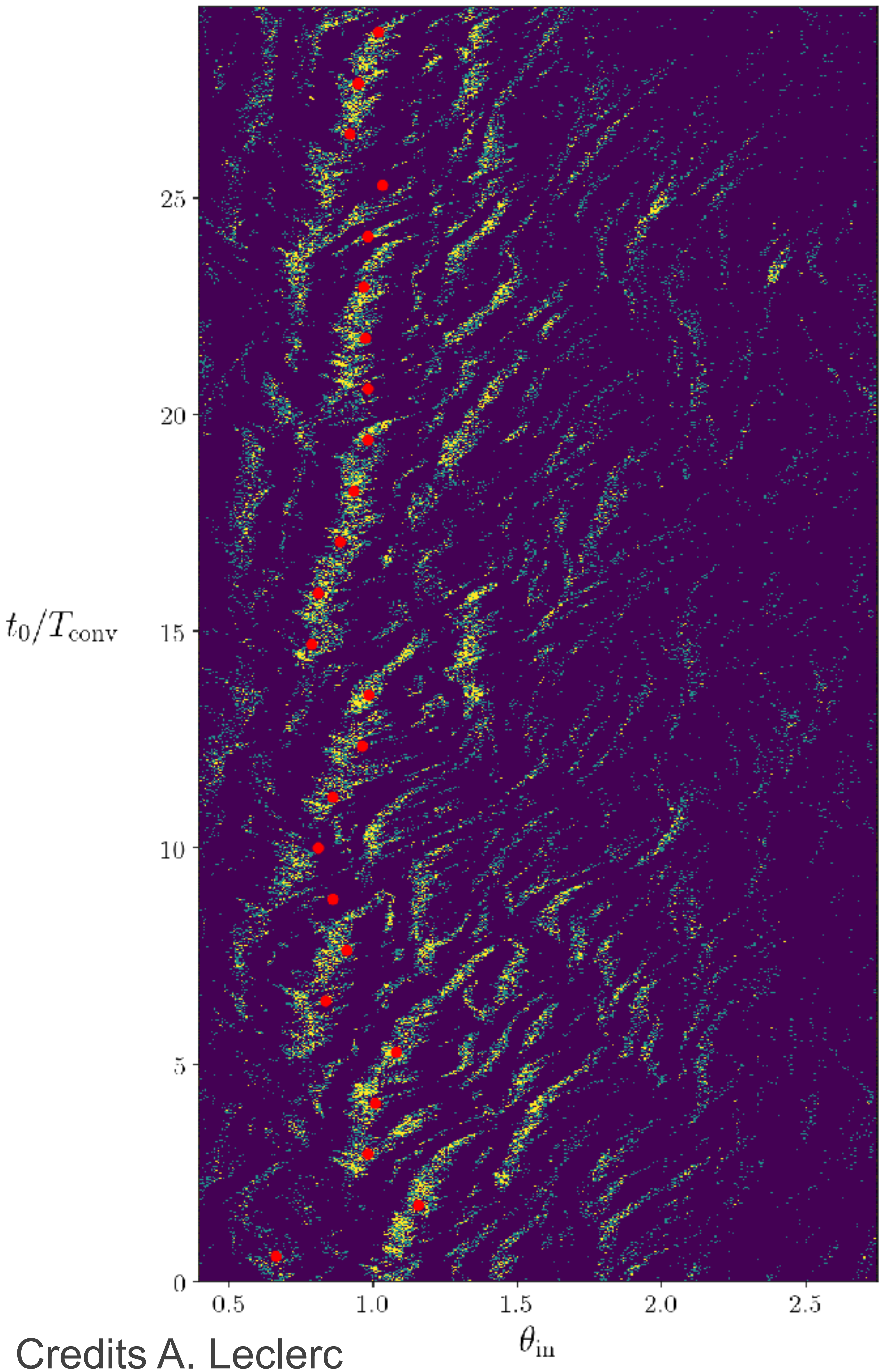
$$\frac{\omega}{N} = \pm \frac{k_h}{k} = \pm \cos(\alpha)$$

Position of the penetrative plume identified



Plume characterisation





Credits A. Leclerc

Good match between position of convective penetration and plumes excitation region

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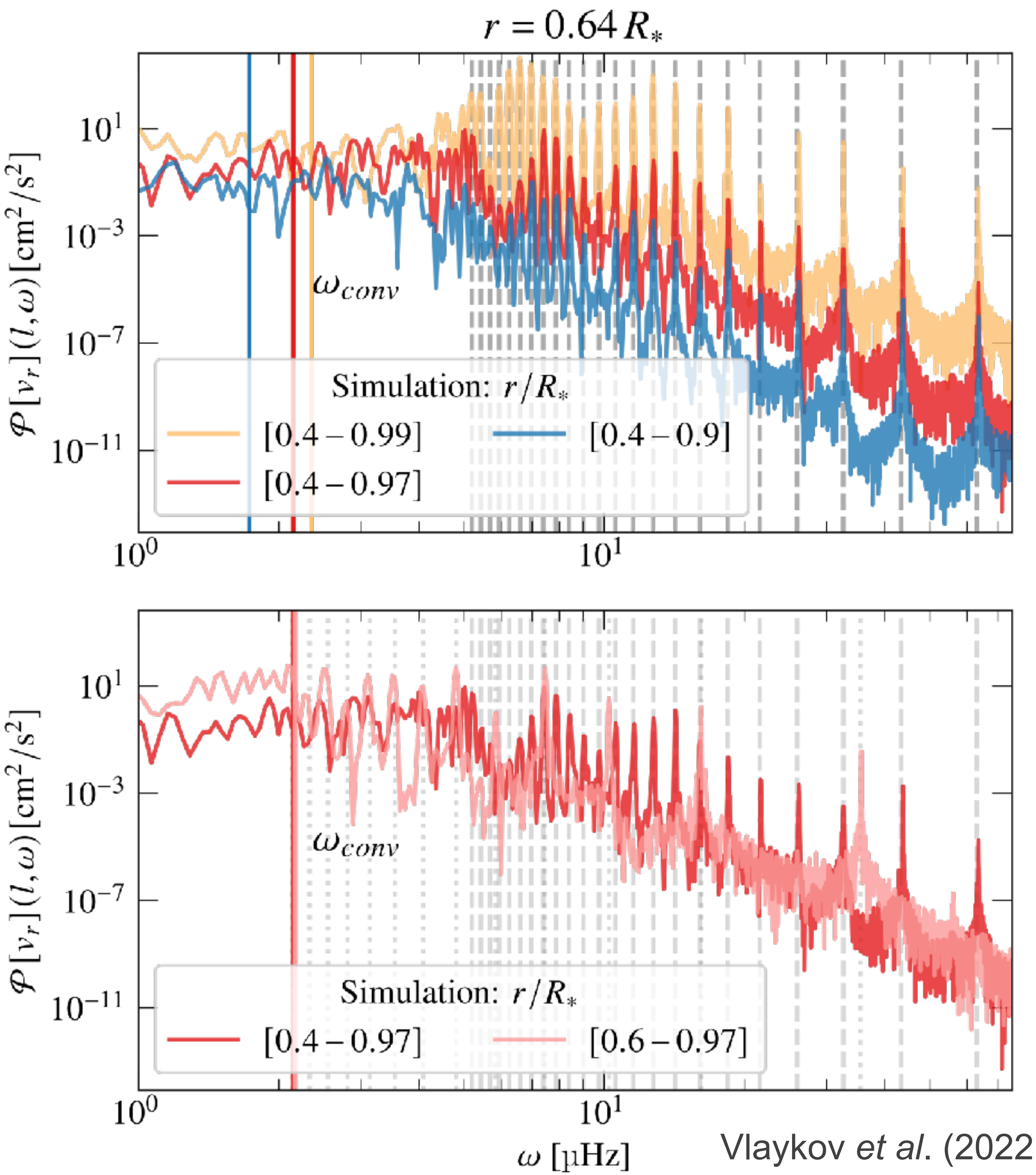
Impact of radial truncation

- Internal gravity waves (IGWs)
  - $r_{\max}$ dependence
    - ω_{conv} increases
    - slope flattens
    - amplitude grows
  - $r_{\min}$ dependence
    - change of g-mode frequency with height of resonant cavity



Vlaykov *et al.* (2022)

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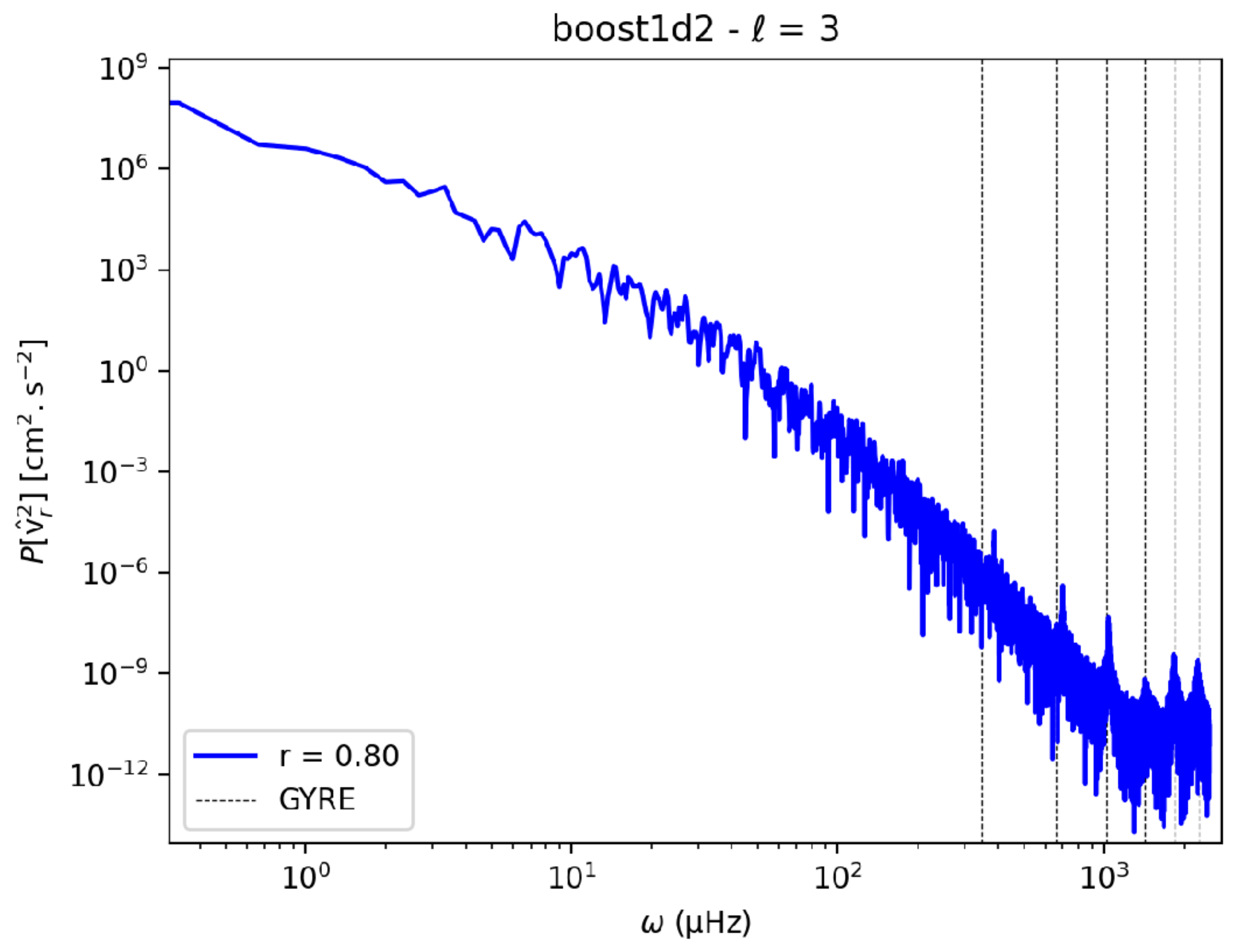
Acoustic modes

- p modes for n = 0 to 5



Ongoing work with Jane Pratt (Georgia State Uni.): model and identify mixed modes in a Red Giant star

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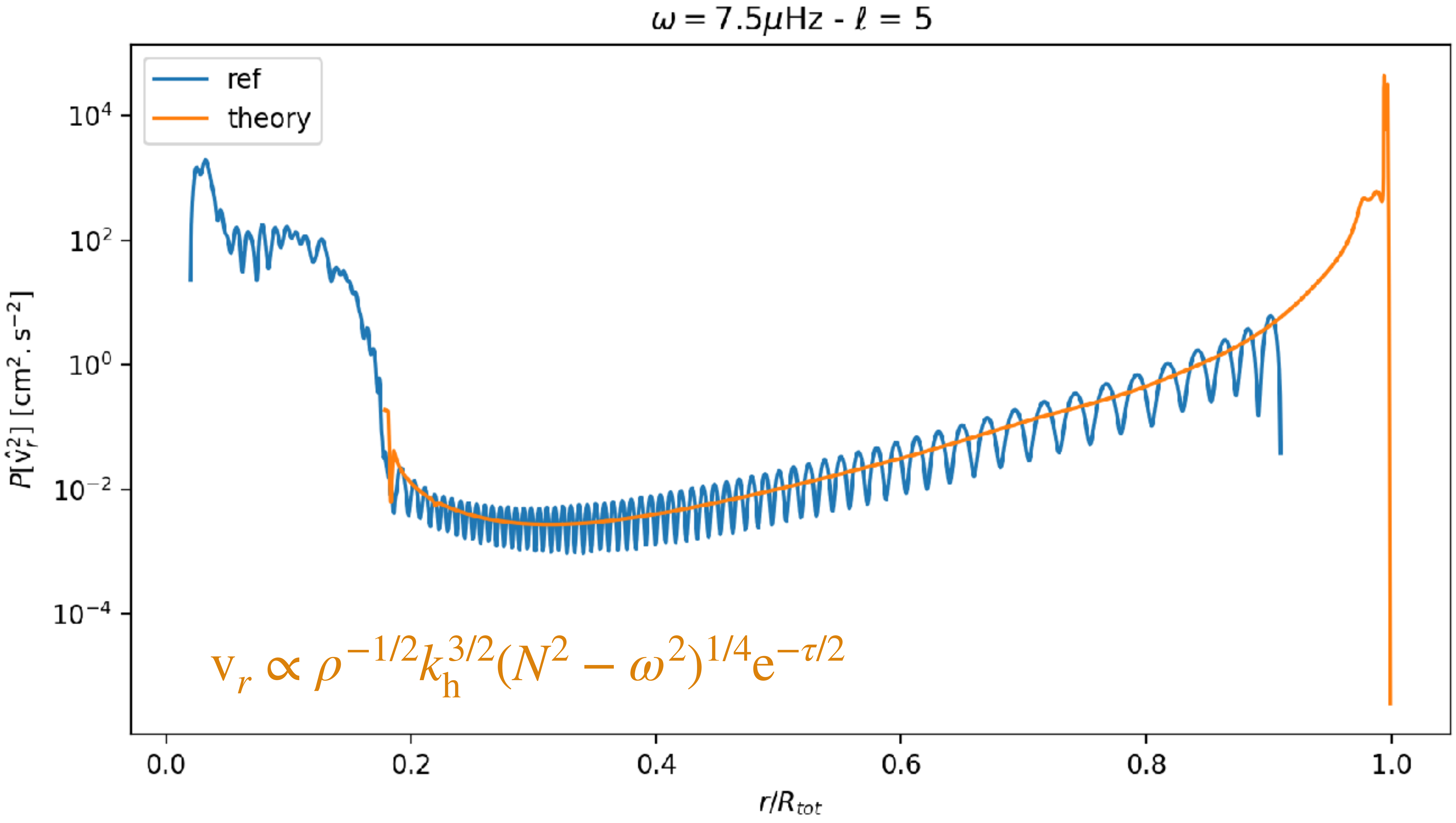
5 solar mass star

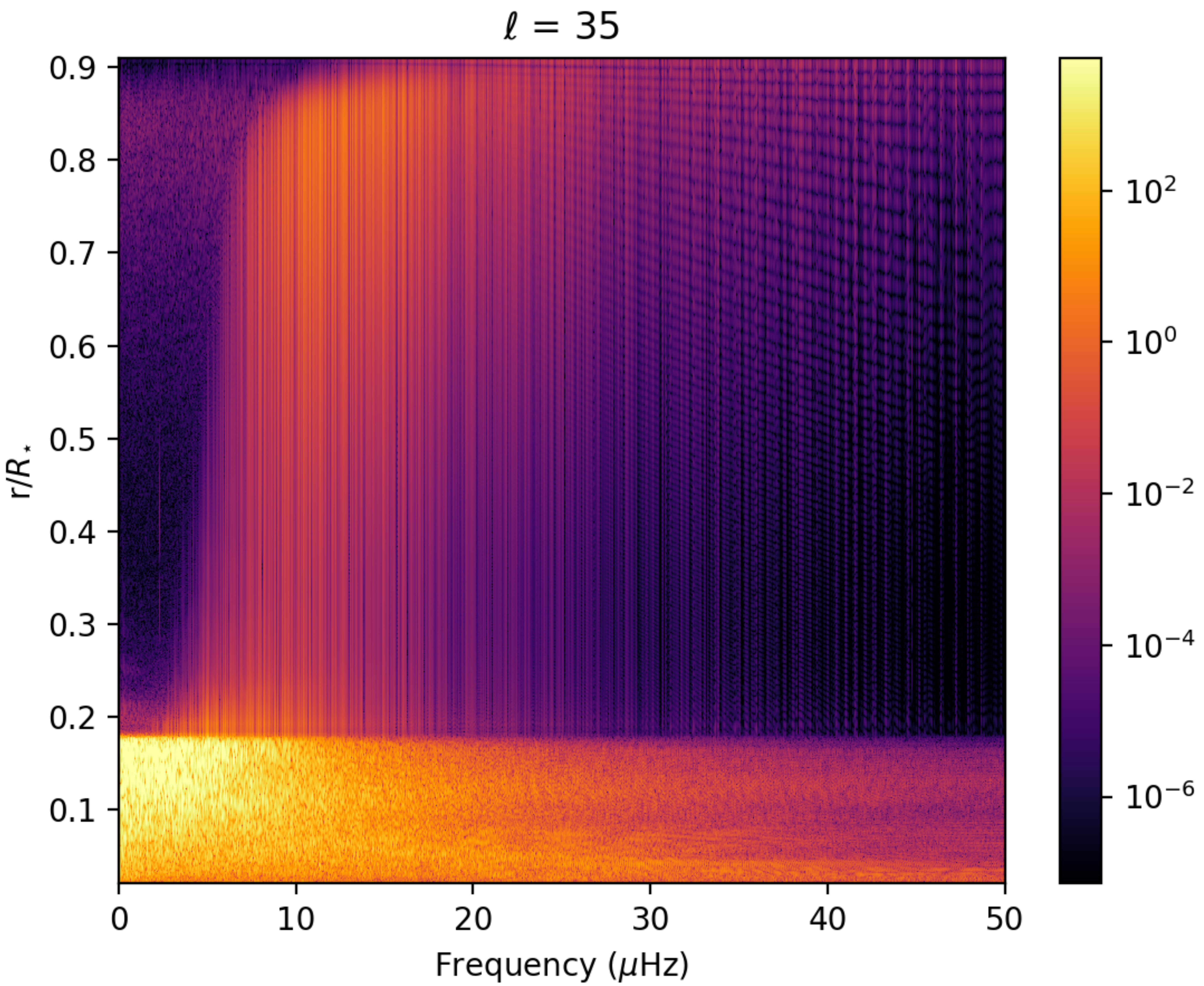
Analytical expression from Press (1981)



$$\mathrm{v}_r \propto \rho^{-1/2} k_\mathrm{h}^{3/2} (N^2 - \omega^2)^{1/4} \mathrm{e}^{-\tau/2}$$

5 solar mass star

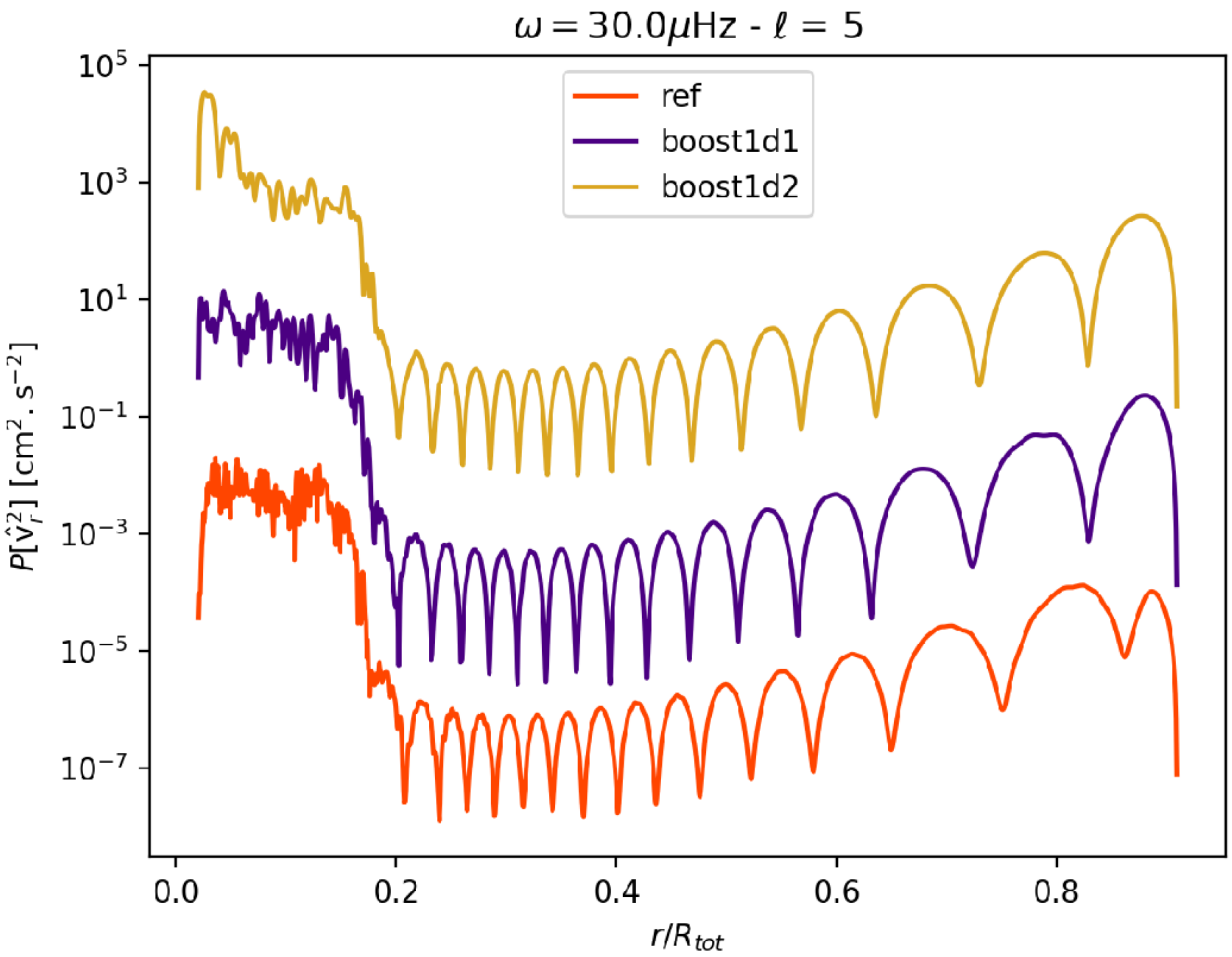
Increase of damping
from r = 0.8 Rstar



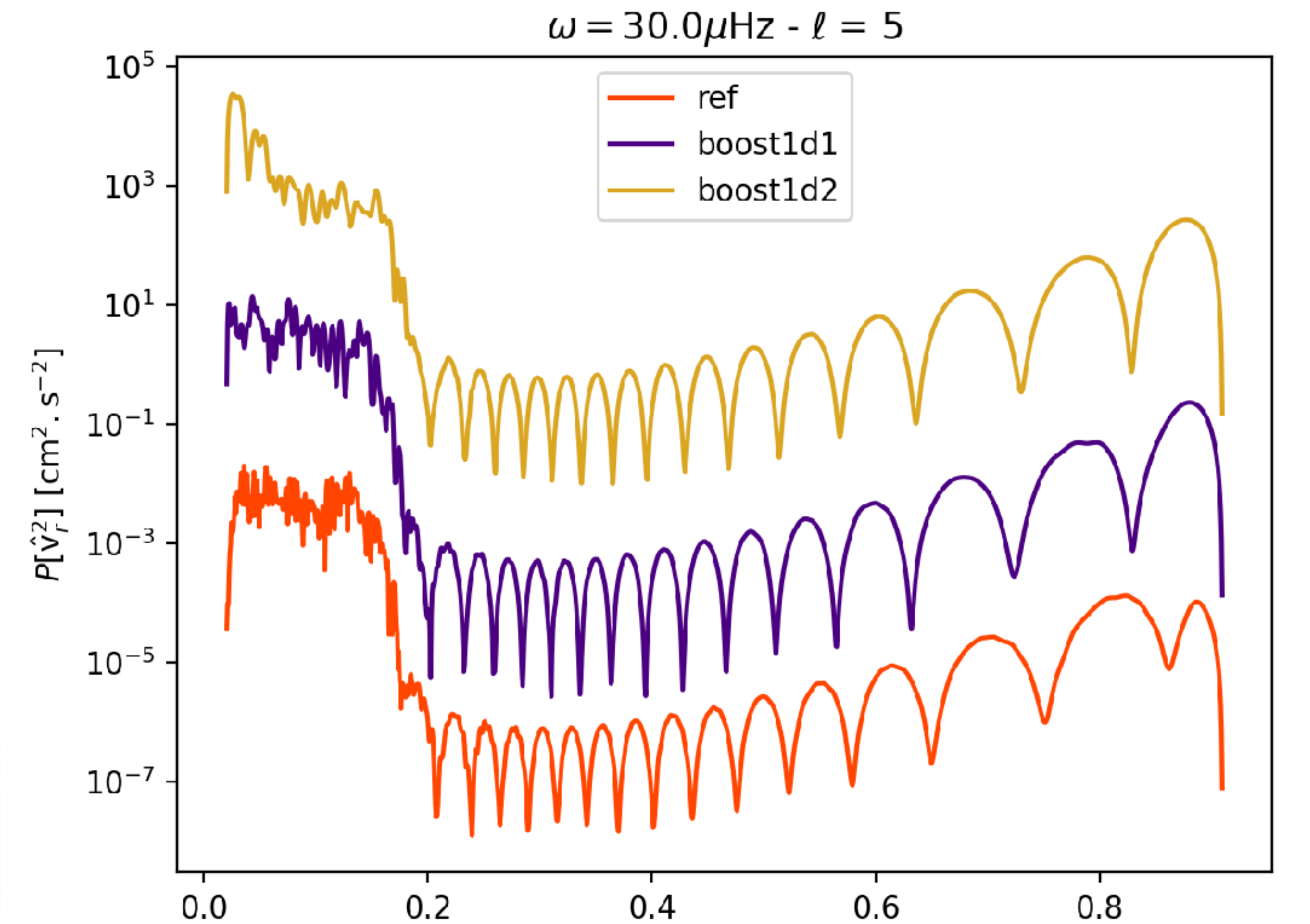
5 solar mass star

Comparison with 2 simulations with artificially enhanced luminosity: 10^1, 10^2 $L_\star$

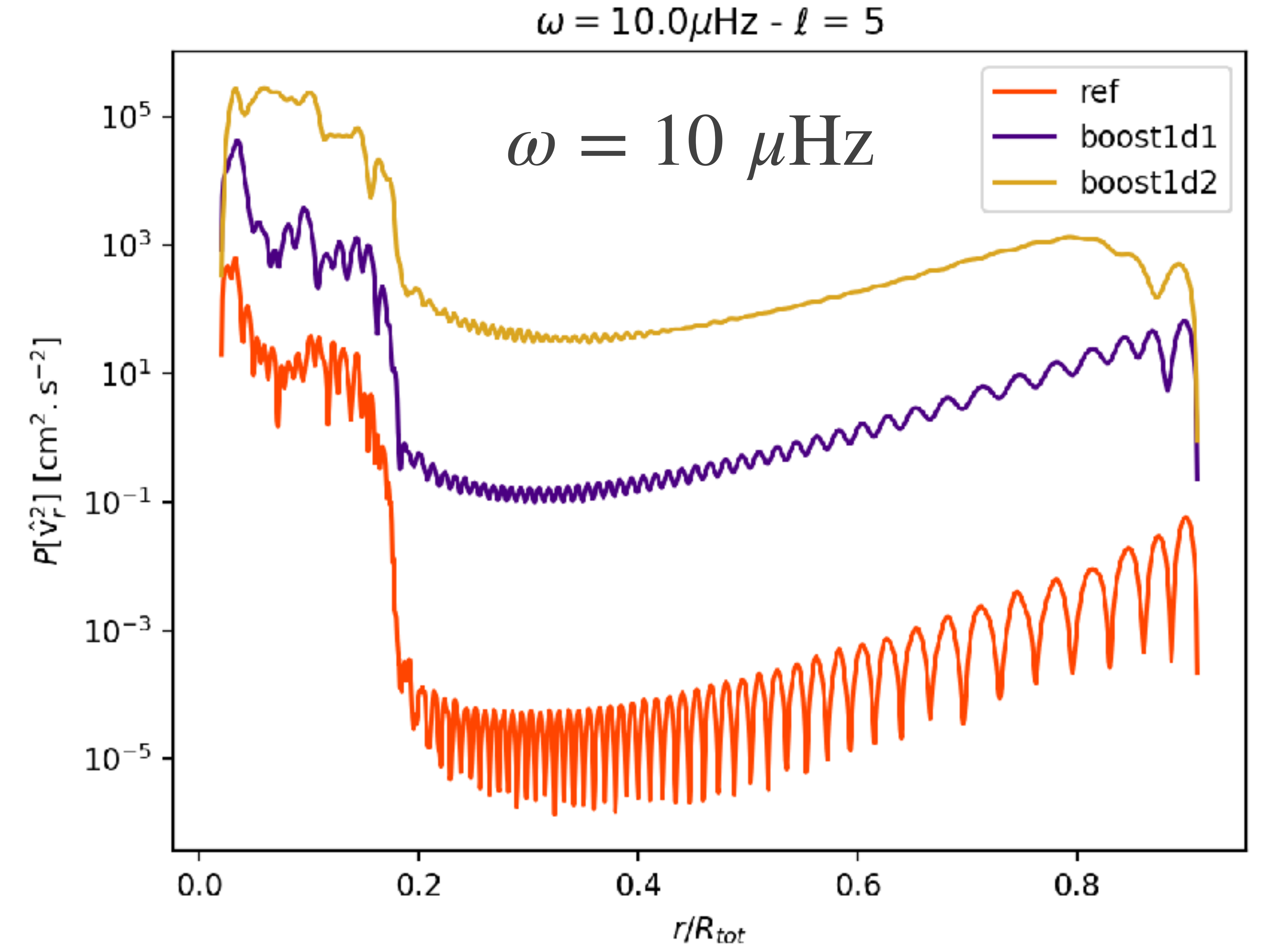
$$\omega = 30\ \mu\text{Hz}$$



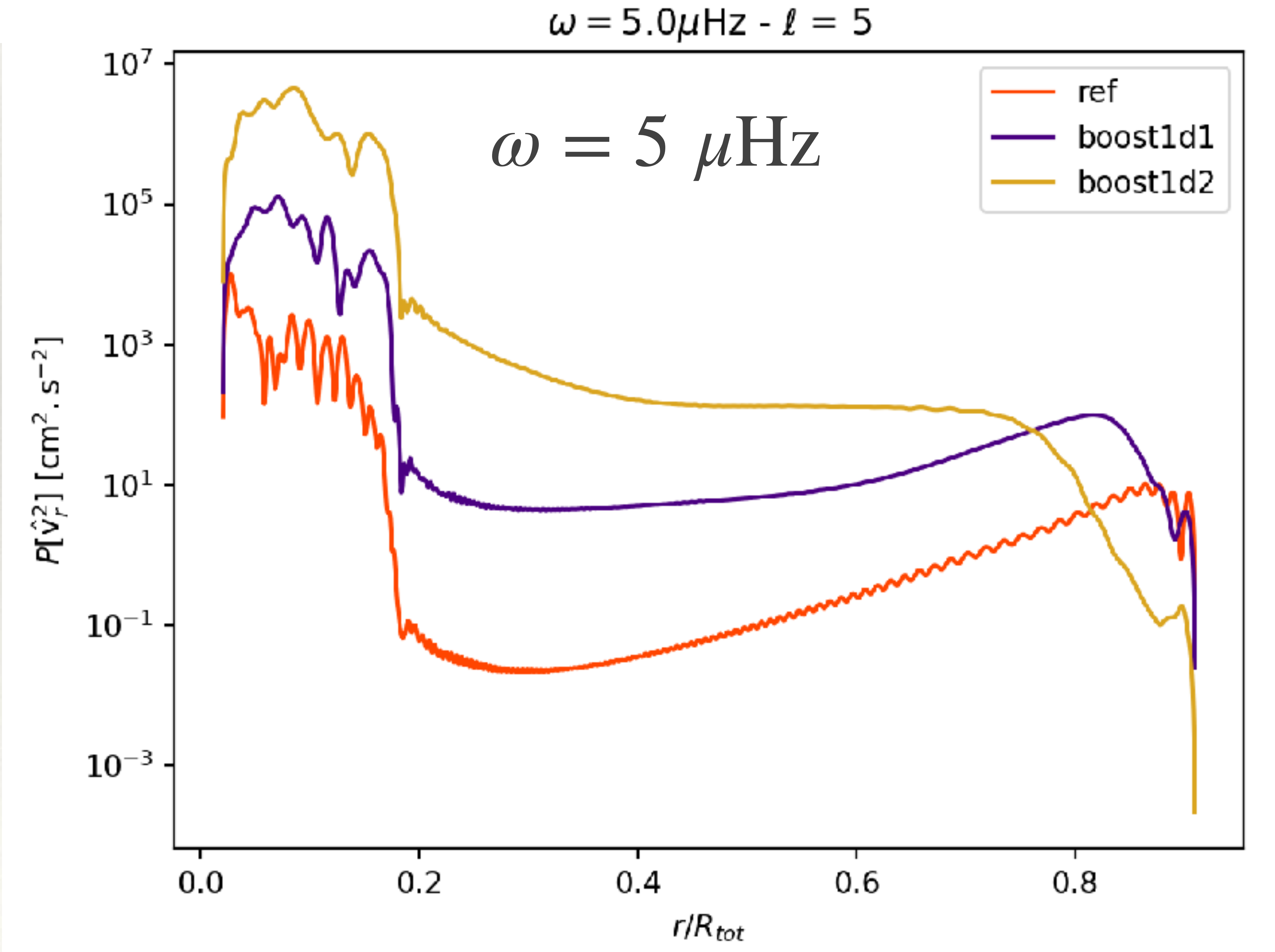
5 solar mass star



$\omega = 30\ \mu$Hz



$\omega = 10\ \mu$Hz



$\omega = 5\ \mu$Hz

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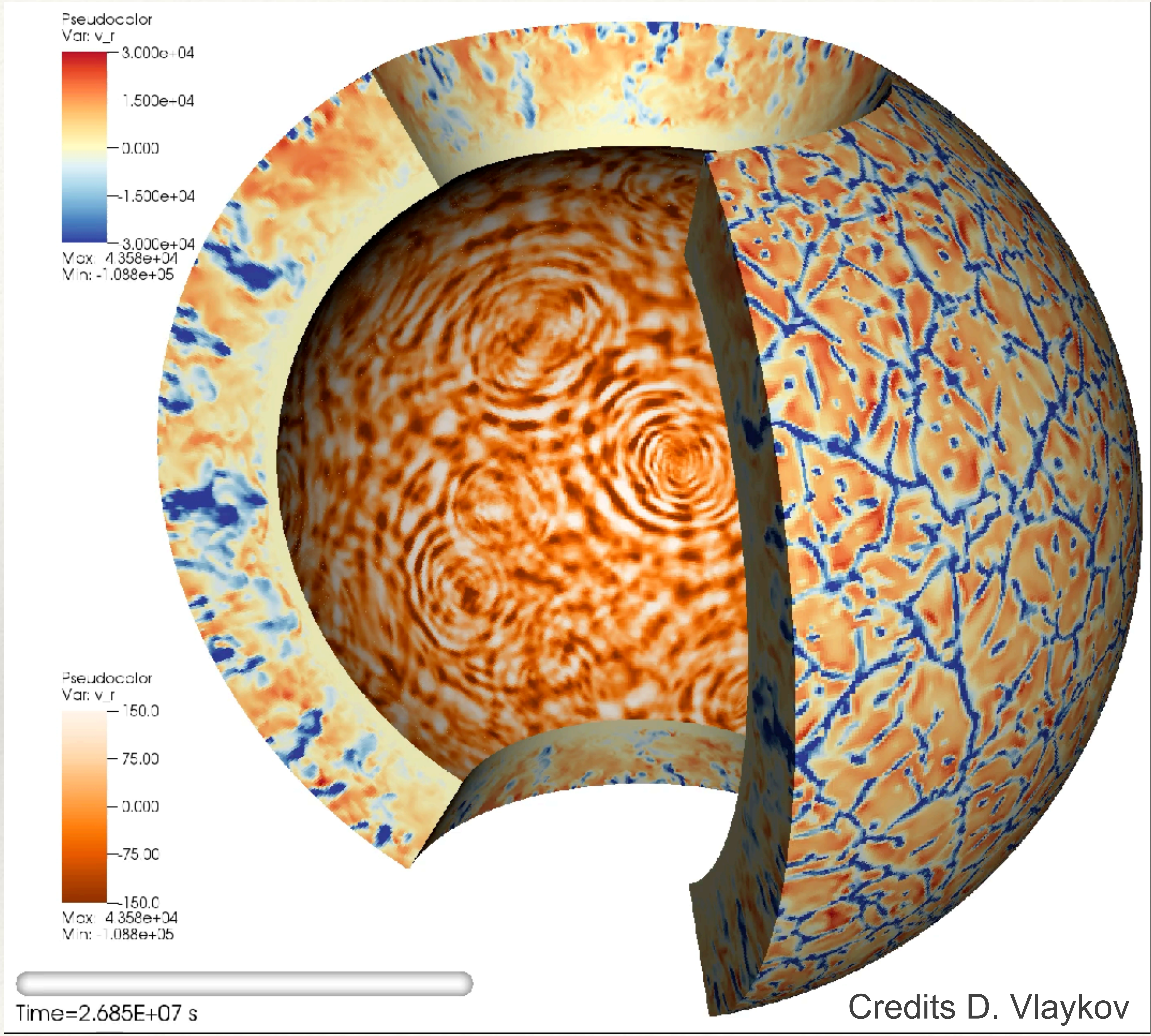
3D models

- Intrinsically 3D effects:
  - rotation
  - magnetic field
  - quantitative analysis
    - length and time scales of convective structures
    - plume lifetimes



Credits D. Vlaykov

3D models

Convection Zone

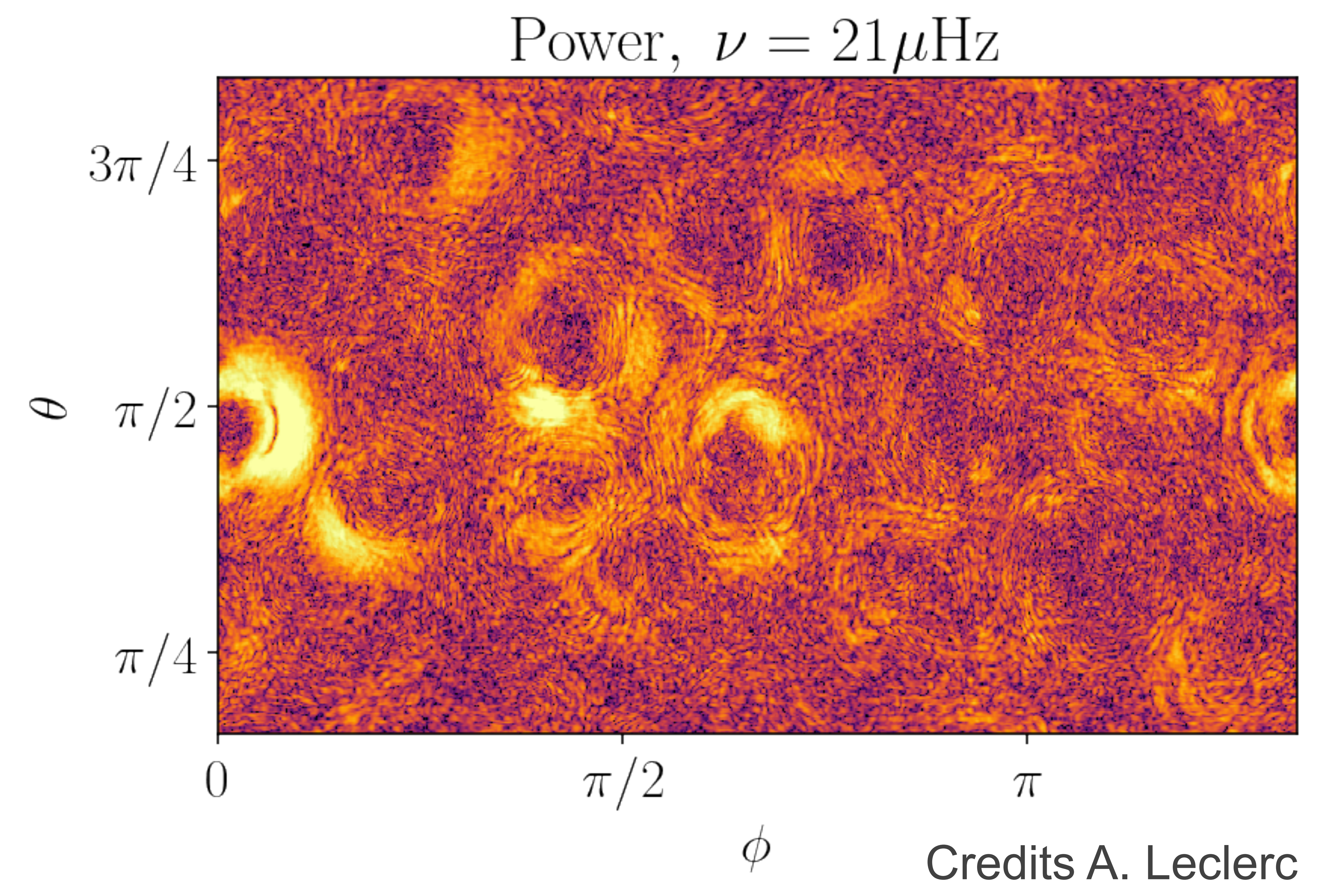
Interface

Radiative Zone

Credits A. Leclerc

3D models

Ongoing work study of IGW in a 3D solar-like star and comparison with 2D



Credits A. Leclerc

Summary

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For more information:
Le Saux *et al.* (2022), A&A 660, A51
Vlaykov et al. (2022), MNRAS 514, 1
Baraffe *et al.* (2021), A&A 654, A126
Baraffe *et al.*, (2022), A&A 659, A53