Code & stellar physics workshop:

Study of intensity spectrum from rapidly rotating stars

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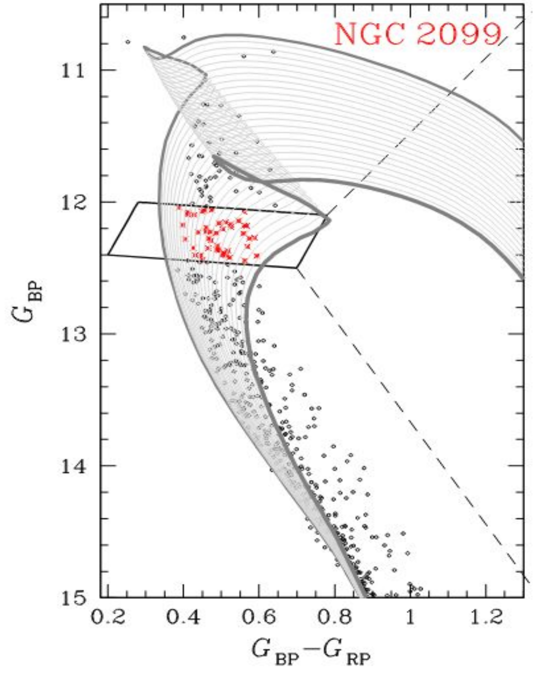
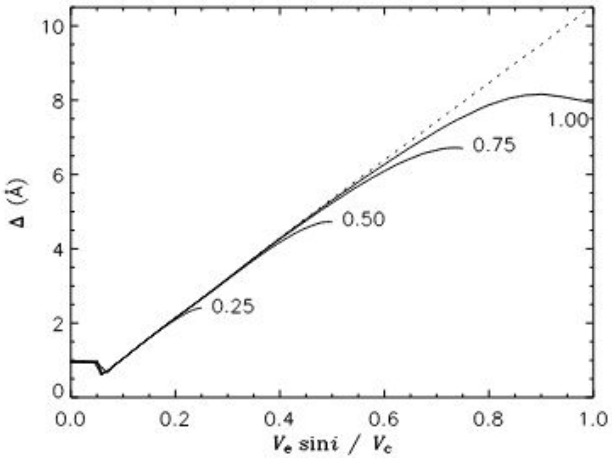
State of the art

1D models

G. Cordonni et al. (2018)
Broadening of the isochron in the vicinity of the turn-of



Townsend (2004) the equatorial projected velocity is underestimated



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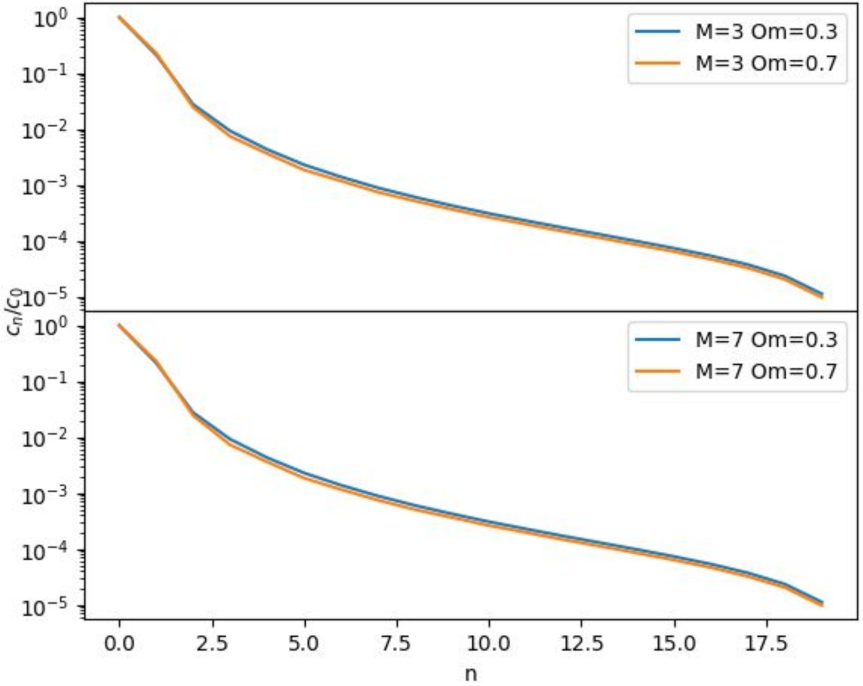
ESTER : robustness of the latitudinal profils



Interpolation of (Teff,logg,R) by Legendre polynomials

=> Polynomials degree correspond to latitudinal resolution of ESTER nθ

Choose nθ such as the interpolated Teff, ∀θ | Teff(θ,nθ)-Teff(θ,20) |/Teff(θ,20) < 1

PHOENIX

=> Solves Radiative Transfer Equation (RTE) under Local Thermodynamic Equilibrium (LTE) and non-LTE (NLTE) assumption(s).

=> Solves Saha-Boltzmann, charge conservation and statistic equilibrium equations

Assumptions: - Spherical symmetry (Matching with plane-parallel when atmosphere thickness<< radius)

- Stationary state

=>Pros: - Give access to I(λ,μ) taking Limb-Darkening (LD) into account.

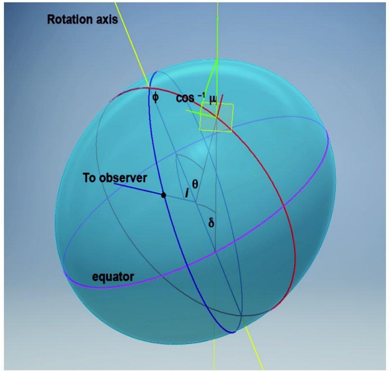
=>Cons: - Don’t take Gravity Darkening (GD) into account

plane-parallel geometry: $\boxed{\frac{dI_\nu}{ds} = -\mu \frac{dI_\nu}{dt}}$ with $dt = - \quad \mu$

spherical geometry:

$$\frac{dI_\nu}{ds} = \frac{\partial I_\nu}{\partial r}\frac{dr}{ds} + \frac{\partial I_\nu}{\partial \mu}\frac{d\mu}{ds}$$

$$\Rightarrow \boxed{\frac{dI_\nu}{ds} = \frac{\partial I_\nu}{\partial r}\mu + \frac{\partial I_\nu}{\partial \mu}\frac{1-\mu^2}{r}}$$



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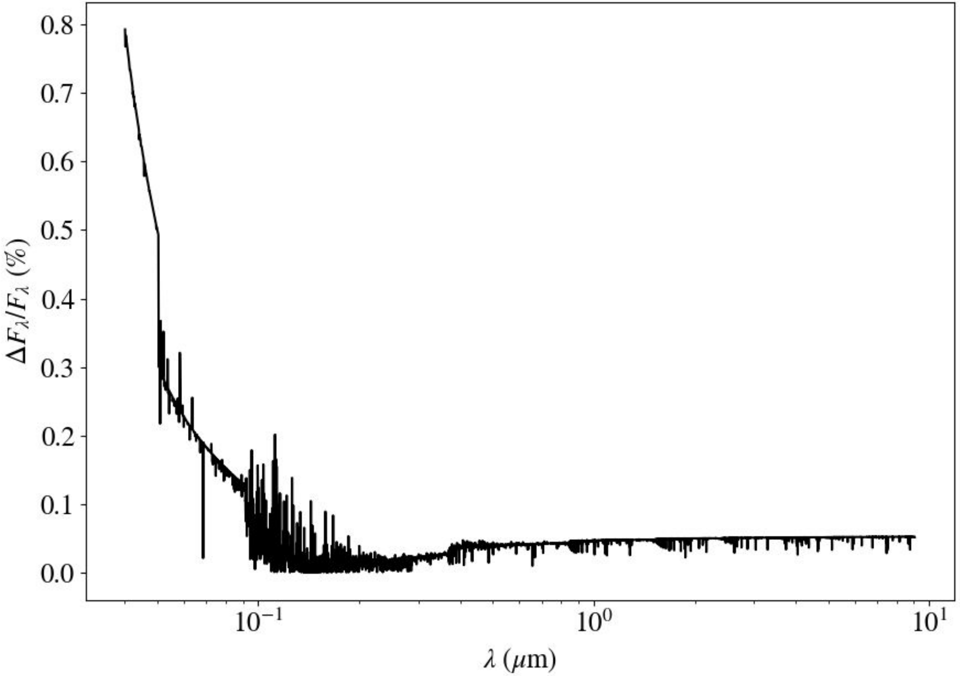
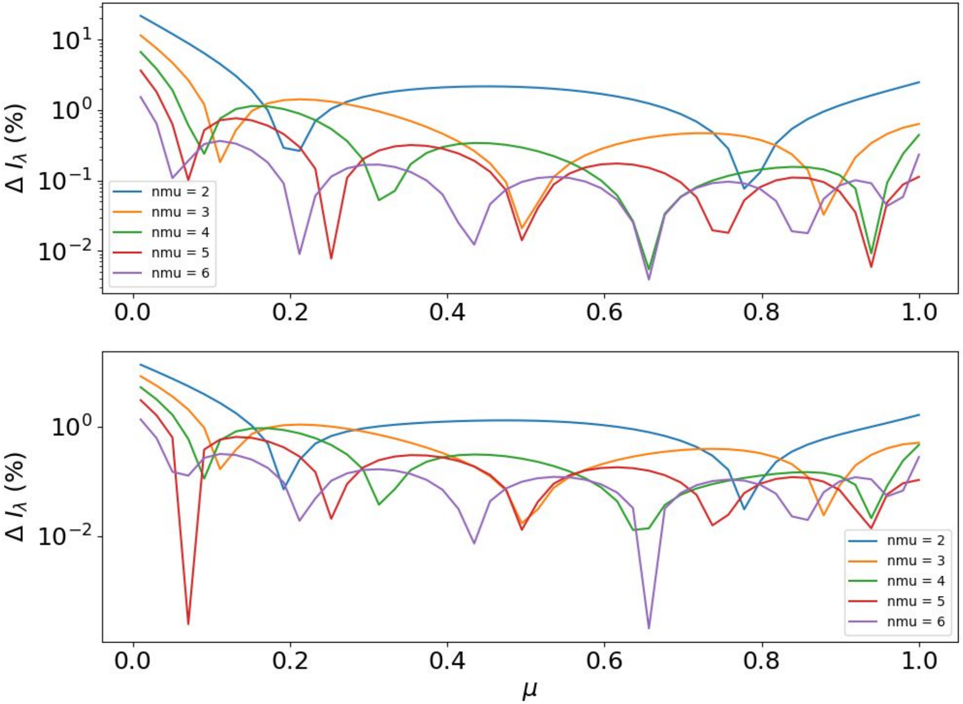
PHOENIX : Parallel plan or spherical model ?



The relative radiative flux difference between a plane-parallel and a spherical model is below 0.1 % in the wavelength interval we use.
We, thus, have done all the calculations with the plane-parallel geometry to reduce the
computing time

PHOENIX : Gauss grid



- N_μ= 4 ensure a numerical precision better than 5% everywhere (less than 1% mostly)

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Rebuilding spectrum:
Spatial grid

$$r_\theta(\theta_k) = \partial r/\partial\theta$$

$$\Delta\phi = 2\pi/N_\phi(\theta_k)$$

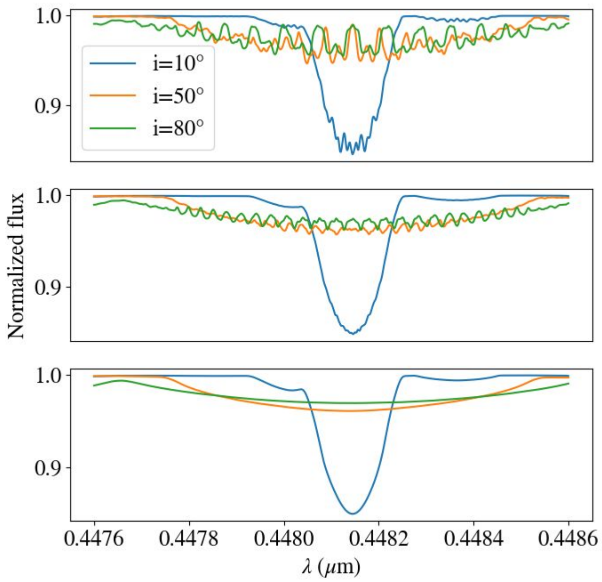
- Surface element: $\Delta S(\theta_k) = r^2(\theta_k)\sqrt{1 + \frac{r_\theta^2(\theta_k)}{r^2(\theta_k)}}\sin\theta_k \Delta\theta\Delta\phi$

=> All elements must have the same ΔS in order to avoid an overestimation of polar contributions and an underestimation of the equatorial ones.
=> We thus define our grid with N_Θ and $N_\phi(\Theta_0)$

- Visible part of the grid : μ>=0

$$\mu(\theta_k, \phi_j) = \frac{1}{\sqrt{r^2 + r_\theta^2}}\left[(r\sin\theta_k - r_\theta\cos\theta_k)\cos\phi_j \sin i + (r\cos\theta_k + r_\theta \sin\theta_k)\cos i\right]$$

Rebuilding spectrum: Spatial grid



$(N_\Theta, N_\phi(\Theta_0))=(10,10)$

$(N_\Theta, N_\phi(\Theta_0))=(20,10)$

$(N_\Theta, N_\phi(\Theta_0))=(100,20)$

- Oscillations du to a lack of spatial resolution which lead to a bad mapping of Doppler effect.

=> Optimal N_Θ and $N_\phi(\Theta_0)$ depend on mass , rotation and inclination

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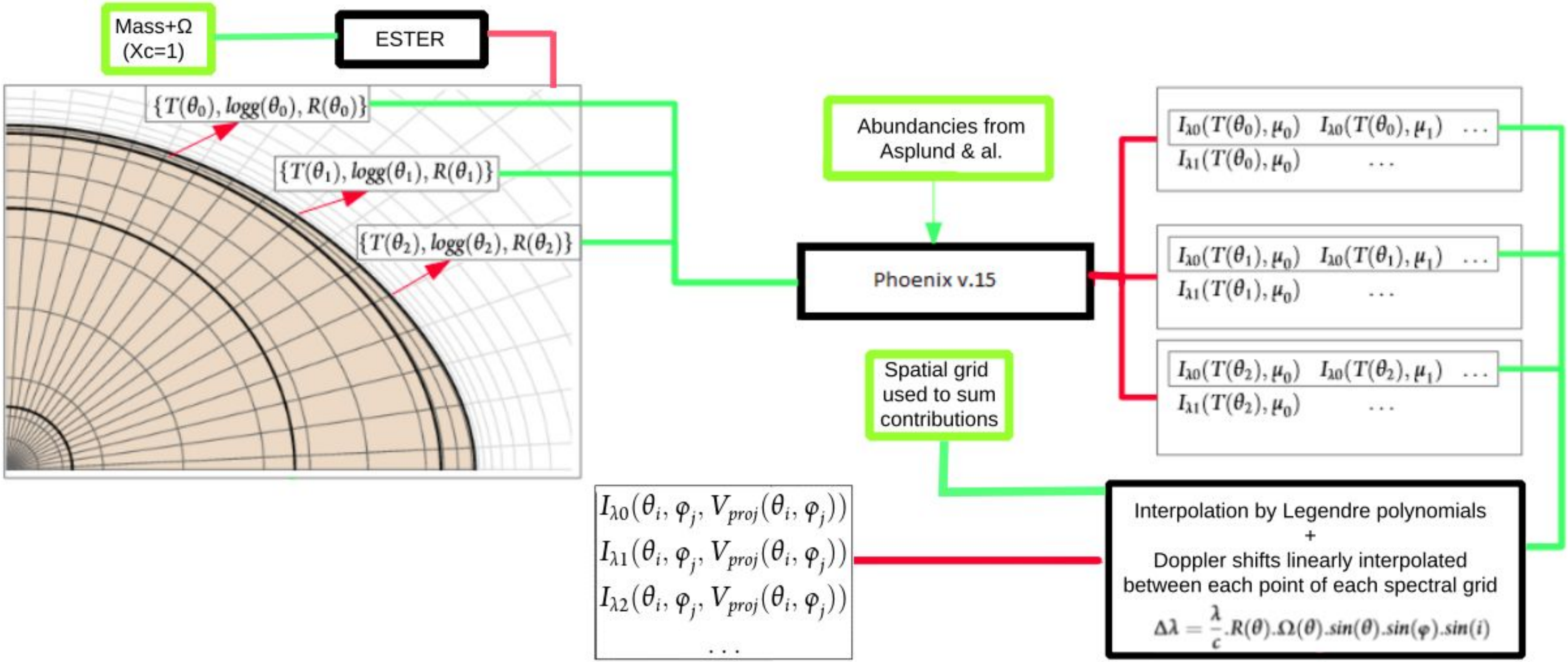
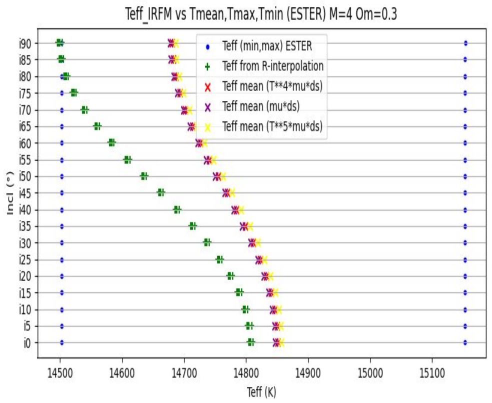
Rebuilding spectrum:



Possible applications

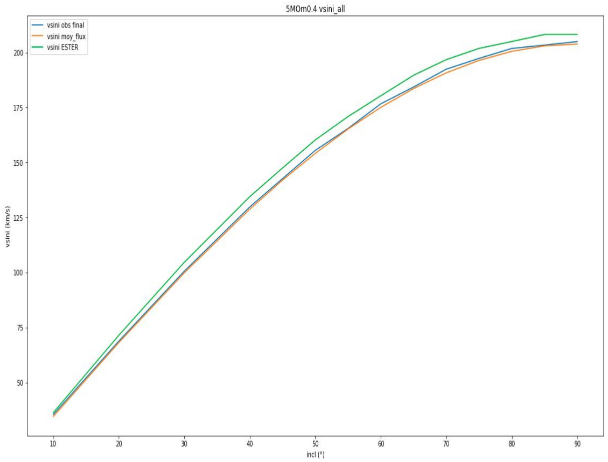
- Photometry:

  Exemple : Study the variation of an observable according the inclination.



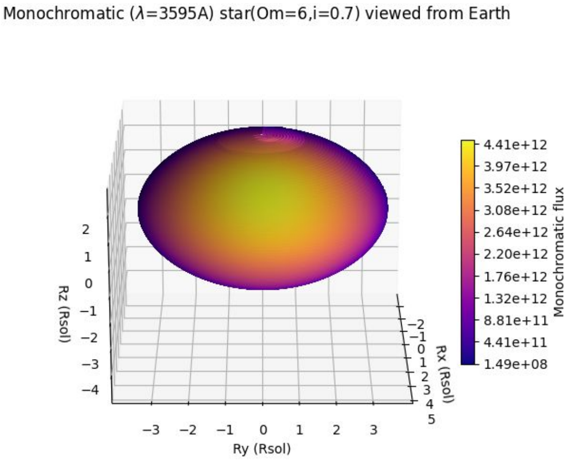
- Spectrometry:

  Exemple : Study of a measured equatorial velocity



- Interferometry:

  Exemple : Study of monochromatic maps of the st

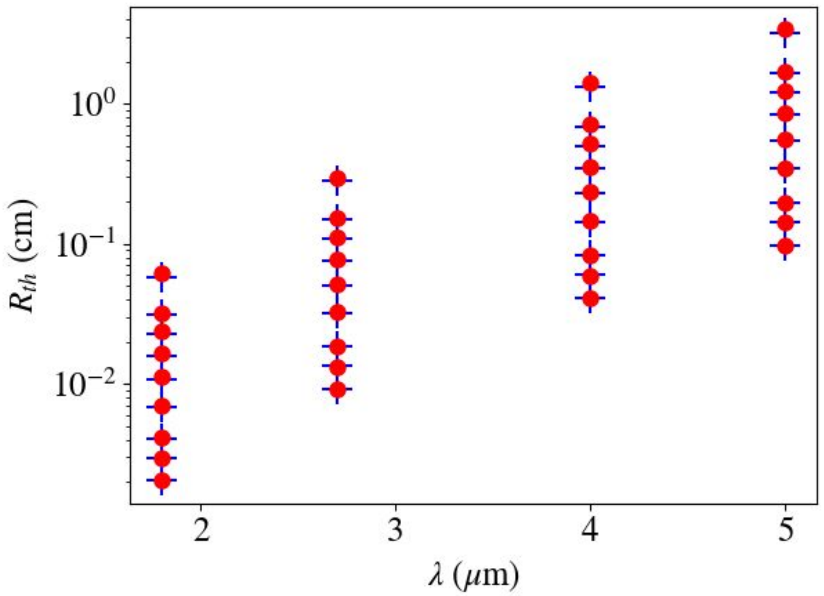


InfraRed Method Flux:
IRFM

Method based on the study of the following ratio:

$$R = \frac{\int_0^{\infty} F_{\lambda} d\lambda}{F_{IR}}$$

with an IR wavelength>2μm to be in the Rayleigh domain.



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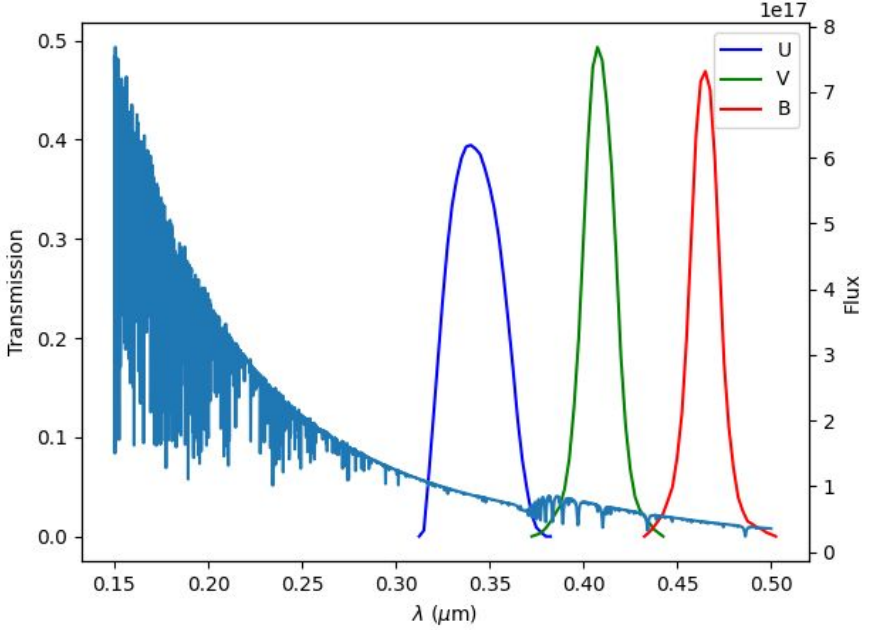
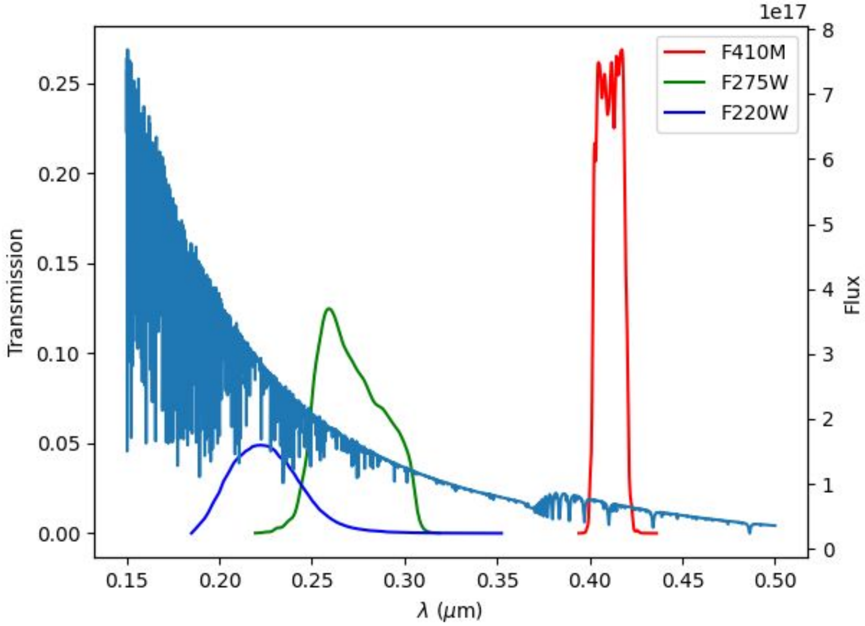
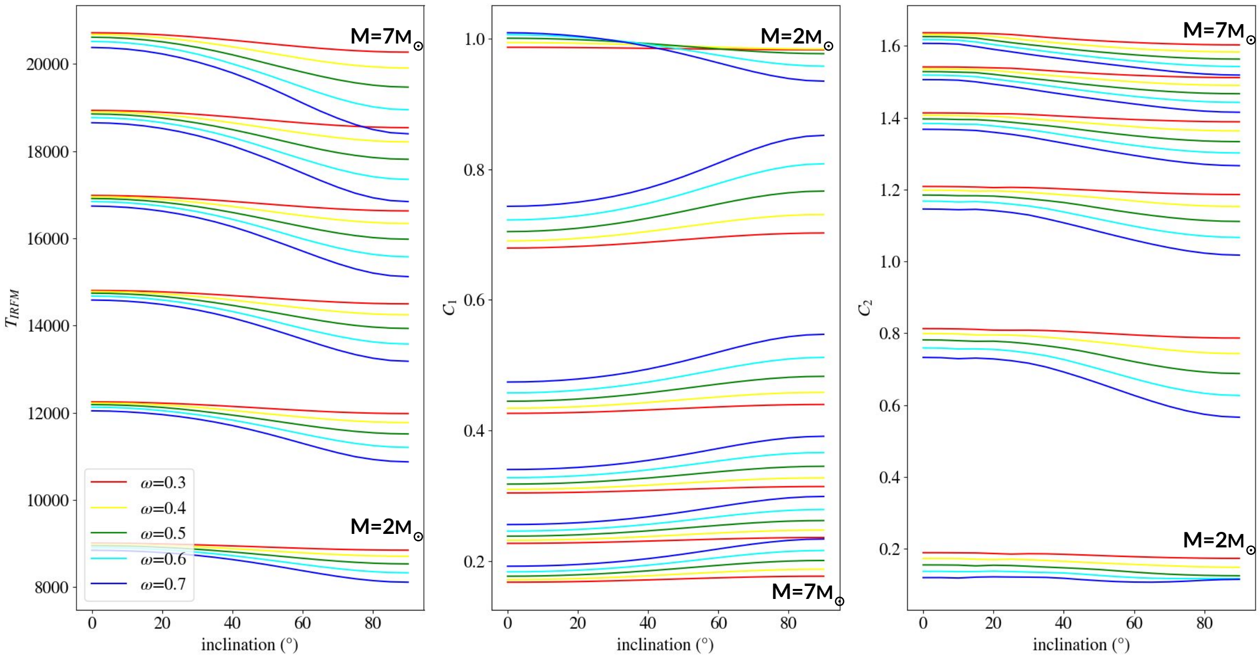
Color indexes





Tricubic interpolation



$$T_{IRFM}(M,\Omega,i) = \sum_{k=1}^{n_M}\sum_{l=1}^{n_\omega}\sum_{m=1}^{n_{\sin i}} a_{k,l,m}^{IRFM} B_{k,l,m}^{IRFM}(T_{\rm sph},\omega,\sin i)$$

$$c_1(M,\Omega,i) = \sum_{k=1}^{n_M}\sum_{l=1}^{n_\omega}\sum_{m=1}^{n_{\sin i}} a_{k,l,m}^{c_1} B_{k,l,m}^{c_1}(T_{\rm sph},\omega,\sin i)$$

$$c_2(M,\Omega,i) = \sum_{k=1}^{n_M}\sum_{l=1}^{n_\omega}\sum_{m=1}^{n_{\sin i}} a_{k,l,m}^{c_2} B_{k,l,m}^{c_2}(T_{\rm sph},\omega,\sin i)$$

=> Interpolation by Tricubic splines

$$T_{sph} = T_p\left(1 - \sum_{k=1}^{n_M} a_k B_k(\omega)\right)$$

=> Orthogonalization of the parameter space

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Initialisation :

Star

L_{app}

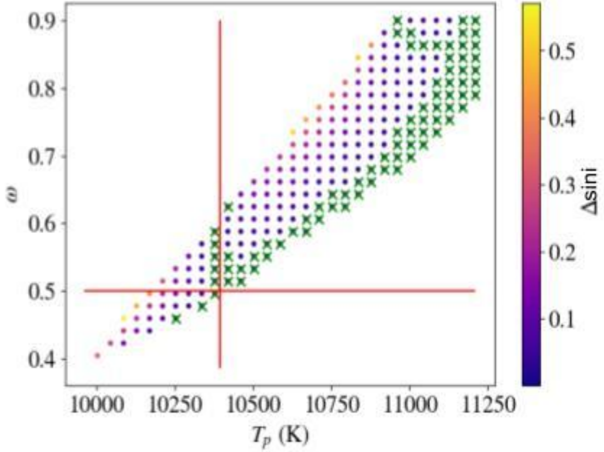
$$T_p(L_{app}) = T_0\left(\frac{L_{app}}{L_\odot}\right)^\beta \pm 10\%$$

Initialization grid

$$\mathcal{G} = \begin{cases} T_p \in [T_p(L_{app})(1-0.1); T_p(L_{app})(1+0.1)] \\ \omega \in [0, 0.9] \end{cases}$$

Conditions:

- Observed values lie in the latitudinal profile
- Inclination determined by piecewise linear interpolation over the 3 profiles associated have too match



Newton method

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Resolution : feasibility map

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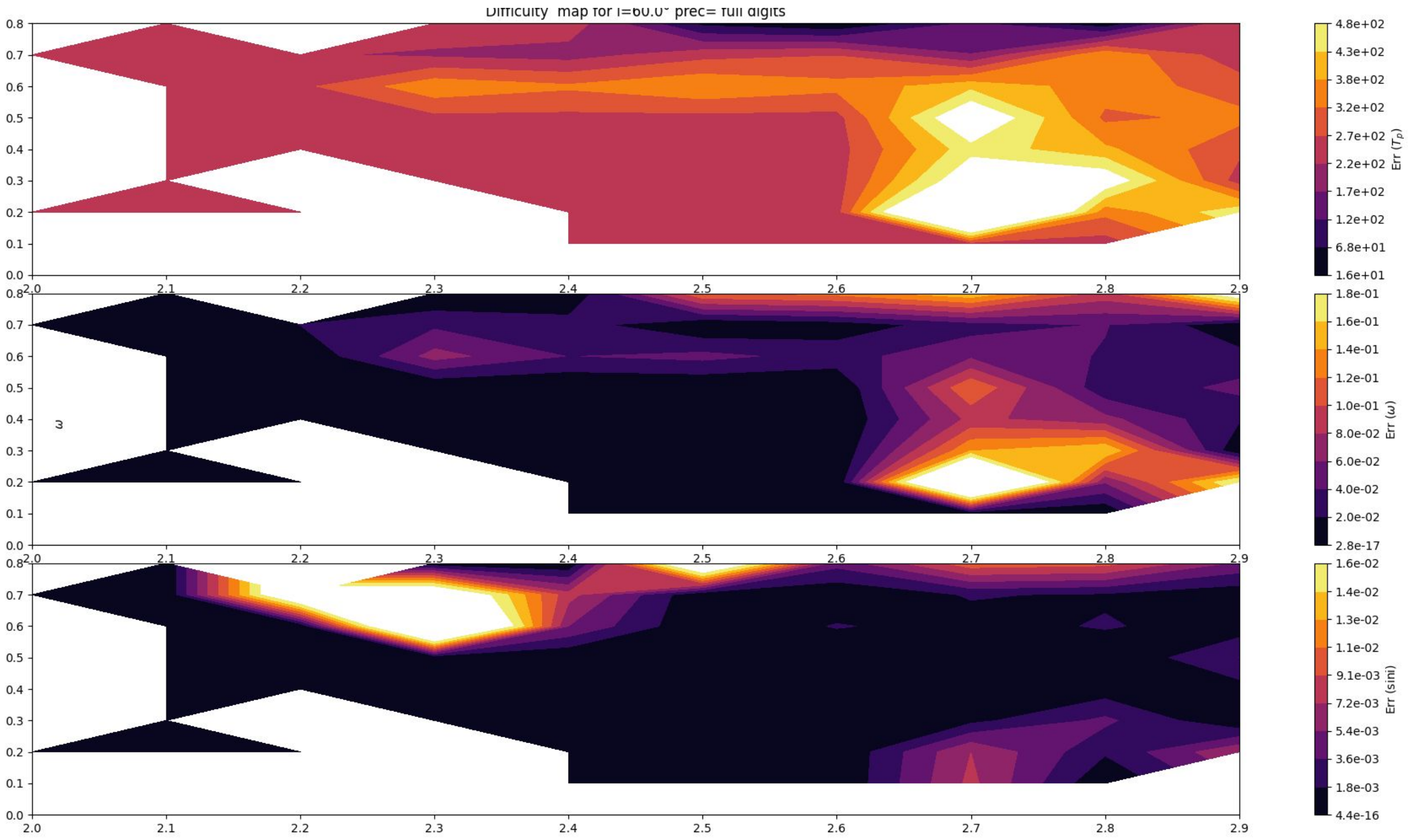
Resolution : error map



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Application to Vega

Grid of models between M=[2 ; 2.9]$M_\odot$ ω=[0 ; 0.8] X_c= 0.268

Vega model :

- -M = 2.36 $M_\odot$
- - ω = 0.51
- - i = 6°

Observable from Vega :

- -T_{IRFM}= 9553 K (Ciardi & al 2001)
- -C_1 = 1.089 (Crawford & Barnes 1970)
- -L_{app} = 40.12$L_\odot$ (Yoon & al 2010)
- -C_2 = 0.681 (Compute from the concordance model from Lara & Rieutord 2013)

Solution:

Tp : 10073 K

Omega : 0.48

sini : 13°

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Thanks for your attention !

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