The ESTER project: modelling fast rotating stars

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Why should we make 2D-models ?

To deal properly with rotation !

Rotation means

- non spherical stars
- baroclinic flows in radiative region
- anisotropic convection

We note that

- 1D rotating models are valid when $\Omega \to 0$
- A lot of physics is condensed inside adjustable (transport) coefficients
- 1D models are not usable in asteroseismology of rapid rotators
- New data from optical/IR interferometry require a 2D view...

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Interferometry : Achernar

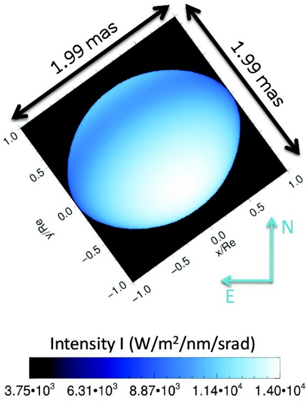

FIGURE – Achernar with VLTI (Domiciano de Souza et al. 2014, AA **569**)

Interferometry : Altair

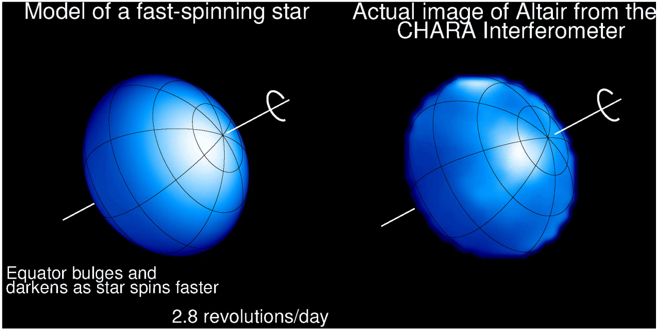

FIGURE – Altair seen by CHARA (Monnier et al. 2007).

An idealization/simplification

- We consider a lonely rotating star
- We are interested in long time-scales
- We discard all magnetic fields.

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$$
\begin{cases}
\Delta\phi = 4\pi G\rho \\
\rho T \vec{v}\cdot\vec{\nabla} S = -\mathrm{Div}\vec{F} + \varepsilon_* \\
\rho(2\vec{\Omega}_* \wedge \vec{v} + \vec{v}\cdot\vec{\nabla}\vec{v}) = -\vec{\nabla}P - \rho\vec{\nabla}(\phi - \frac{1}{2}\Omega_*^2 s^2) + \vec{F}_v \\
\mathrm{Div}(\rho\vec{v}) = 0.
\end{cases}
\tag{1}
$$

$$\begin{cases} P \equiv P(\rho, T) \quad \text{OPAL} \\ \kappa \equiv \kappa(\rho, T) \quad \text{OPAL} \\ \varepsilon_* \equiv \varepsilon_*(\rho, T) \quad \text{NACRE} \end{cases} \tag{2}$$

The energy flux

$$\vec{F} = -\chi_r \vec{\nabla} T - \frac{\chi_{\rm turb} T}{\mathcal{R}_M} \vec{\nabla} S$$

The transport of momentum

$$\begin{aligned} \vec{F}_v = \mu \vec{\mathcal{F}}_\mu(\vec{v}) \quad &= \quad \mu \Big[\Delta \vec{v} + \frac{1}{3} \vec{\nabla} \left(\vec{\nabla} \cdot \vec{v} \right) + 2 \left(\vec{\nabla} \ln \mu \cdot \vec{\nabla} \right) \vec{v} \\ &\quad + \vec{\nabla} \ln \mu \times (\vec{\nabla} \times \vec{v}) - \frac{2}{3} \left(\vec{\nabla} \cdot \vec{v} \right) \vec{\nabla} \ln \mu \Big] \, . \end{aligned}$$

or any mean-field expression of the Reynolds stress.

Boundary conditions

- On pressure

$$P_s = \frac{2}{3}\frac{\overline{g}}{\overline{\kappa}}$$

- On the velocity field

$$\vec{v}\cdot\vec{n} = 0 \quad \text{and} \quad ([\sigma]\vec{n})\wedge\vec{n} = \vec{0}$$

- On temperature (black body radiation)

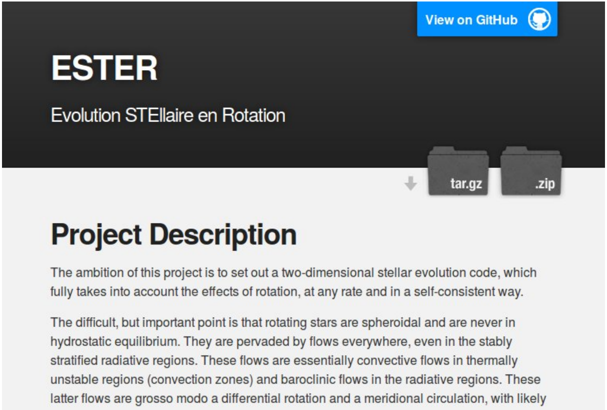
$$\vec{n}\cdot\vec{\nabla}T + T/L_T = 0$$

The last touch

$$\int_{(V)} r \sin\theta \rho u_\varphi \, dV = L$$

or

$$v_\varphi(r = R, \theta = \pi/2) = V_{\rm Eq}$$


View on GitHub

ESTER

Evolution STEllaire en Rotation

tar.gz .zip

Project Description

The ambition of this project is to set out a two-dimensional stellar evolution code, which fully takes into account the effects of rotation, at any rate and in a self-consistent way.

The difficult, but important point is that rotating stars are spheroidal and are never in hydrostatic equilibrium. They are pervaded by flows everywhere, even in the stably stratified radiative regions. These flows are essentially convective flows in thermally unstable regions (convection zones) and baroclinic flows in the radiative regions. These latter flows are grosso modo a differential rotation and a meridional circulation, with likely

FIGURE – Freely available on the www

Mappings

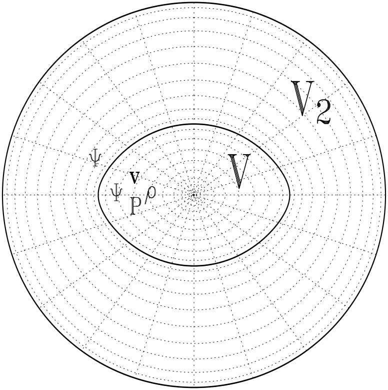
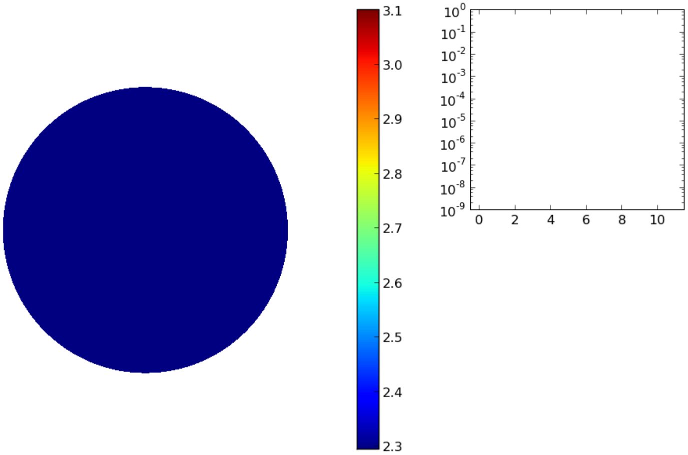
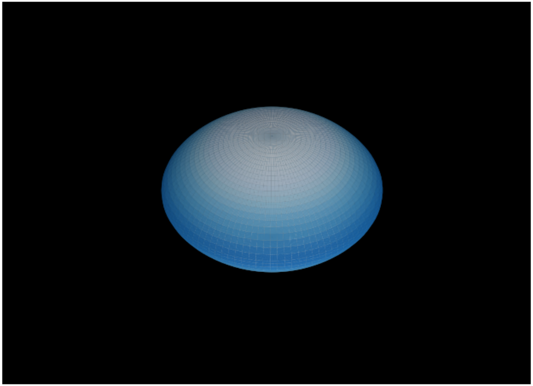


FIGURE – The mapping.

Convergence of iterations



Gravity darkening of Achernar (α Eri)



Gravity darkening exponent : $T_{\text{eff}} \propto g_{\text{eff}}^{\beta}$

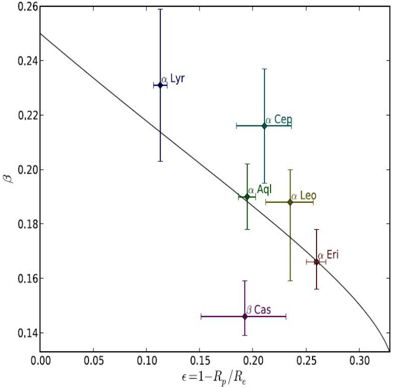


FIGURE – Observed values of β and a simple model of Espinosa Lara & Rieutord (2011).

We have modeled 8 stars of intermediate mass :

Star		M ($M_\odot$)	V_{eq}(km/s)
Altair	α Aql	1.86	313
Alderamin	α Cep	1.9	265
Ras Alhague	α Oph	2.2	242
	δ_A Vel	2.27 & 2.43	150 & 143
Vega	α Lyr	2.4	205
Regulus	α Leo	4.1	335
Achernar	α Eri	6.5	339

δ Vel seen by Kervella et al. 2013 at VLTI with PIONIER

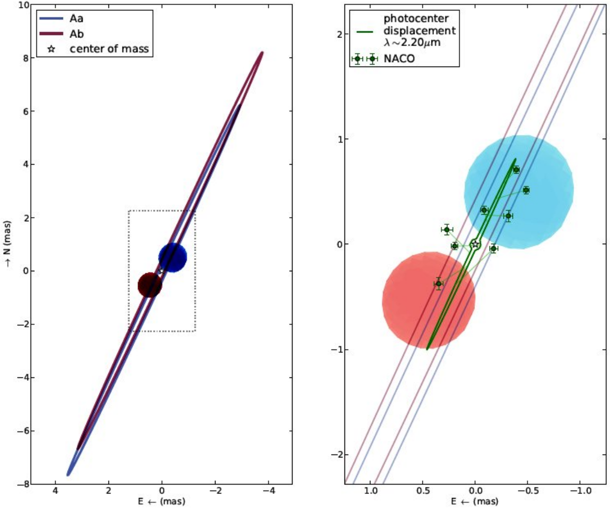
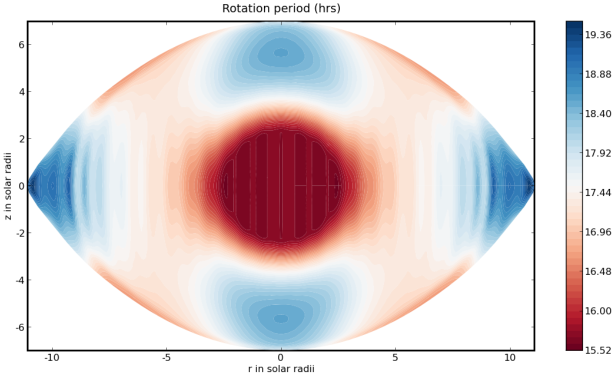


FIGURE – The orbit of delta vel (Kervella et al. 2013).

Star	Delta Velorum Aa		Delta Velorum Ab	
	Obs.	Model	Obs.	Model
Mass ($M_\odot$)	2.43 ± 0.02	**2.43**	2.27±0.02	**2.27**
R_{eq} ($R_\odot$)	2.97±0.02	**2.95**	2.52±0.03	**2.52**
R_{pol} ($R_\odot$)	2.79±0.04	**2.77**	2.37±0.02	**2.36**
T_{eq} (K)	9450	**9440**	9560	**9477**
T_{pol}(K)	10100	**10044**	10120	**10115**
L ($L_\odot$)	67±3	**65.2**	51±2	**48.5**
V_{eq} (km/s)	143	**143**	150	**153**
P_{eq} (days)		**1.045**		**0.832**
P_{pol} (days)		**1.084**		**0.924**
$X_{env.}$		**0.70**		**0.70**
$X_{core}/X_{env.}$		**0.10**		**0.30**
Z		**0.011**		**0.011**

Inside the stars : internal differential rotation

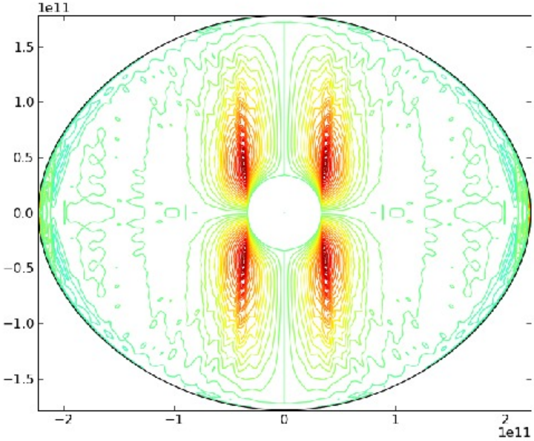
$M=30M_\odot$ at 98% of critical angular velocity



Espinosa Lara & Rieutord (2013) A&A,**552**,A35

Inside the stars : meridional circulation

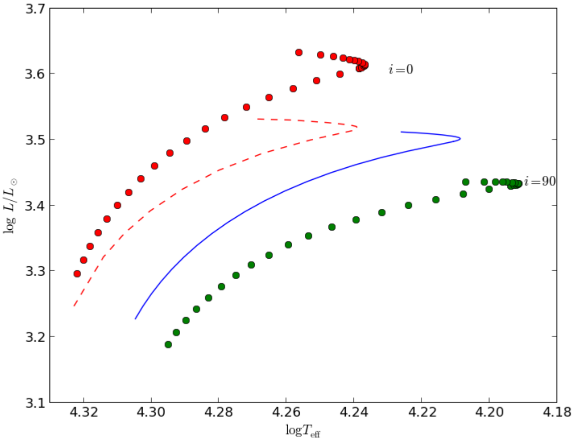
$M=5M_\odot$ at 70% of critical angular velocity



Espinosa Lara & Rieutord (2013)

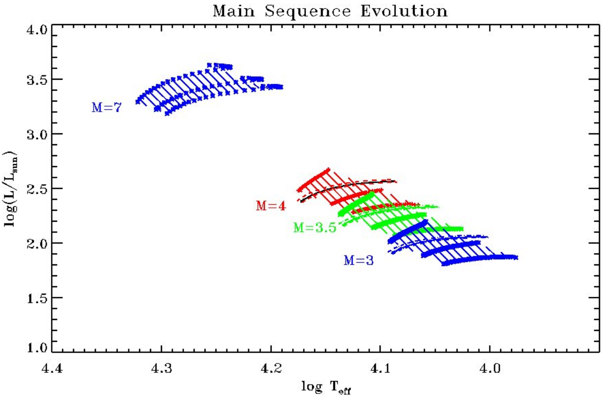
Towards evolution

HR diagram track of a $7M_\odot$ star of constant angular momentum, starting at $\Omega/\Omega_k = 0.5$.

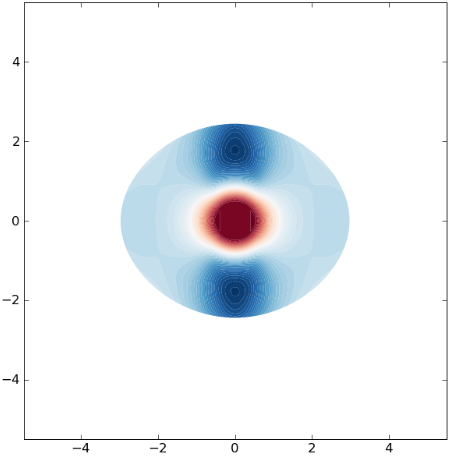


Towards evolution

HR diagram tracks at constant angular momentum



Evolution of a $5M_\odot$ star at constant angular momentum : heading to the Be state

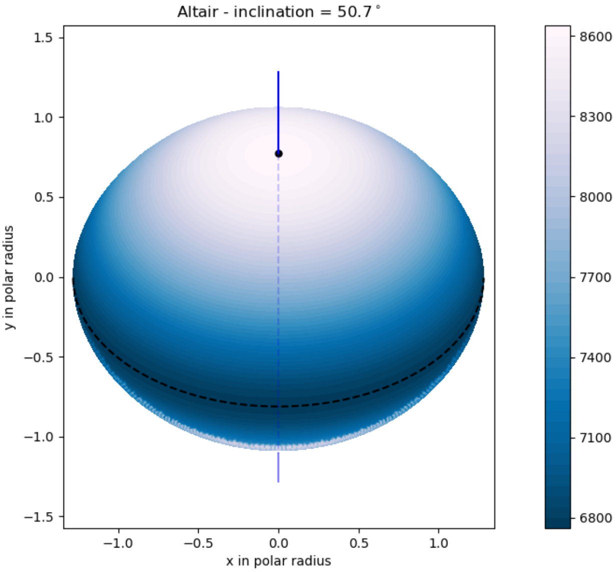


Altair's age

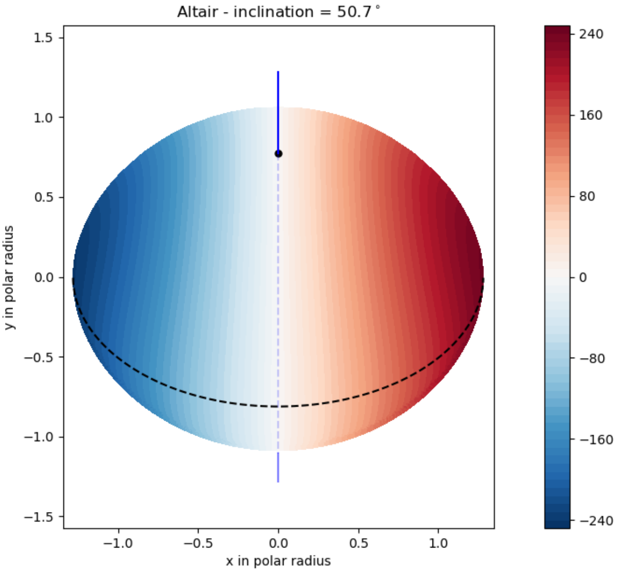
By fitting an ESTER model to interferometric, spectroscopic and asteroseismic data of Altair, Bouchaud et al. (2020) give the concordance model of Altair :

- M=$1.86 \pm 0.03 M_\odot$
- Z= 0.019
- V_{eq} = 313 km/s, flat=0.22
- Age $\simeq 100$ Mys

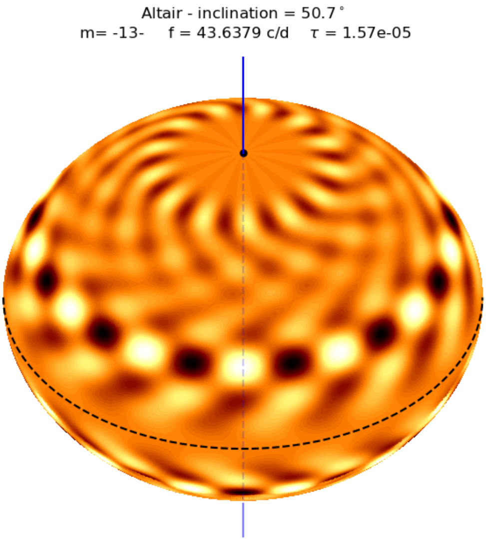
Altair's view



Altair's view



Altair's view



Last developments and road map

- Portability improved, github management
- Documentation strongly improved (95 pages)
- Low mass stellar models under construction

Next :

1. Implement nuclear evolution on MS
2. Implement thermal evolution (PMS and post-MS)
3. Make it more user-friendly

Domain Specific Languages – DSL

1. Flexibility
2. Expressiveness
3. Compactness

Example : the LSB solver

Developed to solve coupled differential equation

```
INPUT DATA:
nz*        = nz          [ i3 ]
nn         = nn          [ i3 ]

VARIABLES:
f

EQUATIONS:

eqf;
 x(i)**2  * f''  + x(i) * f'  -nn**2  * f   =  -x(i)**2   * f

BOUNDARY CONDITIONS:

bcf left  : f = 0
bcf right : f = 0

ORDER VARIABLES:
[ f ]

ORDER EQUATIONS:
# [equation name + (b.c.  left) + (b.c. right) ]
[ eqf + (bcf) + (bcf) ]
```

Conclusions

Points to take away :

1. ESTER 2D models are ripe to face observational data in
   - asteroseismology (coupled with TOP)
   - interferometry (coupled with CHARRON)

   for early-type stars.

2. low-mass fast rotating stellar model should come soon...

Some references

- Rieutord, Espinosa Lara & Putigny (2016), J. Comput. Phys. 318, 277
- Espinosa Lara & Rieutord (2013), A&A,**552**, A35
- ESTER website : http ://ester-project.github.io/ester/