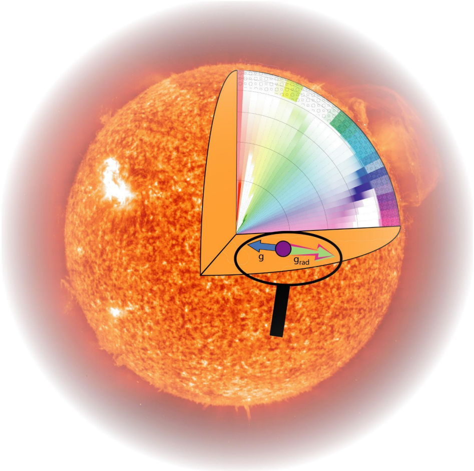
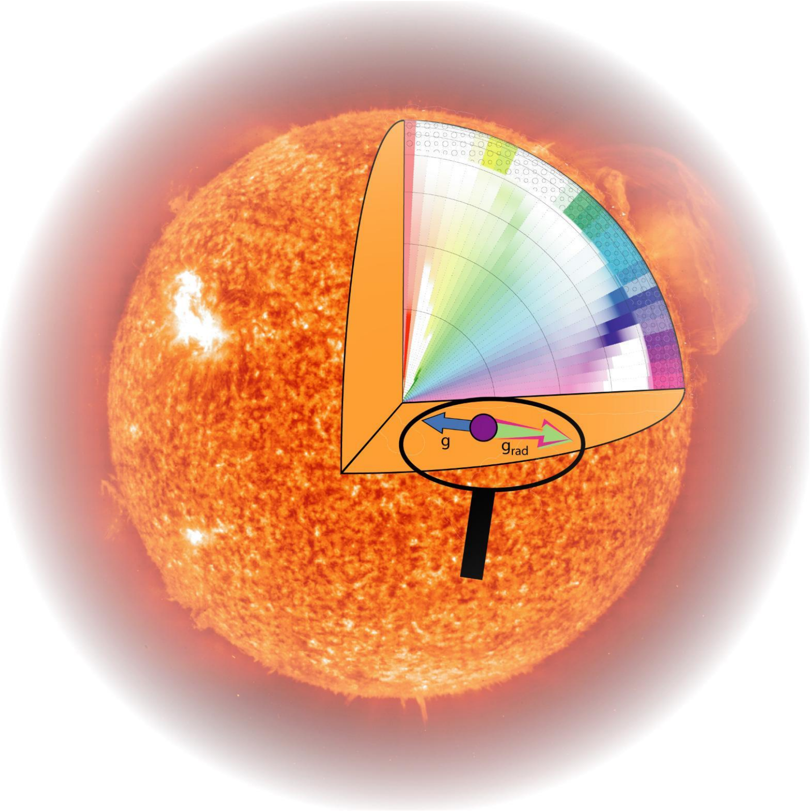
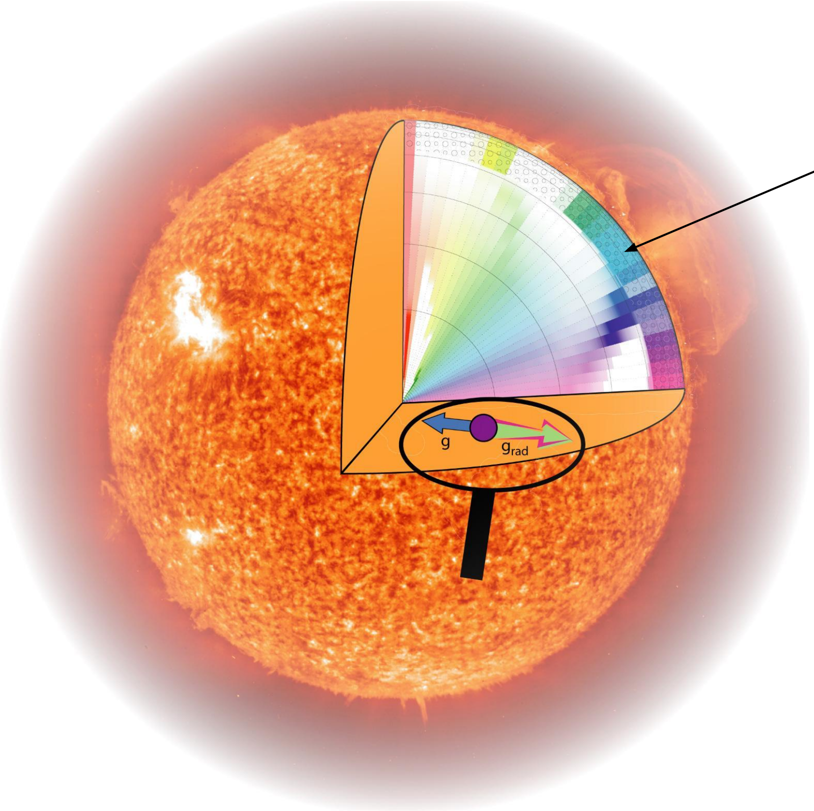
CESTAM: Transport of chemical elements

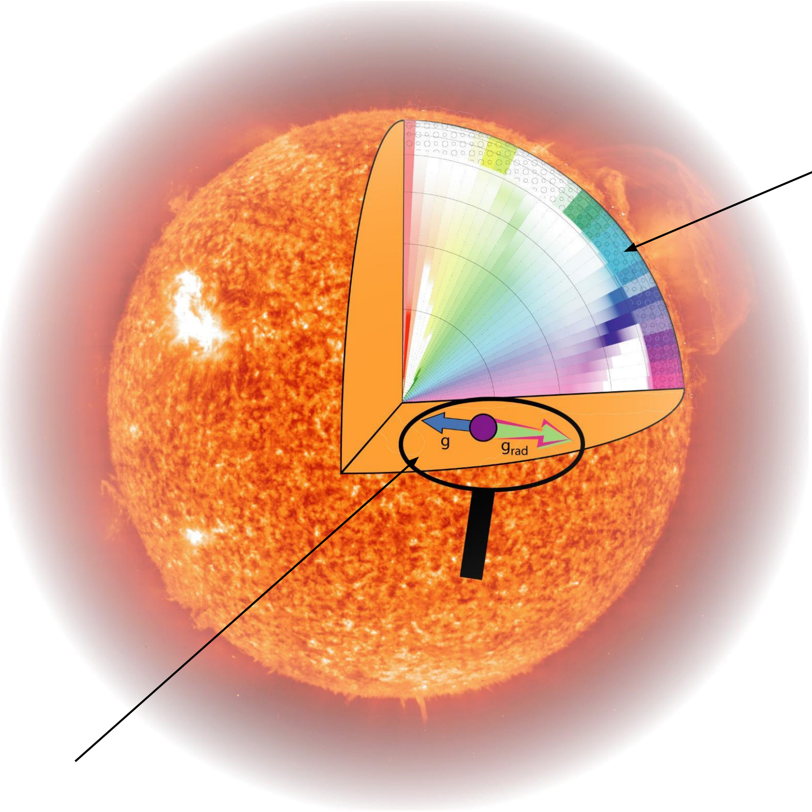


Morgan Deal


g
g_{rad}

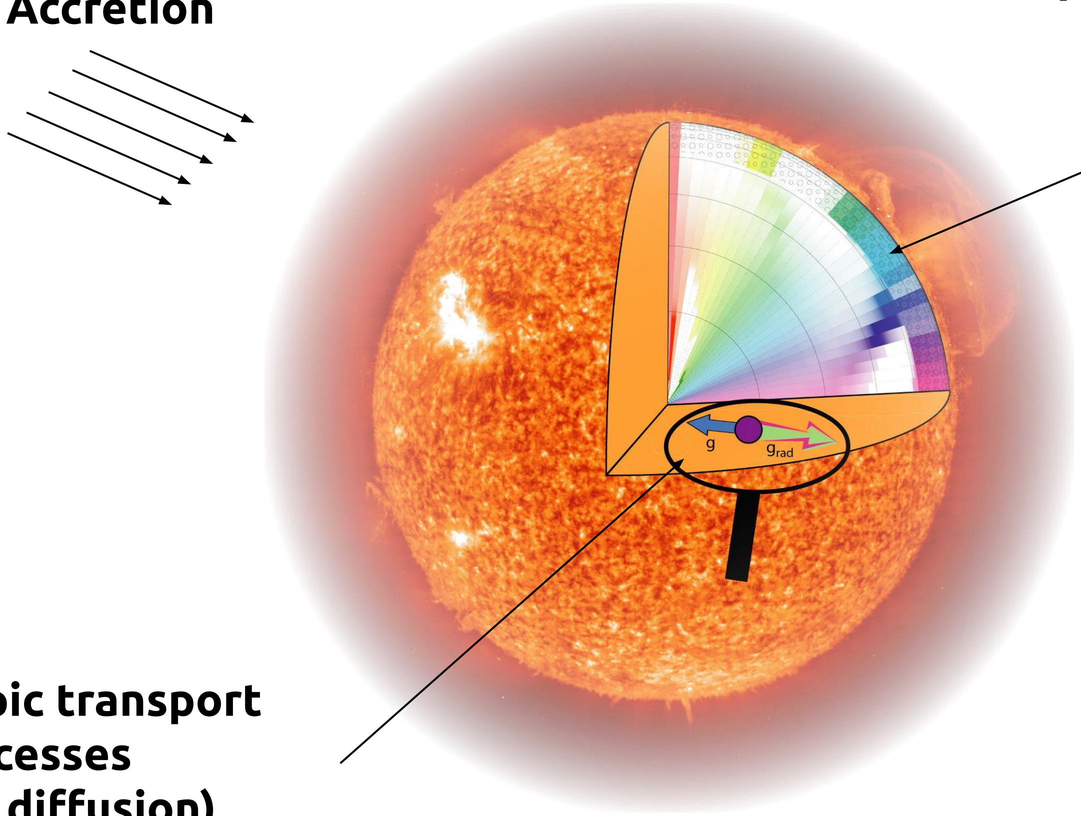


Macroscopic transport processes



Macroscopic transport processes

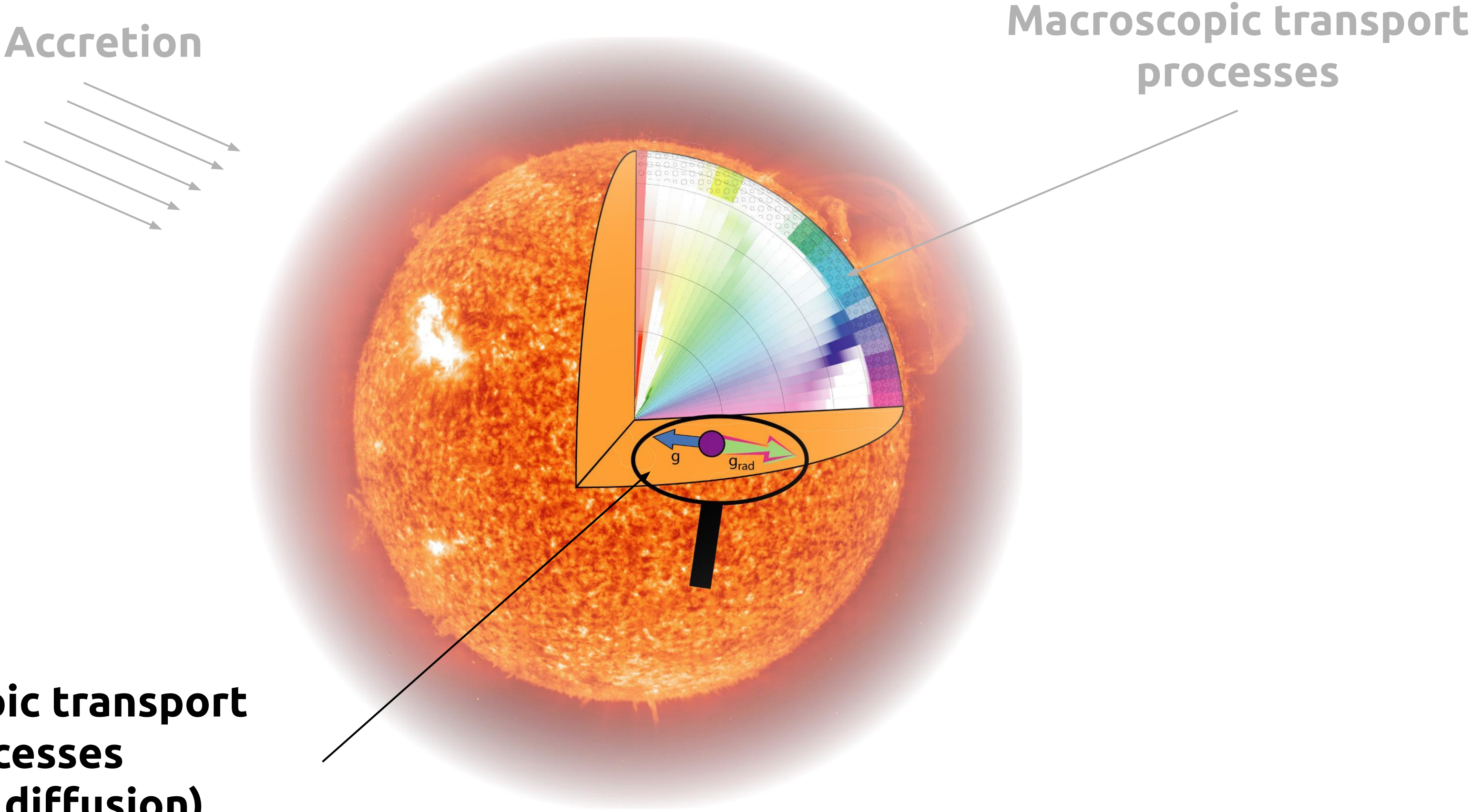
Microscopic transport processes (atomic diffusion)



Accretion

Macroscopic transport processes

Microscopic transport processes (atomic diffusion)



Microscopic transport processes (atomic diffusion)

Diffusion equation

$$\rho\frac{\partial X_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho\boxed{D_{\mathrm{turb}}}\frac{\partial X_i}{\partial r}\right] - \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho\boxed{v_i}\right] + A_i m_p\left[\sum\nolimits_j \boxed{(r_{ji} - r_{ij})}\right]$$

Turbulent processes (macroscopic)

Diffusion velocity (microscopic)

NACRE + LUNA

`Angulo 1999`

`Imbriani et al. 2004`

- ➔ Resolution of the equation: **finite elements**
- ➔ approximation of the scalar product: **Gauss integration**
- ➔ Iteration of **Newton-Raphson** for convergence
- ➔ **H, He, Li, Be, B, C, N, O, Ne, Na, Mg, Al, Si, S, Ca, Fe** and isotops

Diffusion velocity in the trace element case

$$v_i = \boxed{D_{ip}} \left[- \frac{\partial \ln X_i}{\partial r} + \frac{A_i m_p}{kT} \left(\boxed{g_{rad,i}} - g \right) + \frac{(\boxed{\bar{Z}_i} + 1) m_p g}{2kT} + \kappa_T \frac{\partial \ln T}{\partial r} \right]$$

$\propto Z_i^{-2}$

Collision integrals

`Paquette et al. 1986`

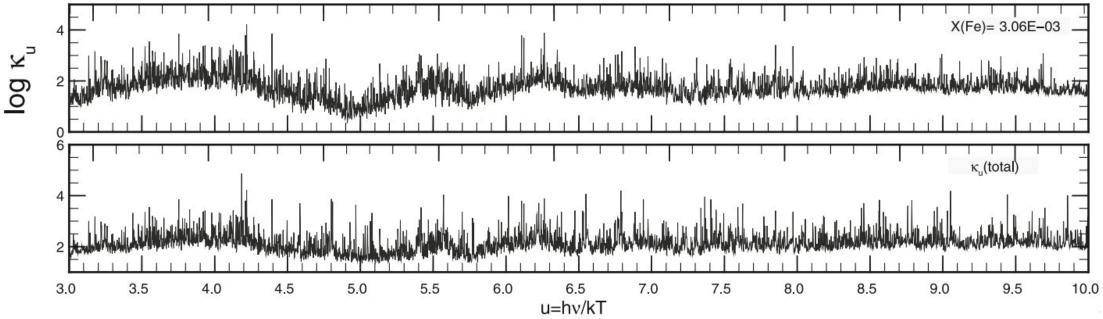
Radiative acceleration

- ➔ Diffusion velocities from `Burgers 1969` or `Michaud & Proffitt 1993`
- ➔ Partial ionisation: **Saha equation**

Radiative accelerations

$$g_{rad,i} = \frac{1}{4\pi r^2}\frac{L^{rad}\boxed{\kappa_R}}{cX_i}\int_0^\infty \boxed{\frac{\kappa_{\nu,i}}{\kappa_\nu(total)}}P(u)du$$

Monochromatic opacities



- ➔ Need for monochromatic opacities: **OPAL** or **OP** Seaton 2005
- ➔ **Expensive to compute** (10 time faster with Hui-Bon-Hoa 2021 procedure)

i : ionisation state
E : element

Alecian & LeBlanc 2004, 2020

SVP approximation

$$g_{i,line} = q\boxed{\varphi_i^*}(1+\boxed{\xi_i^*}c_i)\left(1+\frac{c_i}{b\boxed{\psi_i^*}^2}\right)^{\boxed{\alpha_i}}$$

$$g_{i,cont} = 7,16\times 10^{-26}\frac{N_e T_{\text{eff}}^4}{T^{3/2}}\left(\frac{R}{r}\right)^2\frac{1}{A_i^2}\Theta_i\left(\frac{\chi}{1+\chi}\right)^{\boxed{b_i}}$$

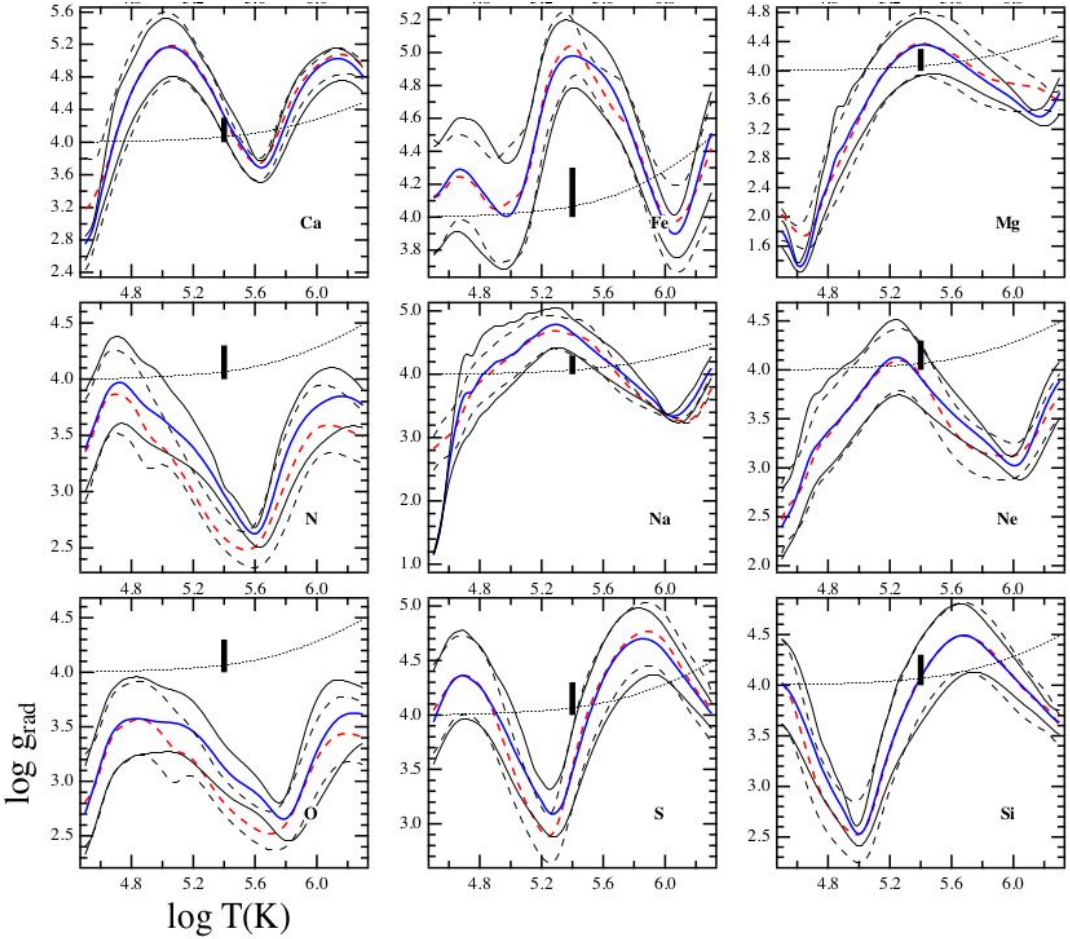
$$\Theta_i \approx \boxed{a_i}\frac{N_{i-1,0}\,p_{i-1}}{N_{i-1}\,p_i\,g_0}\sum_k n_k g_k Q_k$$

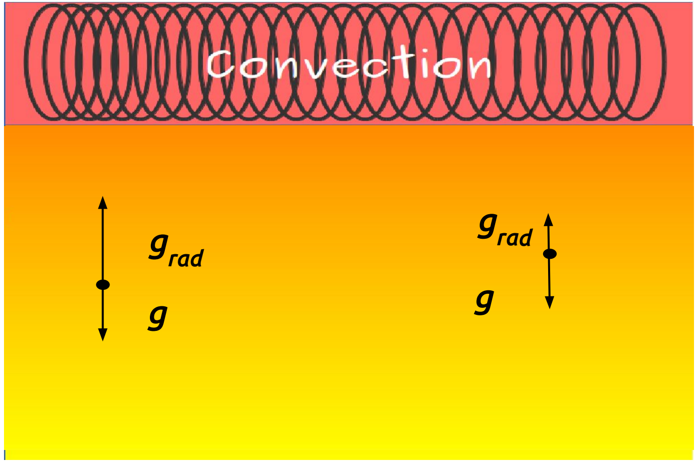
$$g_{rad,E} = \frac{\sum_i N_i(g_{i,cont}+g_{i,line})}{\sum_i N_i}$$

➔ Single-Valued Parameter (6 tabulated parameters)

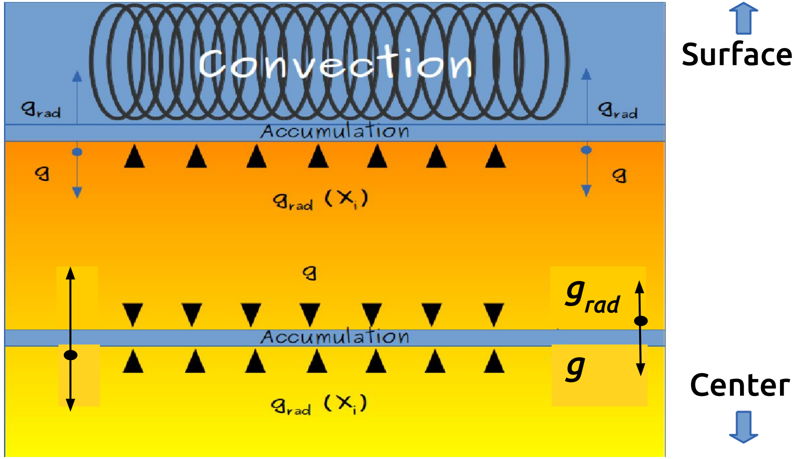
➔ **Fast to compute**

Alecian & LeBlanc 2004, 2020





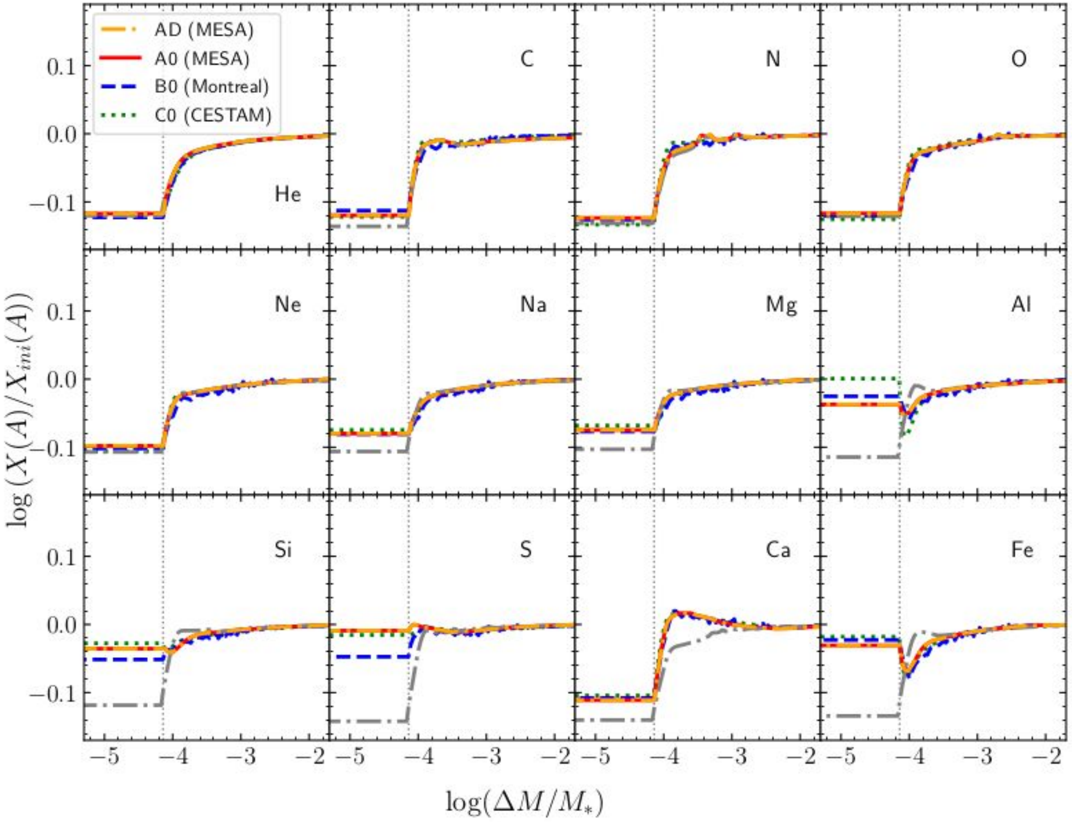
Leads to accumulation of some elements

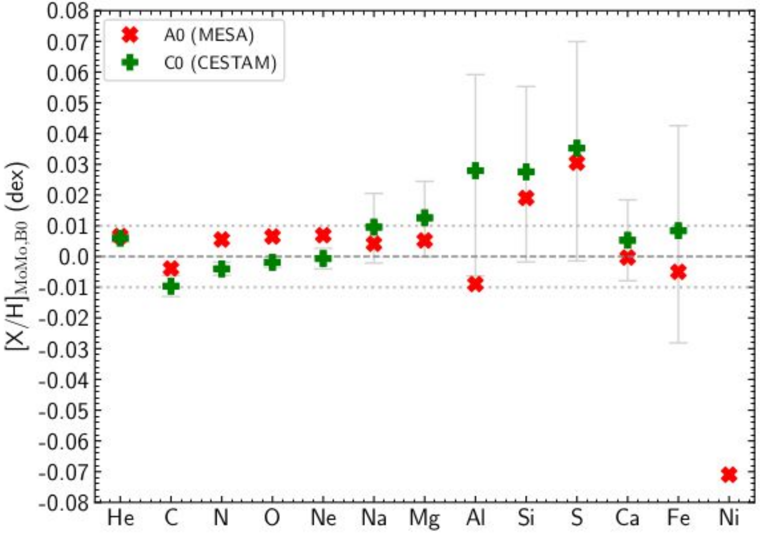


These effects are different **for each element** and depend on :
- the **abundance** of the element
- the **ionisation state**
- the **photon flux**

→ **Direct influence** on stellar **structure** and **surface abundances**

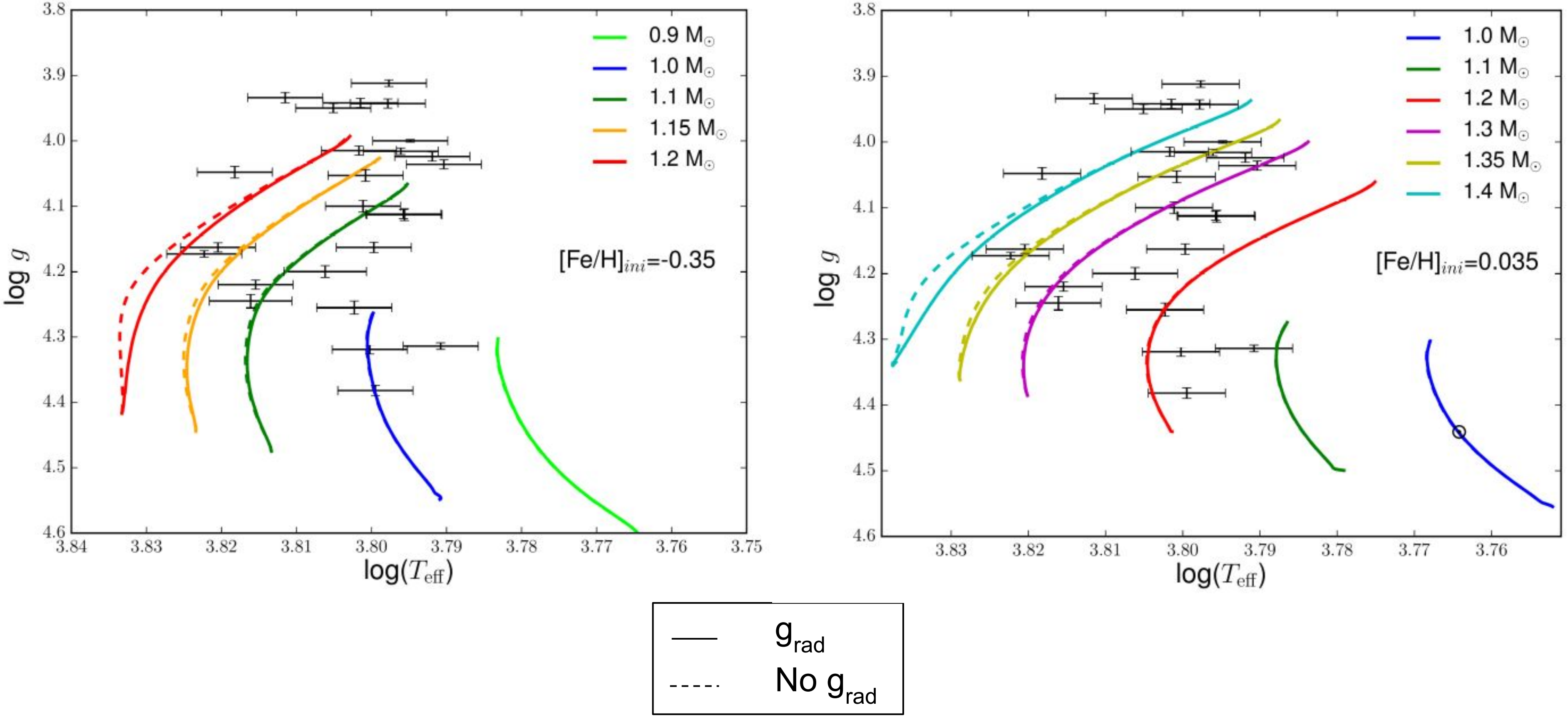
➔ 1.4 $M_\odot$, 400Myr



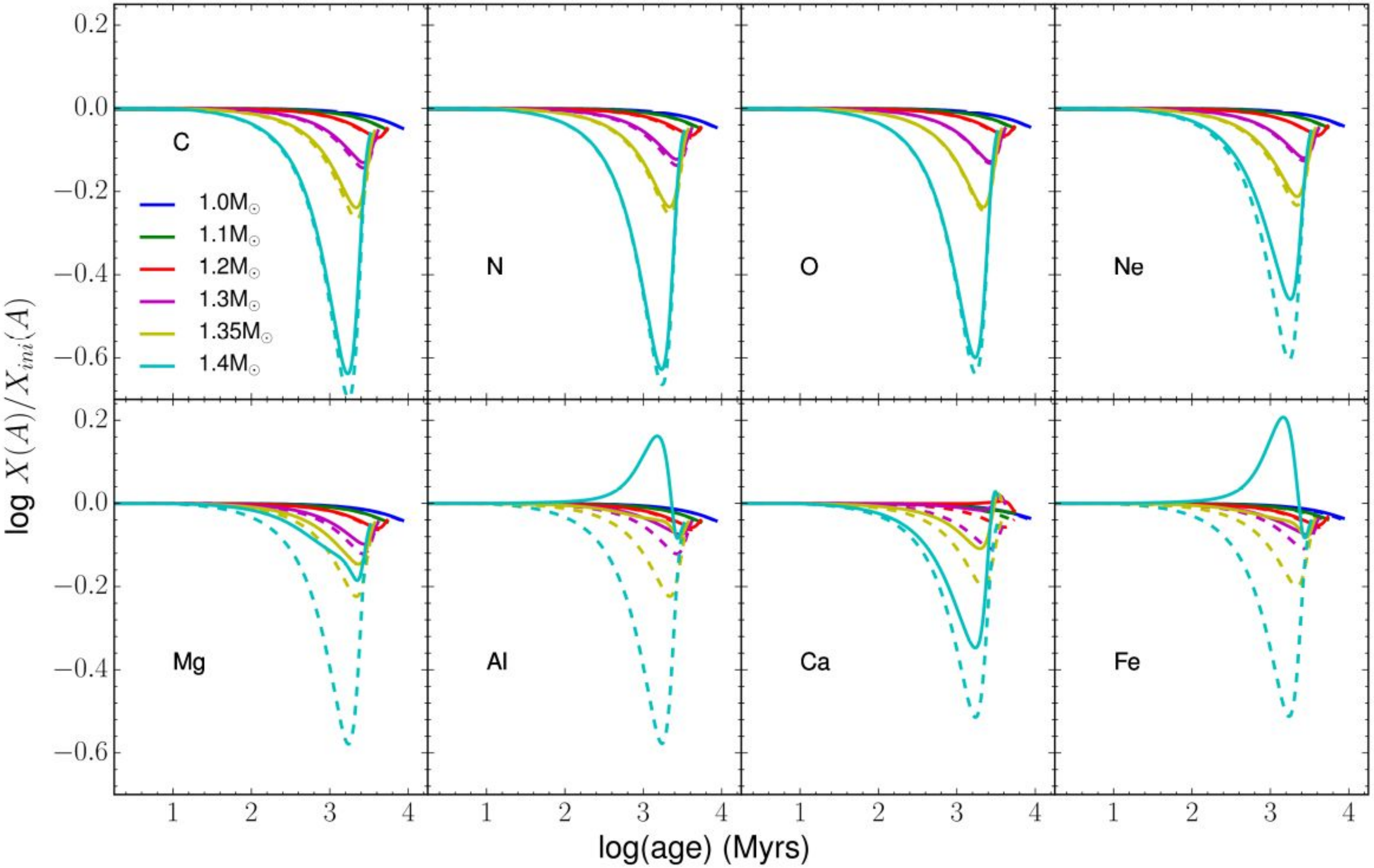


Campilho, Deal et al. 2022

Deal et al. 2018

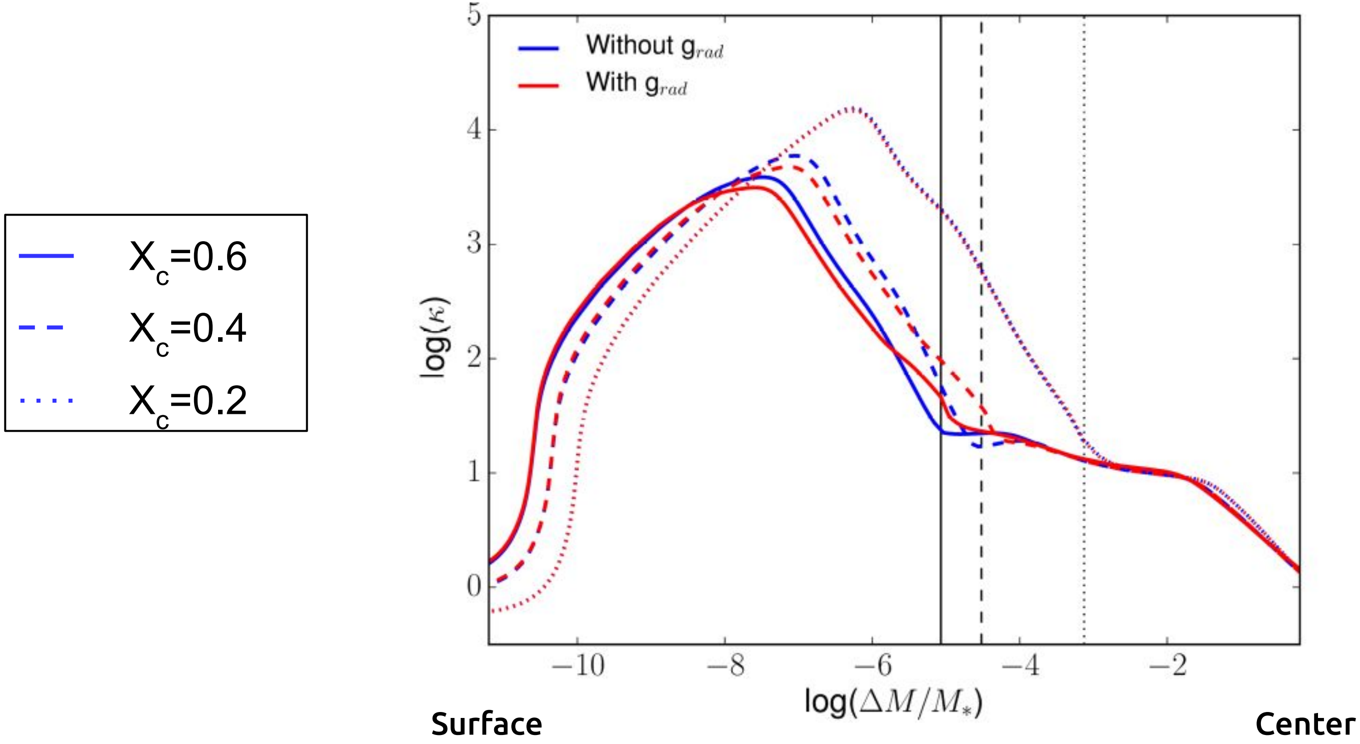


Deal et al. 2018



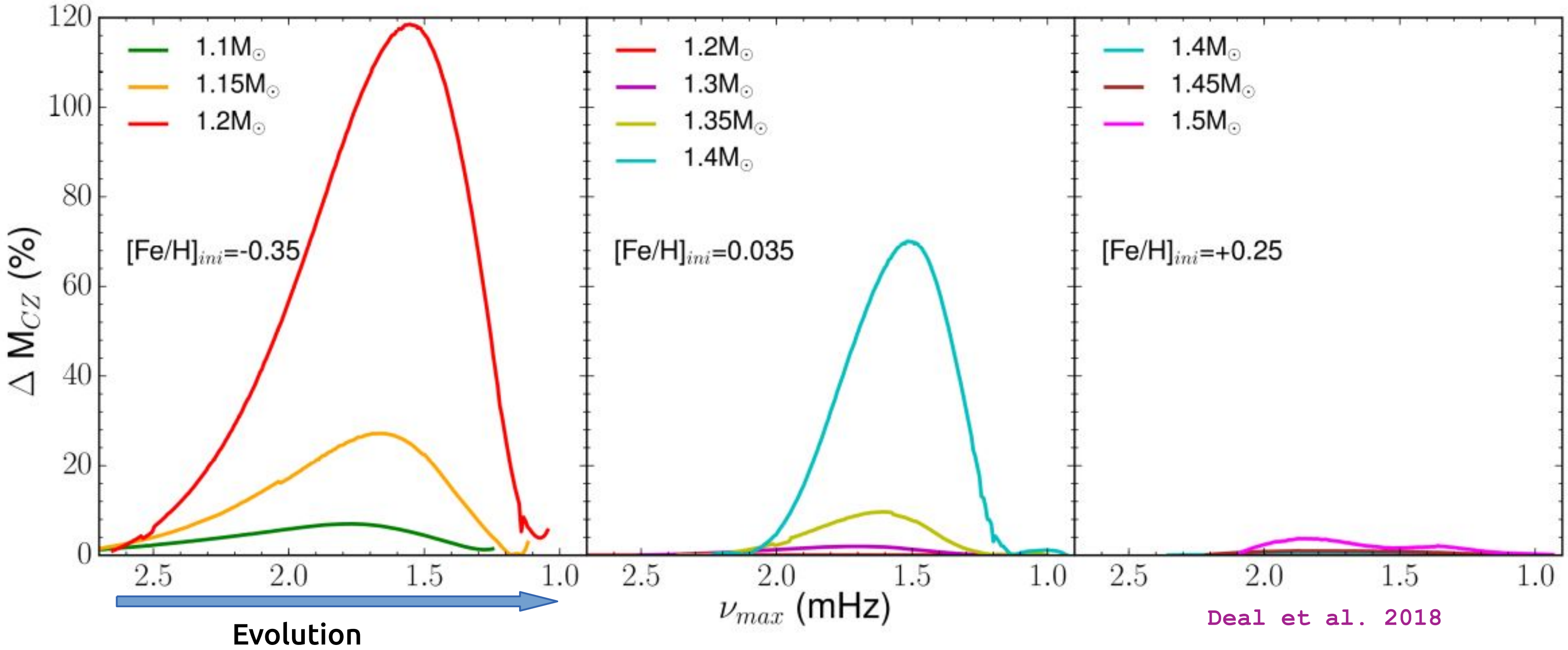
Local increase of the opacity due to iron accumulation

1.4 $M_\odot$, [Fe/H]ini=0.035



Deal et al. 2018

Difference of M_{cz} between models with and without radiative accelerations



Deal et al. 2018

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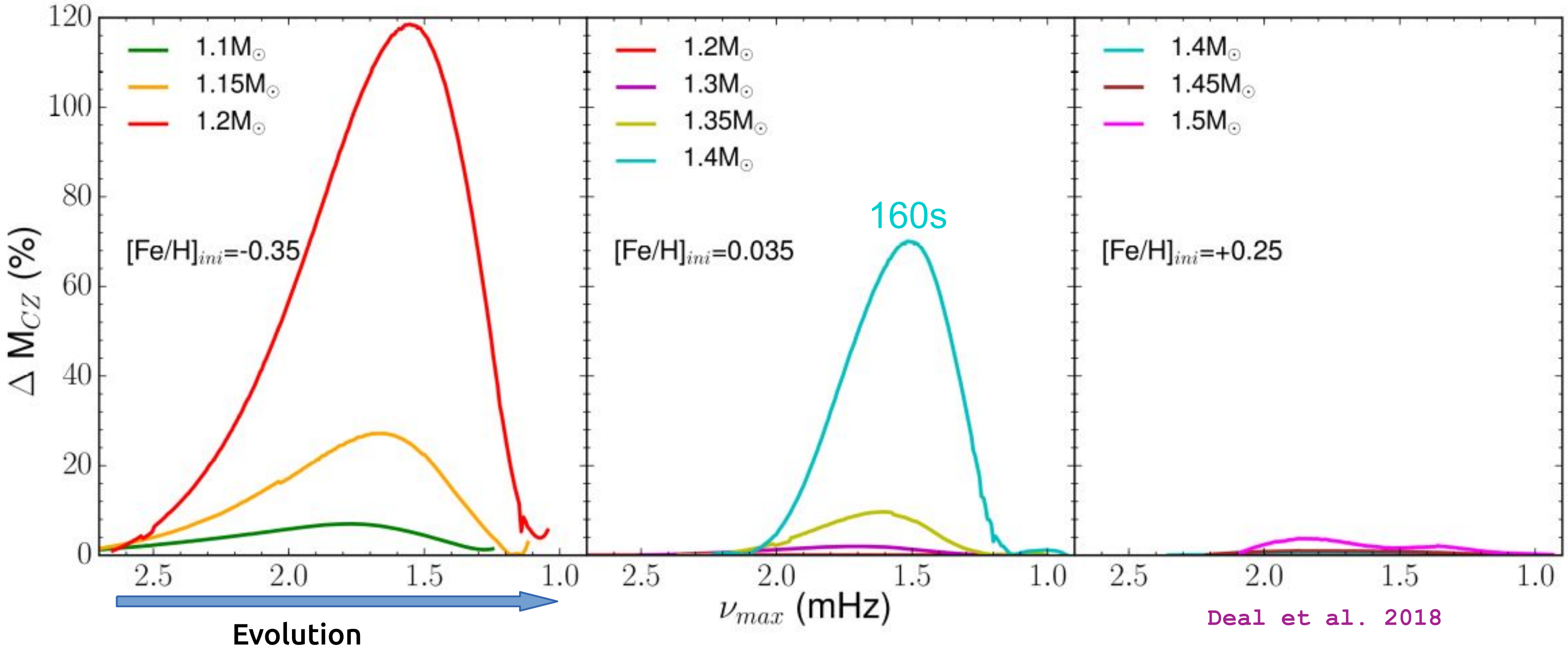
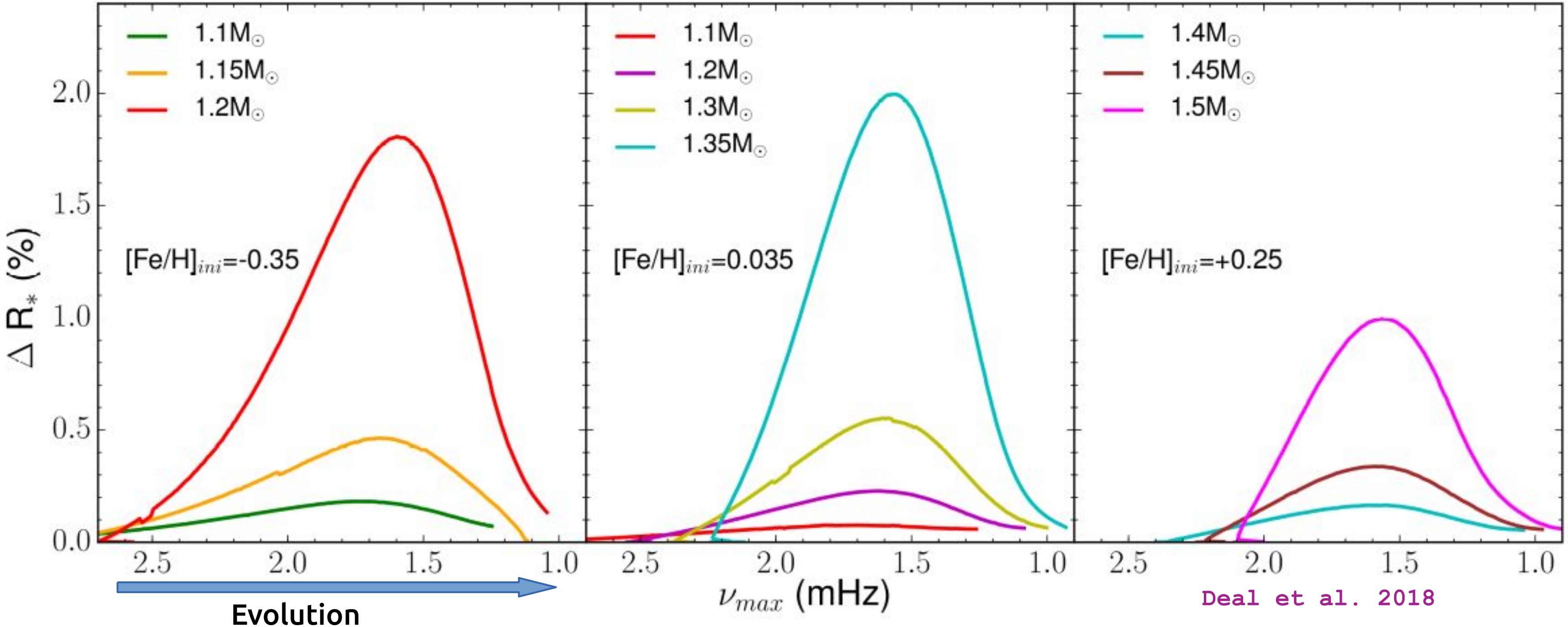


Deal et al. 2018

Larger than the uncertainty on the acoustic depth of surface convective zone of some F type stars from ***Kepler*** (Verma+ 2017)

Difference of radius between models with and without radiative accelerations



Evolution

Deal et al. 2018

2% in radius at maximum

Atomic diffusion (with radiative accelerations)

- First estimation

94 Ceti A: age difference of 4% (Deal et al. 2017)

PLATO: 10% on ages, 5% on masses and 2% on radii

Atomic diffusion (with radiative accelerations)

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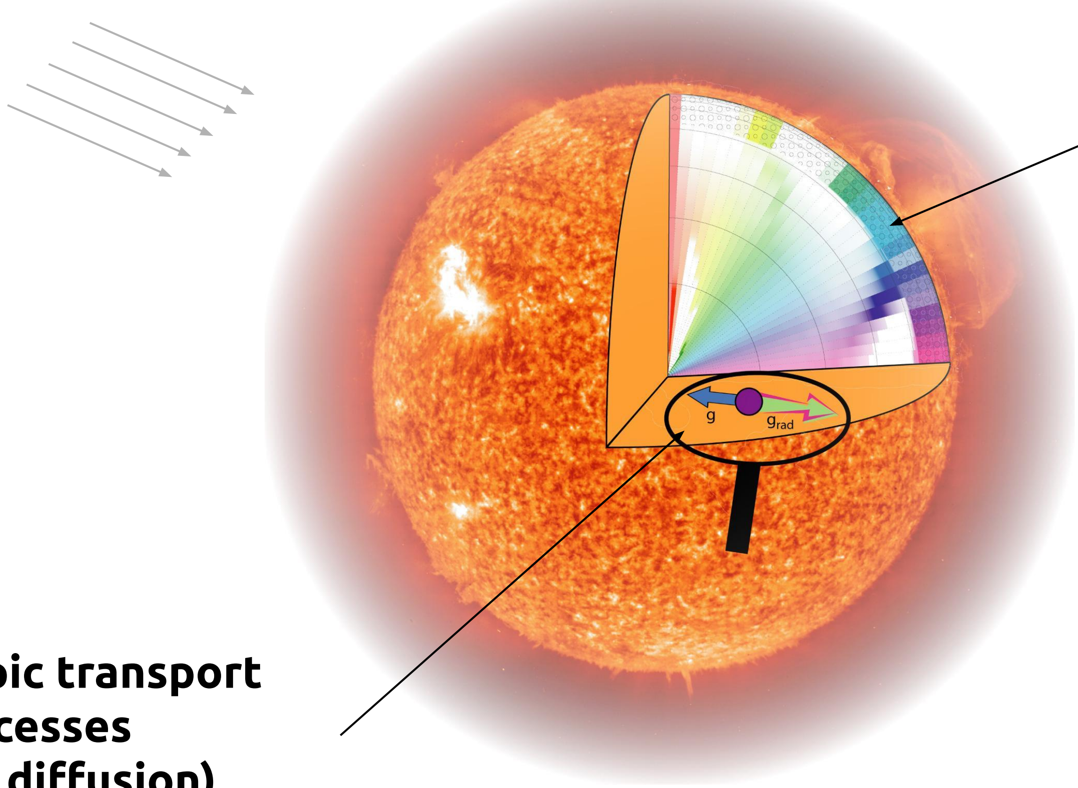
- Using optimization method **(AIMS,** Lund & Reese 2017, Rendle et al. 2019**)** : Classical + seismic constraints

1.4 $M_\odot$ at solar metallicity: difference in age of 10-15%, in mass of 1-4%, and in radius of 1%
(Deal et al. 2018, 2020)

PLATO: 10% on ages, 5% on masses and 2% on radii

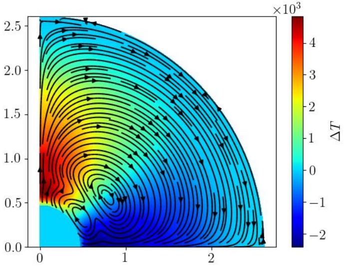
Accretion

Macroscopic transport processes

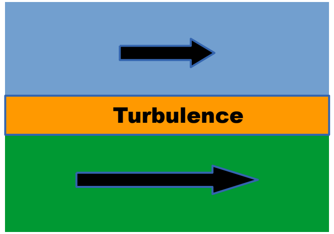


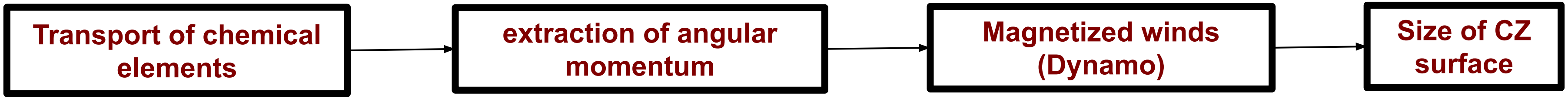
Microscopic transport processes (atomic diffusion)

Meridional circulation



Shear turbulence





Transport of chemical elements → **extraction of angular momentum** → **Magnetized winds (Dynamo)** → **Size of CZ surface**

Transport induced by the rotation

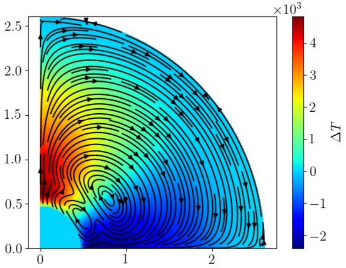
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right)_{M_r} = \frac{1}{5\rho r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega U_2\right) + \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right)$$

$$D_{\rm eff} = \frac{(rU_2)^2}{30 D_h} \qquad\qquad D_v = \nu_v$$

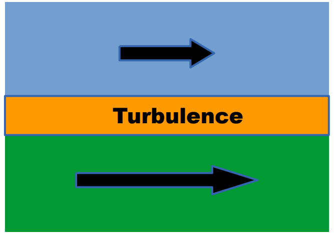
$$\boxed{D_{\rm turb,rota} = D_v + D_{\rm eff}}$$

- ➔ **D_h**: Mathis et al. 2004, 2018
- ➔ **D_v**: Talon & Zahn 1997 (+ possibility to include an additional vertical viscosity)
- ➔ **Am loss**: Matt et al. 2015
- ➔ Transport of chemicals directly linked to the transport of angular momentum

Meridional circulation

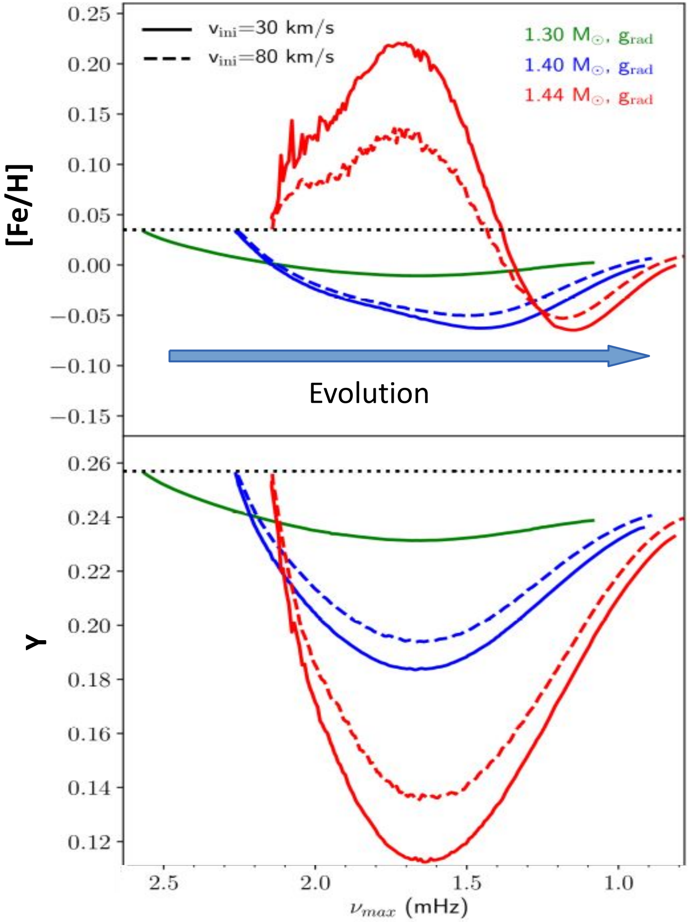


Shear turbulence



Is atomic diffusion negligible in rotating stars?

What is the combined effect of atomic diffusion and rotation on stellar parameters?

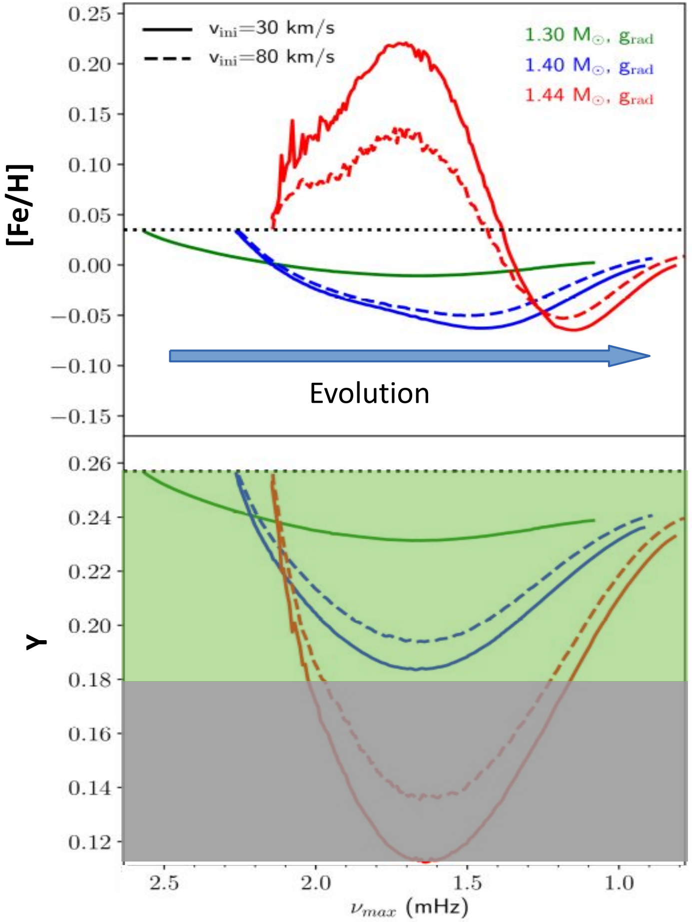


Deal et al. 2020

D_h: Mathis et al. 2004

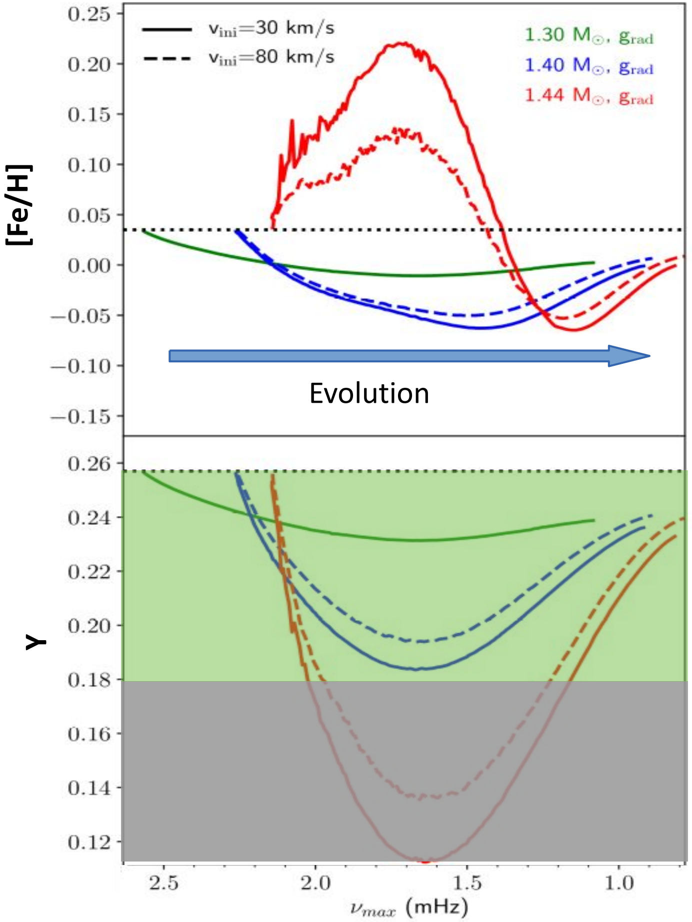
D_v: Talon & Zahn 1997

Extract. AM: Matt et al. 2015, 2019



Deal et al. 2020

Helium consistent with observations up to 1.4$M_\odot$
(Verma et al. 2019)



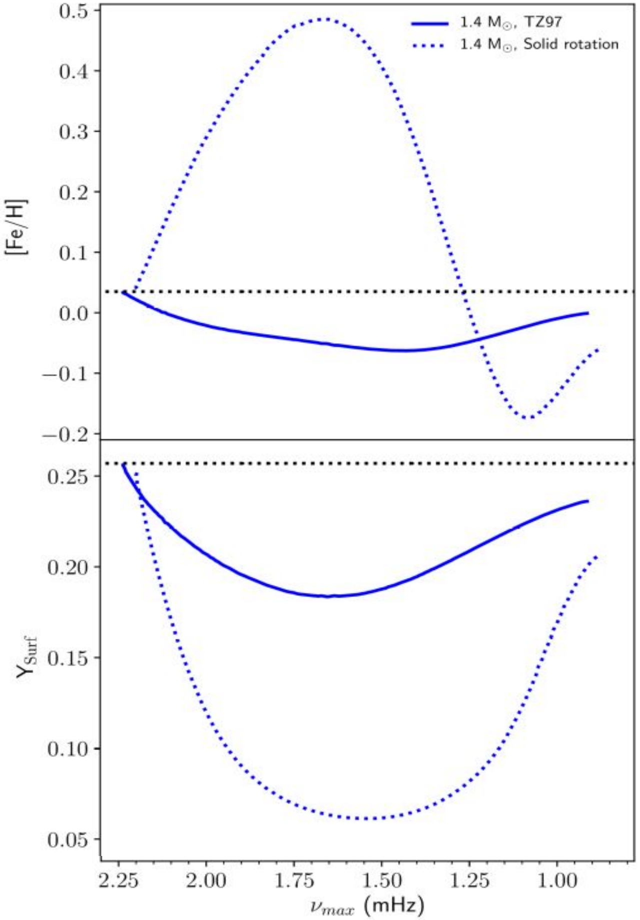
Deal et al. 2020

What processes occur for larger masses?

Helium consistent with observations up to 1.4$M_\odot$
(Verma et al. 2019)

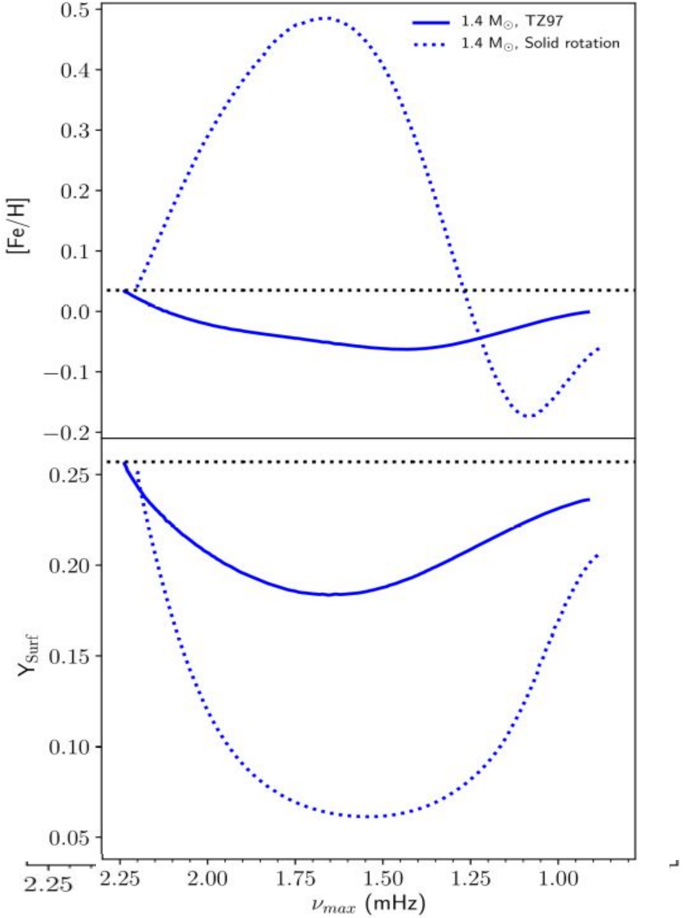
Deal et al. 2020

Solid rotation

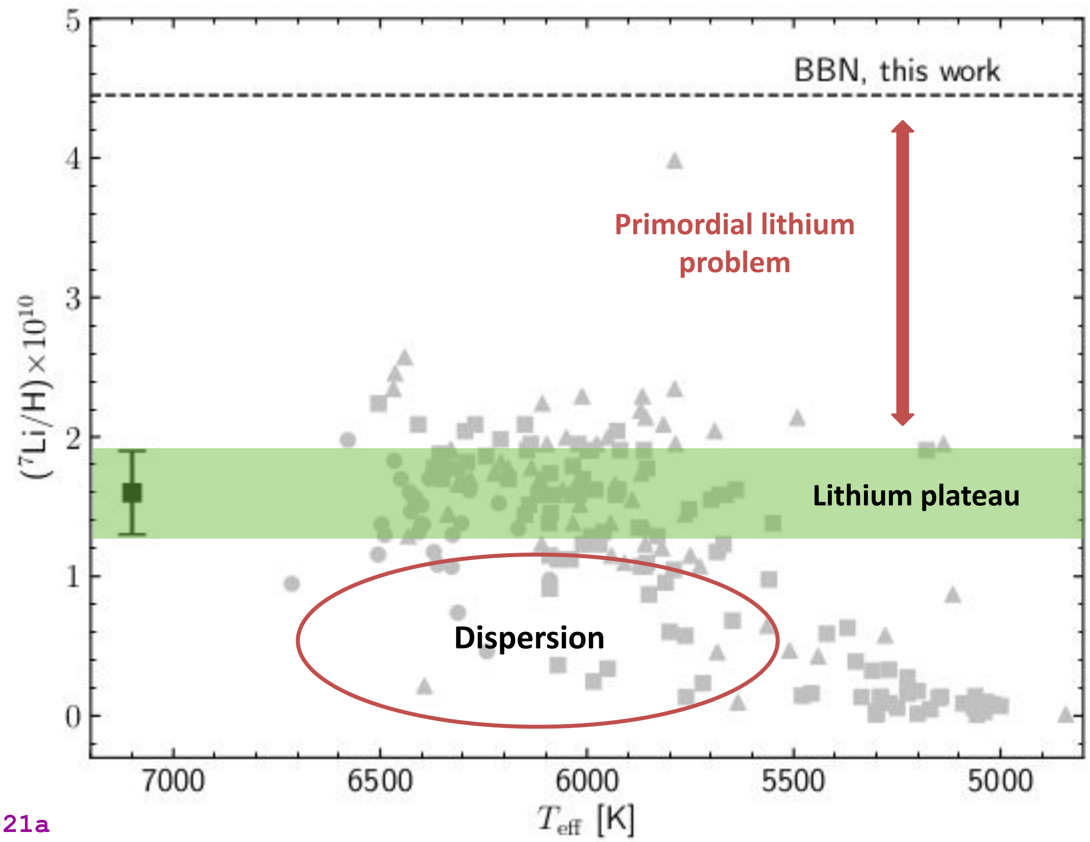


Deal et al. 2020

Solid rotation



What are the other processes?



see also Deal et al. 2021a
(CEMP-s stars)

Deal & Martins 2021

Perturbative approach of the BBN model

➔ α: fine-structure constant, varies between Big Bang and now

The whole problem cannot be solved by α variation due to the constraint on deuterium

Deal & Martins 2021

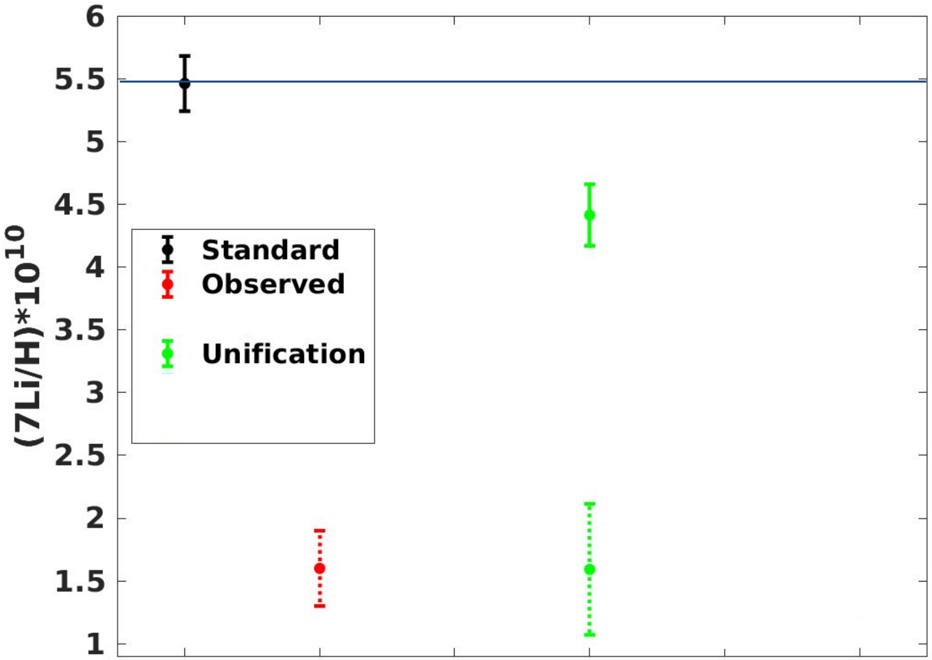
Perturbative approach of the BBN model

- ➔ α: fine-structure constant, varied between Big Bang and now
- ➔ Δ: depletion factor ⇨ ${}^7\mathrm{Li}_{\mathrm{obs\ Pop\ II}}=(1-\Delta)\,{}^7\mathrm{Li}_{\mathrm{primordial}}$

The whole problem cannot be solved by α variation due to the constraint on deuterium

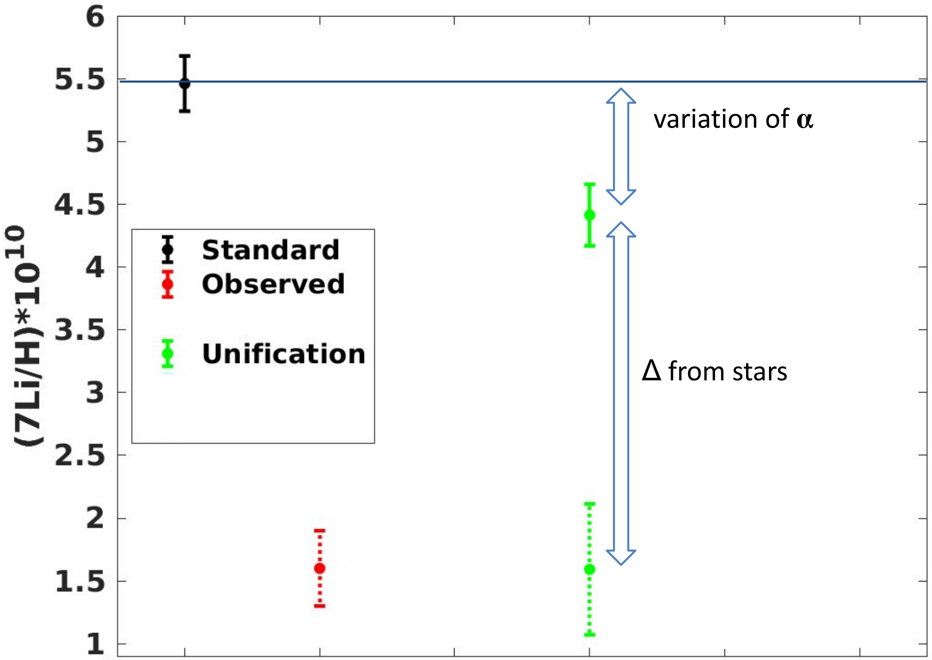
Deal & Martins 2021

Perturbative approach of the BBN model



Deal & Martins 2021

Perturbative approach of the BBN model



Deal & Martins 2021

Perturbative approach of the BBN model

➔ α: fine-structure constant, varies between Big Bang and now

➔ Δ: depletion factor ⇨ $^{7}\mathrm{Li}_{\mathrm{obs\ Pop\ II}} = (1-\Delta)^{7}\mathrm{Li}_{\mathrm{primordial}}$

- ~ **20%** of the lithium problem can be solve by a variation of α
- ~ **80%** of the lithium problem can be solve by a **depletion occurring in stars**

Deal & Martins 2021

Perturbative approach of the BBN model

➔ α: fine-structure constant, varies between Big Bang and now

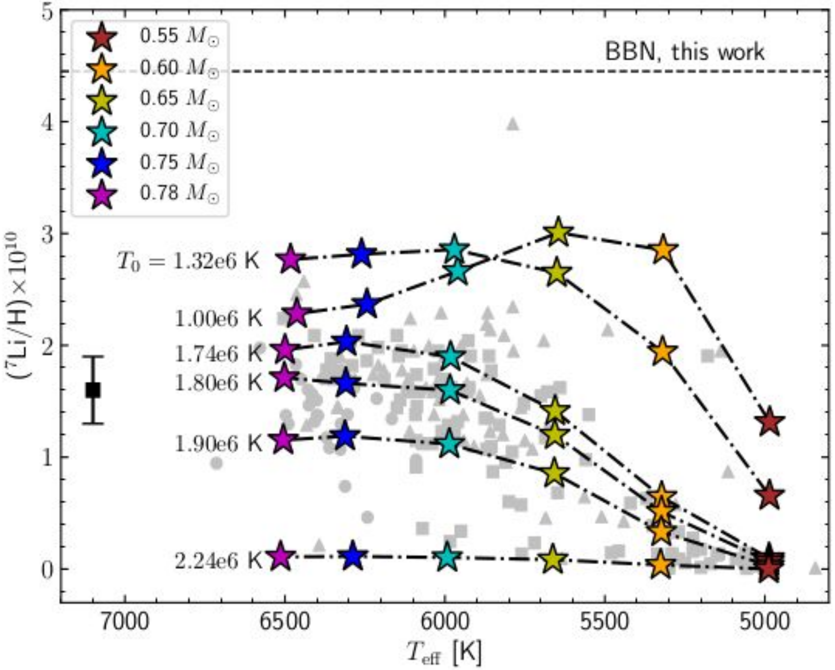
➔ Δ: depletion factor ⇨ $^{7}Li_{obs\ Pop\ II}=(1-\Delta)^{7}Li_{primordial}$

- ~ **20%** of the lithium problem can be solve by a variation of α
- ~ **80%** of the lithium problem can be solve by a **depletion occurring in stars**

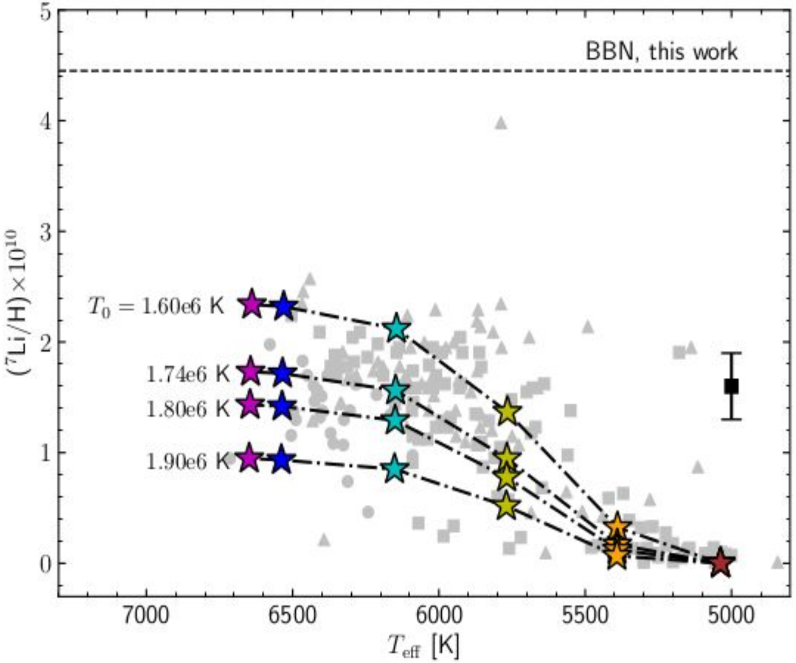
What are the processes responsible of the depletion in stars?

Deal & Martins 2021

$$D_{\text{turb}} = \omega D(He)_0 \left(\frac{\rho_0}{\rho}\right)^n$$



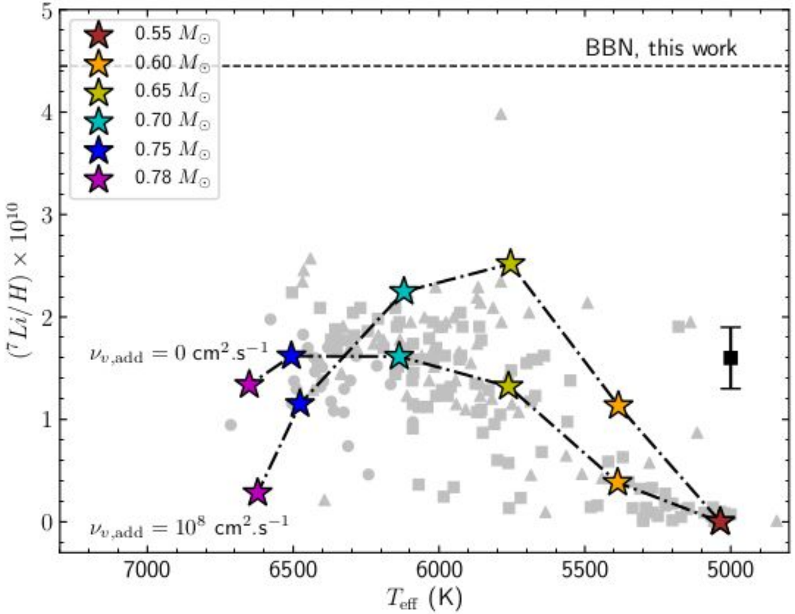
Montpellier/Montréal



CESTAM

Deal & Martins 2021

Atomic diffusion + Rotation



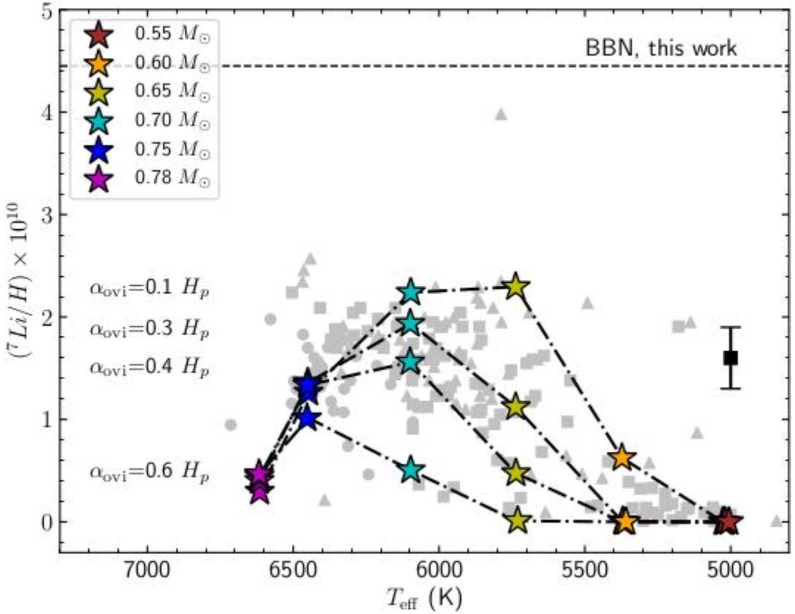
D_h: Mathis et al. 2018

D_v: Talon & Zahn 1997

Extract. AM: Matt et al. 2015, 2019

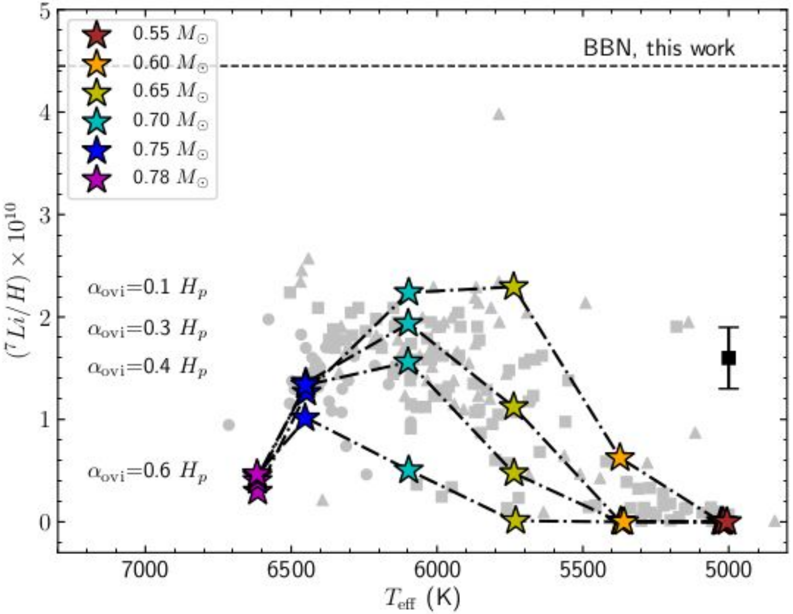
Deal & Martins 2021

Atomic diffusion + Rotation + simple penetrative convection

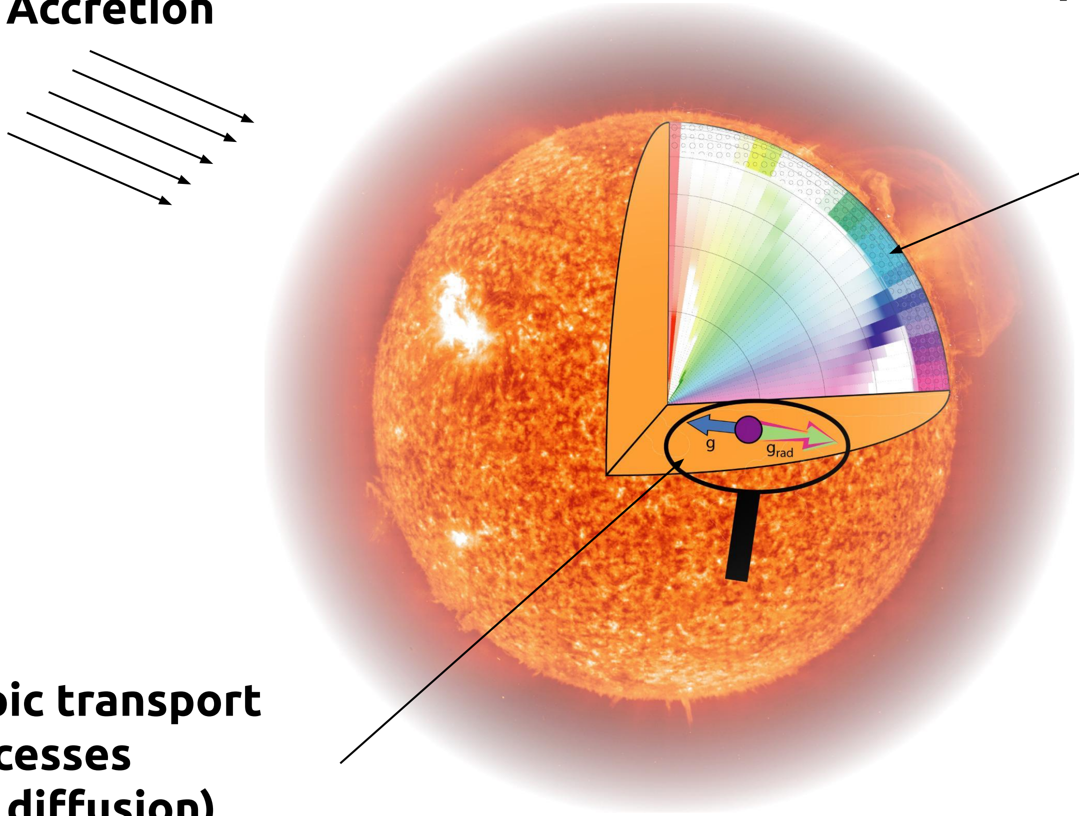


Deal & Martins 2021

Atomic diffusion + Rotation + simple penetrative convection



Next step: use more realistic modelling of the penetration convection, similarly to Dumont et al. 2020

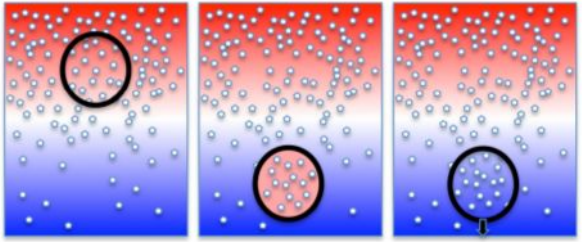


Accretion

Macroscopic transport processes

Microscopic transport processes
(atomic diffusion)

➔ Thermohaline convection



Garaud 2014

- unstable mean molecular weight gradient
- stable temperature gradient

1D prescriptions (from 2D and 3D simulations):

Kippenhahn et al. 1980, Denissenkov et al. 2010, Traxler et al. 2011,
Brown et al. 2013

$$D_{\mathrm{fing}} = Nu_\mu \kappa_\mu$$

Apply on stellar cases : Planetary matter accretion, elements accumulation due to radiative accelerations, evolved stars, ...

Conclusions

- ➔ A proper modelling of the transport (angular momentum and chemicals) **is mandatory** for an **accurate** inference of stellar parameters
- ➔ Atomic diffusion **is not negligible** in solar-like rotating stars
- ➔ An accurate modelling of the **transport of chemicals** requires an accurate modelling of the **transport of angular momentum**
- ➔ We are still **missing transport processes** to explain surface abundances of solar-like stars (the parameters we infer are then still uncertain)
- ➔ **Atomic diffusion**, **rotation** and **penetrative convection** may explain **lithium abundances in Population II stars**, starting with an initial primordial abundance

Diffusion velocities
(for mixture of metals)

- `Burger 1969`
- `Chapman & Cowling 1970`
- `Michaud & Proffitt 1993`
- `Thoul+94`

Radiative accelerations

- direct use of atomic data (atmospheres)

- use of opacity tables with fixed frequency grids (Montréal/Montpellier code: `Turcotte+98`, OPCD: `Seaton 2005`)

- Single-Valued Parameter approximation (`LeBlanc & Alecian 2004`)

Opacities

- OP monochromatic opacities (OPCD, `Seaton 2005`)

- OPAL monochromatic opacities (Montréal/Montpellier code, tables not public)

🟥 CESTAM
🟩 Montréal/Montpellier code
🟦 TGEC
🟨 MESA

Diffusion velocities
(for mixture of metals)

- 🟩 - Burger 1969
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Radiative accelerations

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Opacities

- 🟨🟦🟥 - OP monochromatic opacities (OPCD, Seaton 2005)
- 🟩 - OPAL monochromatic opacities (Montréal/Montpellier code, tables not public)

Atomic diffusion (with radiative accelerations)

- **First estimation**

94 Ceti A: age difference of **4%** (Deal et al. 2017)

- **Using optimization method (AIMS,** Lund & Reese 2017, Rendle et al. 2019**)** : Classical + seismic constraints

1.4 $M_\odot$ at solar metallicity: difference in age of **10-15%**, in mass of **1-4%**, and in radius of **1%**
(Deal et al. 2018, 2020)

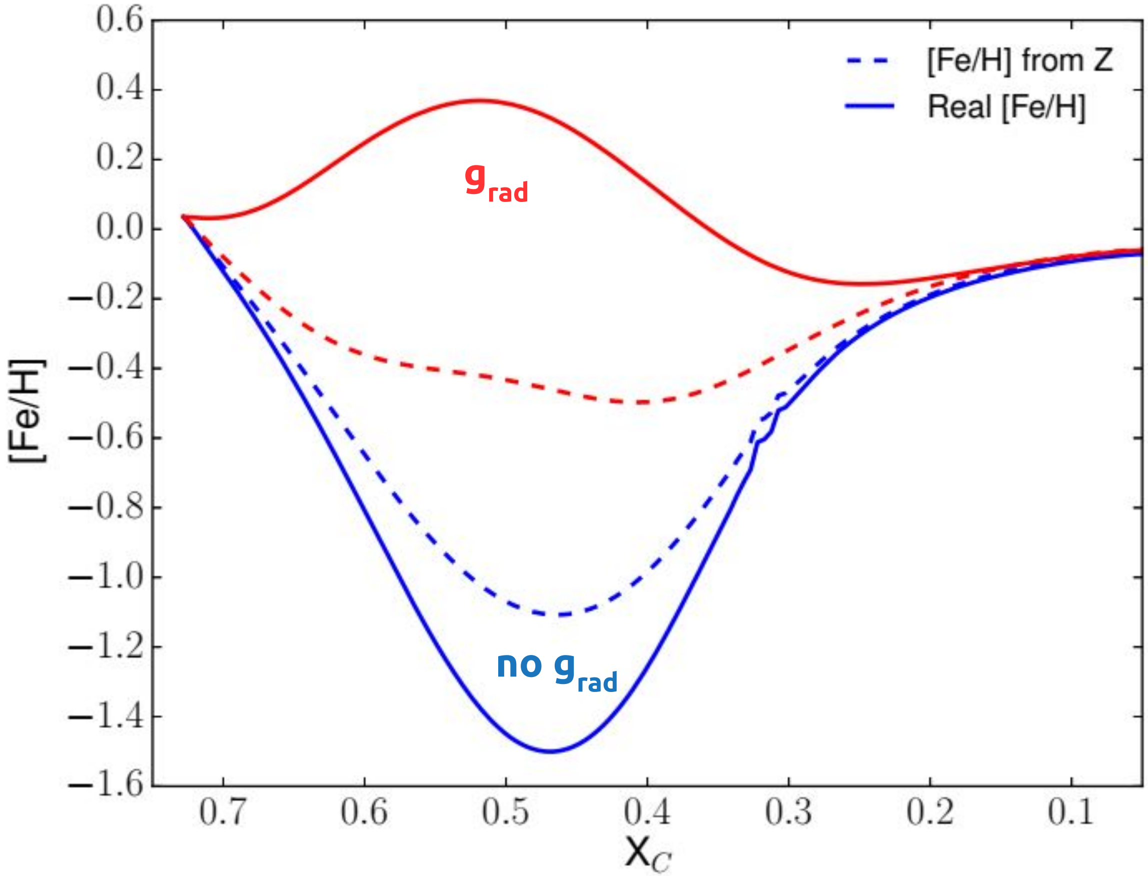
Rotation + atomic diffusion

1.4 $M_\odot$ at solar metallicity: difference in age of **25%**, in mass of **2-5%**, and in radius of **2%**
(Deal et al. 2020)

PLATO: **10%** on ages, **5%** on masses and **2%** on radii

1.4 $M_\odot$, $[Fe/H]_{ini}$=0.035

Deal et al. 2018



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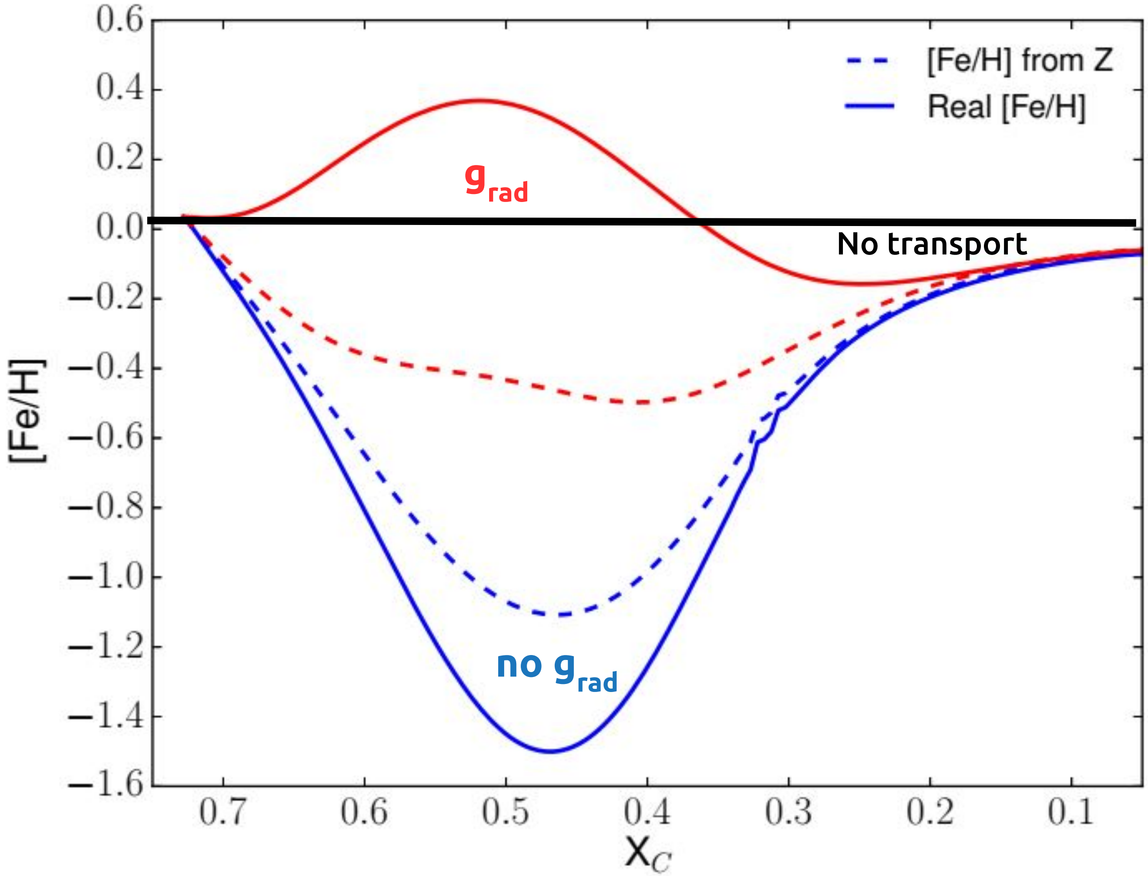
Deal et al. 2018



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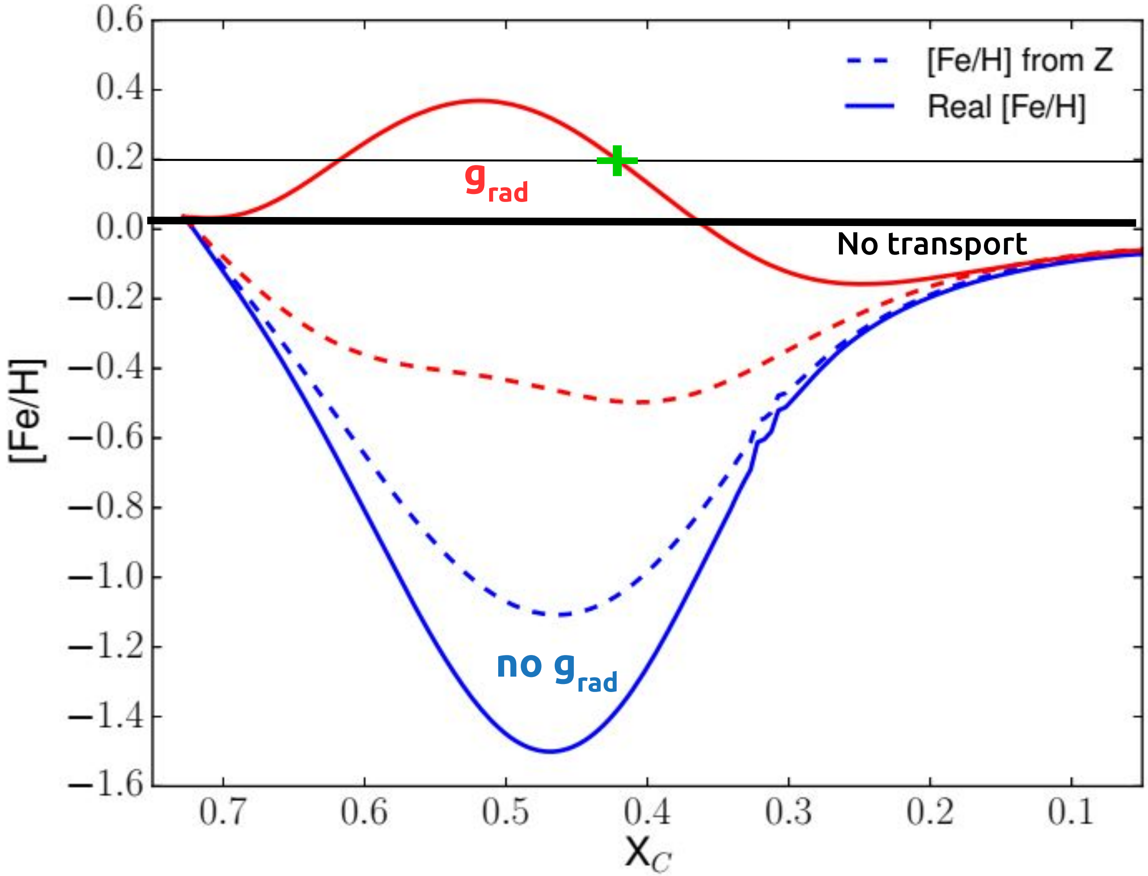
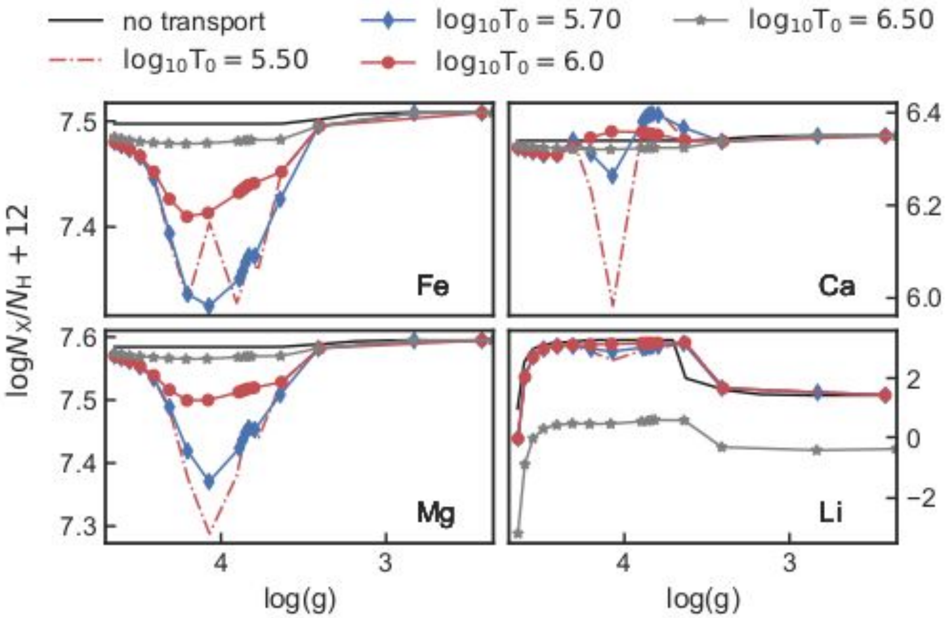


Calibrating the missing process(es) with ah-hoc prescription:

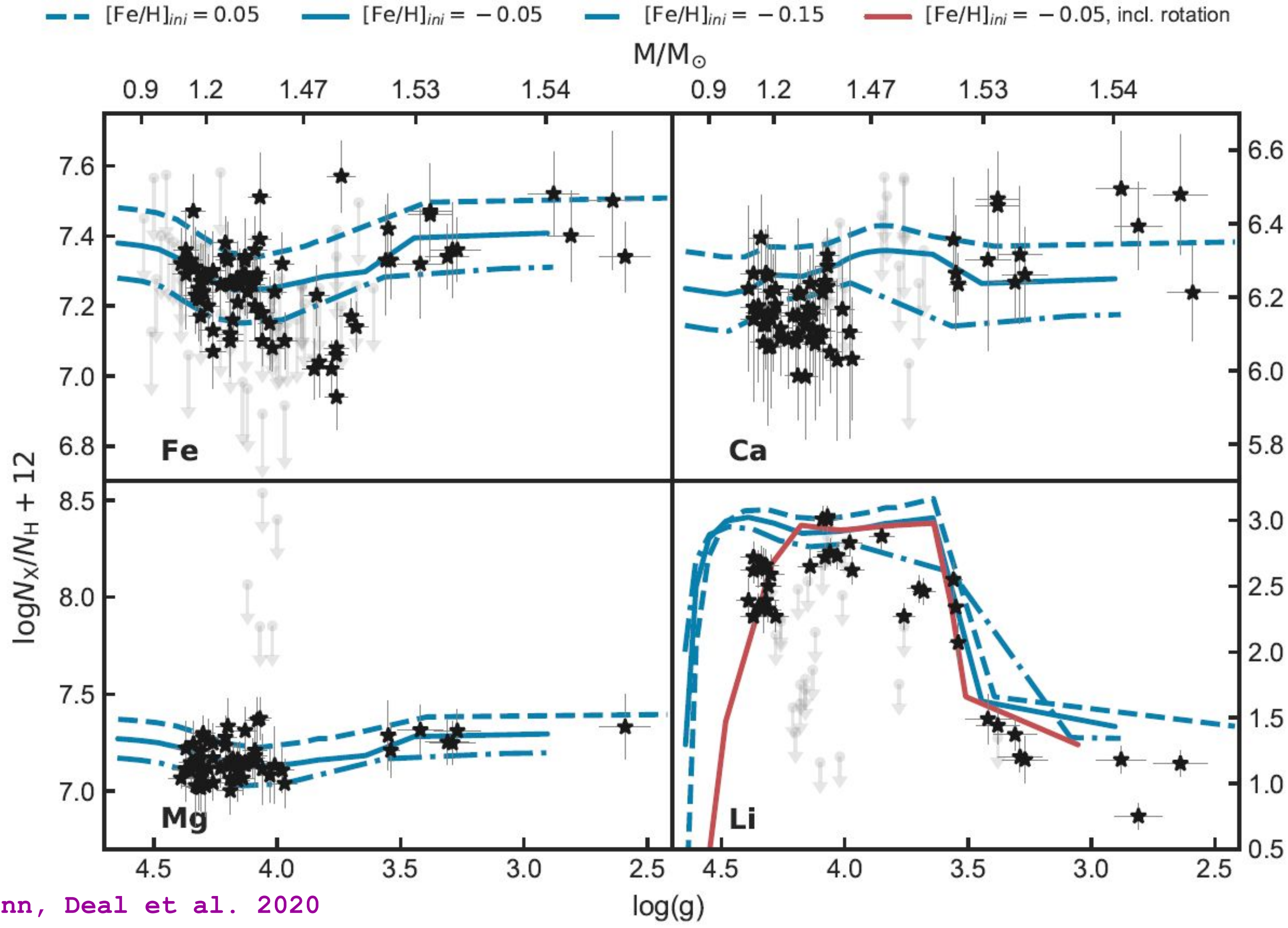
$$D_{\rm turb} = \omega D(He)_0 \left(\frac{\rho_0}{\rho}\right)^n$$

Richer et al. 2000; Richard et al. 2001; Michaud et al. 2011; Semenova, Bergemann, Deal et al. 2020

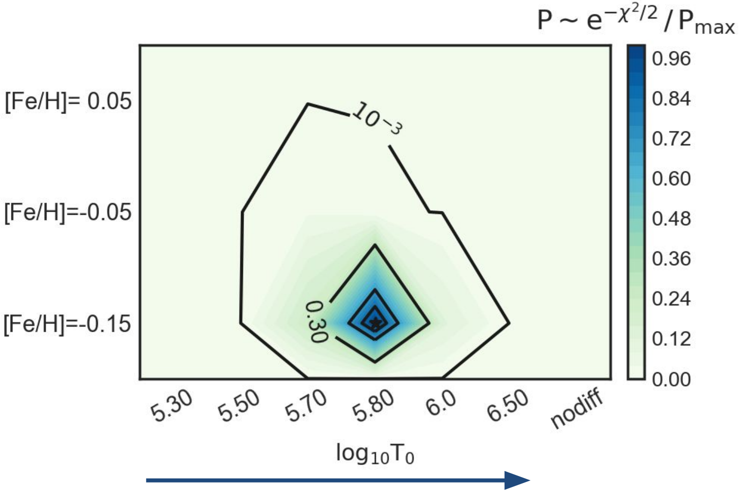
NGC2420: ~2.5 Gyr, [Fe/H] = -0.10 +/- 0.1 dex



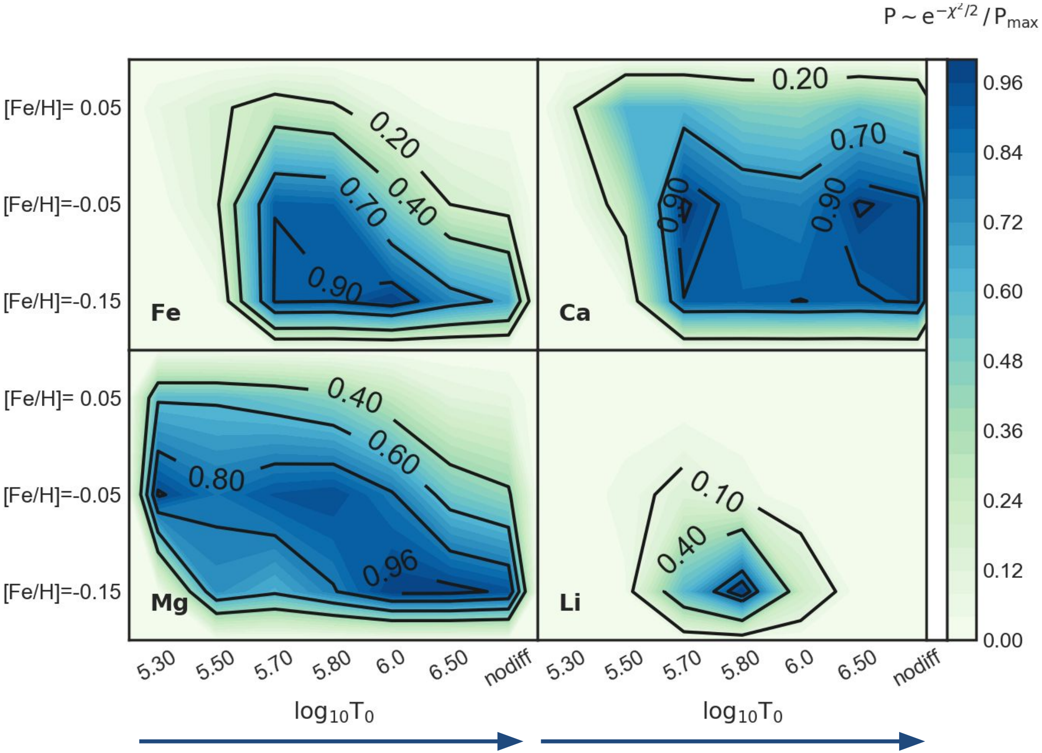
Semenova, Bergemann, Deal et al. 2020



Semenova, Bergemann, Deal et al. 2020



Depth of the mixing



Depth of the mixing

Semenova, Bergemann, Deal et al. 2020