Multi-dimensional stellar structure models with the fully compressible time implicit code MUSIC

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Development of MUSIC "Multidimensionnal Stellar Implicit Code"

(Viallet et al. 2011, 2013, 2016; Geroux et al. 2016; Pratt et al. 2016; Goffrey et al. 2017)

ANR blanche; ERC "TOFU" + "COBOM"

- Cartesian - **Spherical geometry** (2D or 3D)

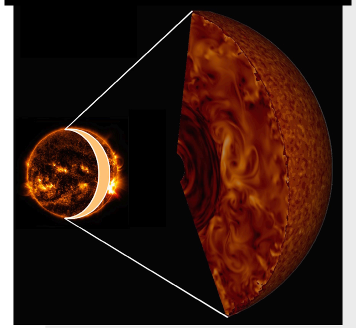
- **Fully compressible** hydrodynamics

$$\frac{\partial}{\partial t}\rho = -\nabla \cdot (\rho \vec{u})$$

$$\frac{\partial}{\partial t}\rho e = -\nabla \cdot (\rho e \vec{u}) - P\nabla \cdot \vec{u} + \nabla \cdot (\chi \nabla T)$$

$$\frac{\partial}{\partial t}\rho \vec{u} = -\nabla \cdot (\rho \vec{u} \otimes \vec{u}) - \nabla P + \rho \vec{g}$$

Spherical wedge



- **Thermal conductivity** (radiative transport)
  κ is the gas opacity (OPAL tables)

$$\chi = 16\sigma T^3 / 3\kappa\rho$$

- **Realistic equation of state** (ionisation, partial degeneracy, mixture of composition, etc…)

- **Implicit Large Eddy simulations** (ILES; numerical viscosity due to truncation errors of scheme)

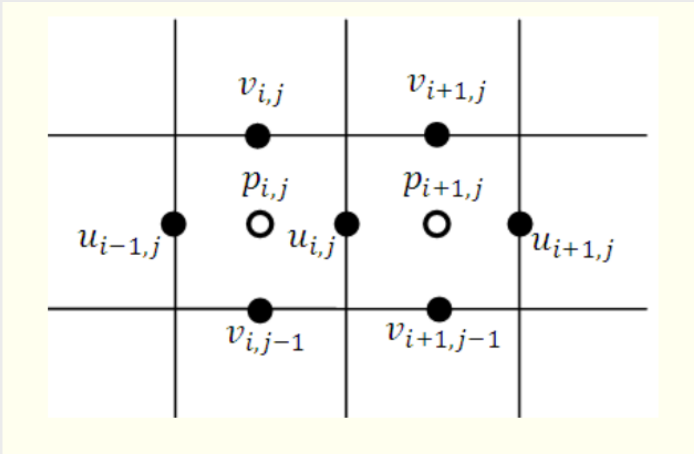
- Benchmark tests (Rayleigh-Taylor, Kelvin Helmholtz, Taylor-Green vortex)
  - ➤ ***Accurate for a wide Mach number range M ~ 10^{-6} - 1***

  (Goffrey et al. 2017)

- Finite volume method on a staggered grid

  (***helps for hydrostatic equilibrium*** $\nabla P = -\rho \mathbf{g}$

  No need for a well balanced scheme)



- **Initial model from 1D stellar evolution calculation**
  - ➤ *interface with Lyon code (Baraffe et al.) and MESA (Paxton et al.)*

- Solution to treat various stiff scales

☛ **Time implicit integration** *(no stability limit on the time-step)*

$$\frac{du(t)}{dt} = f(u(t)) \longrightarrow u^{n+1} = u^n + \Delta t f(u^{n+1})$$

➤ low storage Jacobian-Free-Newton-Krylov solver (Knoll & Keyes 2004)

(Jacobian is not stored and matrix-vector products are estimated with finite-differencing)

(Viallet et al. 2016; Goffrey et al. 2017)

- **Additional and on-going developments** (*Thomas Guillet*)
  - Rotation (coriolis + centrifugal force)
  - Wave analysis (spherical harmonics projection + FT) (*Arthur Le Saux's talk)*
  - Viscosity
  - Passive and active scalar (advection and chemical diffusion)
  - Lagrangian tracer particles
  - MHD (in progress)

Motivation for MUSIC: improve phenomenological approaches used in 1D stellar evolution codes to describe major hydro/MHD processes.

I. First application: Effect of accretion on the structure of very young low mass, convective stars *(Geroux et al. A&A, 2016)*

Problem: Phenomenological treatment of accretion in 1D stellar evolution codes based on major assumptions of instantaneous redistribution of **accreted mass** and **energy** in the interior
(Baraffe et al. 2009, 2012; Hosokawa et al. 2011; Kunitomo et al. 2017; Sigurd & Haugbolle 2017; Haemmerle et al. 2019, etc...)

Effect of amount of accretion energy absorbed $\mathbf{L_{acc} = \alpha (GM\dot{M})/R}$

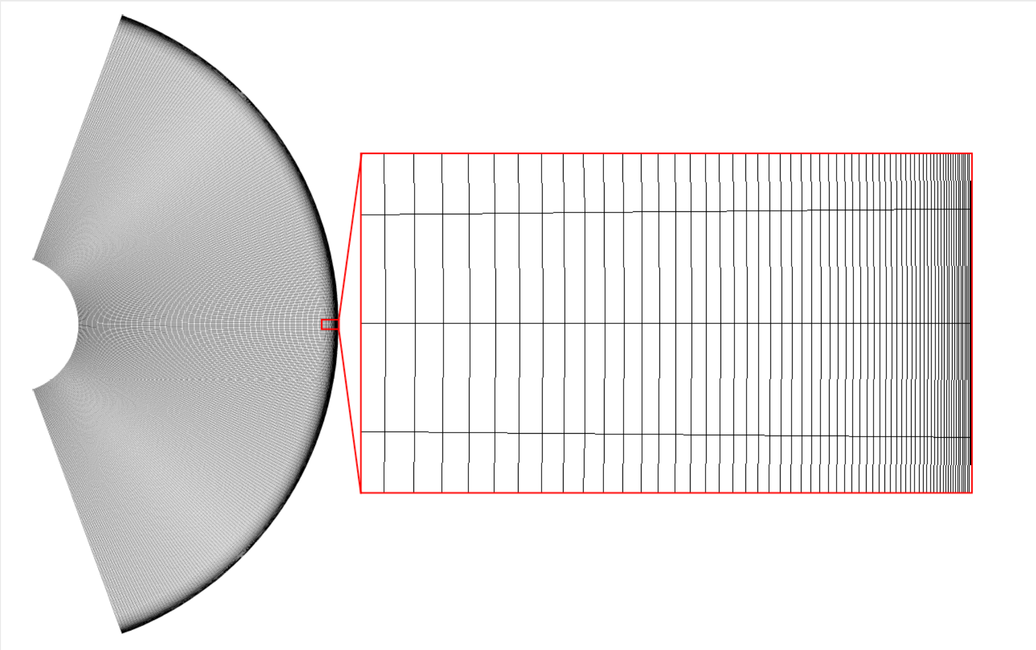
$\alpha \sim 0$ —> "cold" accretion

$\alpha > 0$ —> "hot" accretion

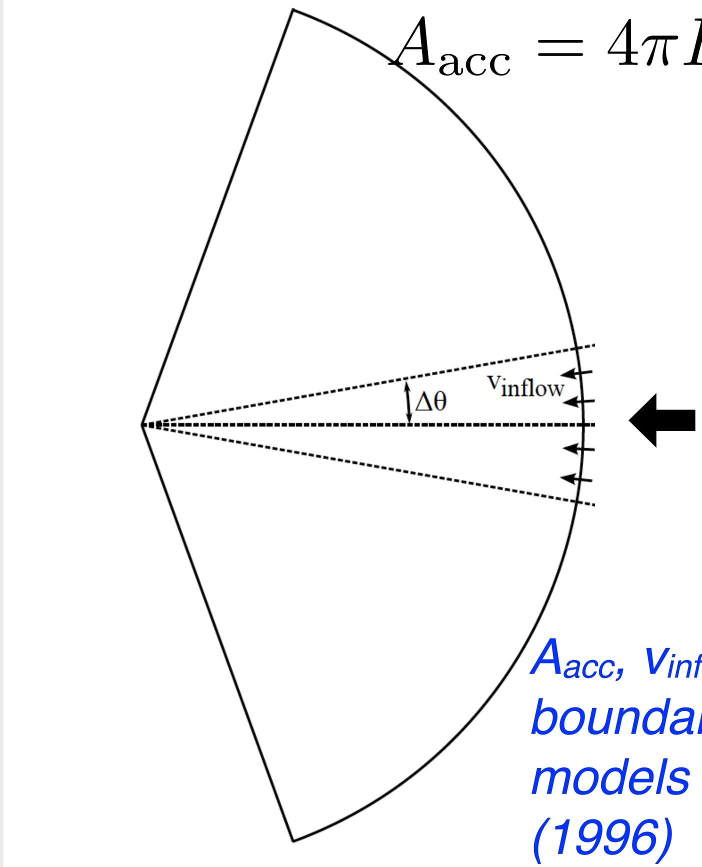
☛Test of these assumptions with MUSIC

Treatment of the surface must be realistic with $F_{surf} = \sigma T^4$

Use of a spliced grid to resolve smaller scales/steep gradients at the surface



$$\rho_{acc} = \dot{M}/(A_{acc} v_{inflow})$$
$$A_{acc} = 4\pi R^2 sin(\Delta\theta)$$



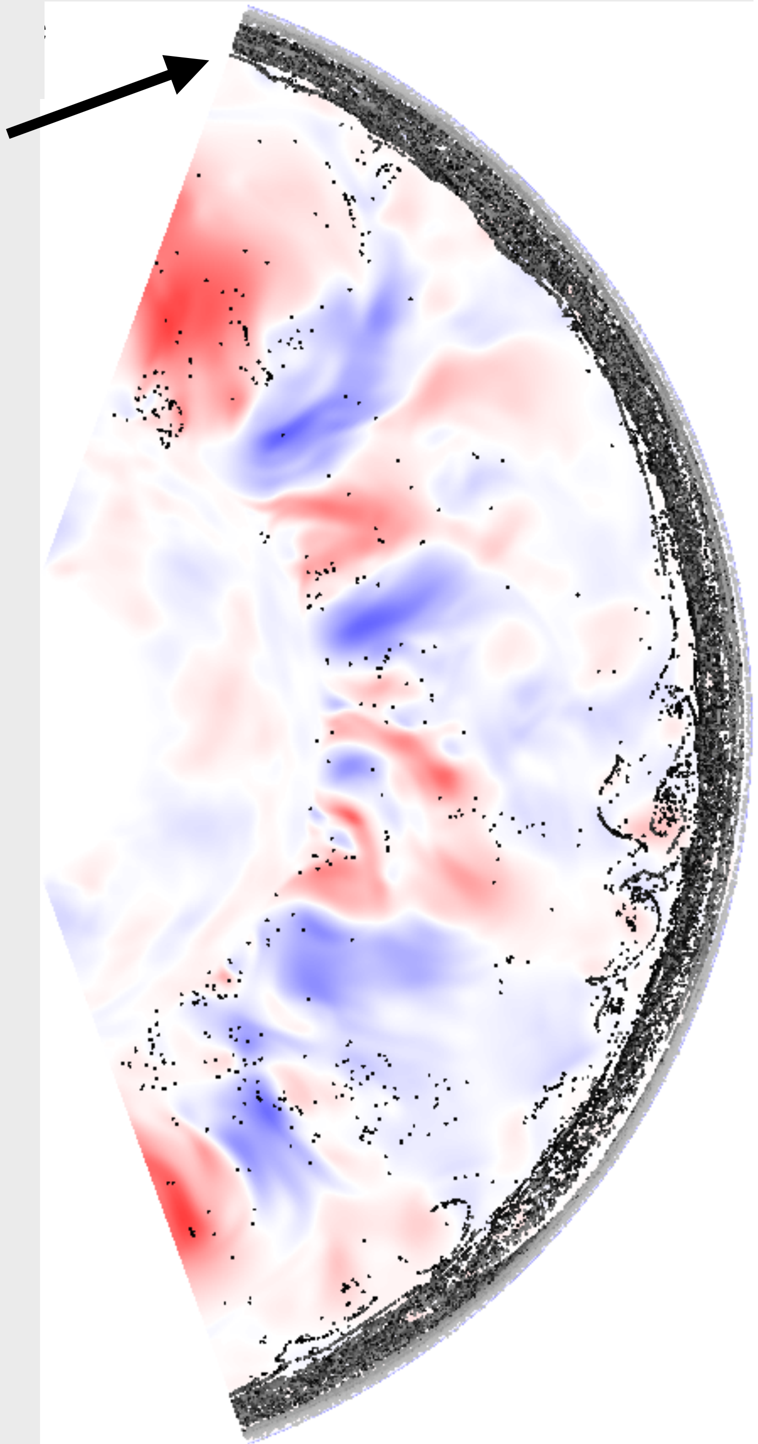
A_{acc}, v_{inflow}, $\Delta\theta$ from boundary layer models of Kley & Lin (1996)

One main result:

- For hot accretion ($\alpha \gtrsim 0.1$ with $L_{acc} = \alpha\,(GM\dot{M})/R$), formation of a hot surface layer (**no deep mixing** of accretion energy)

- Assumption in 1D codes of redistribution of accretion energy deep in the interior **overestimates the effect** on the structure for $\alpha \gtrsim 0.1$ (expansion of accreting object)

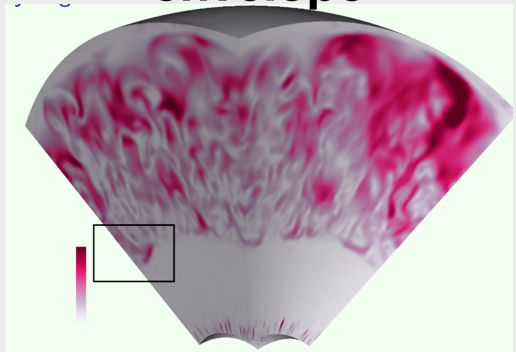
- Use of an accretion boundary condition $\mathbf{L_{surf} = L_{acc}}$ is more realistic in 1D codes



II. A numerical survey of convective penetration/overshooting in stars

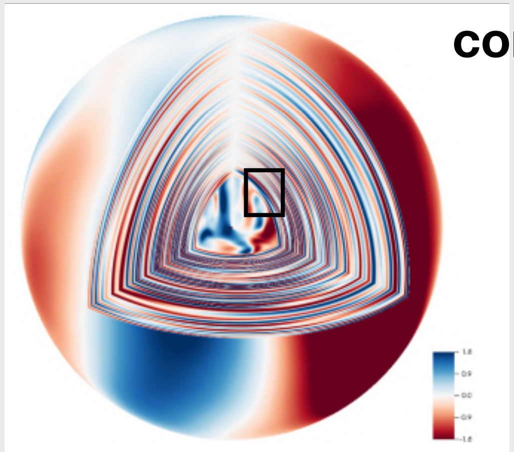
☛ **Extra mixing at a convective boundary due to convective penetrating flows or "plumes": process of overshooting or *penetration***

(Roxburgh 1965; Shaviv & Salpeter 1973; Schmitt et al 1984; Zahn 1991, etc…)



Velocity magnitude during steady-state convection, in a 128^3 wedge of our sun at ~ 1 Myr, simulated with a the three-dimensional, time-implicit, compressible LES, stellar evolution code MUSIC (ERC Advanced grant, University of Exeter).

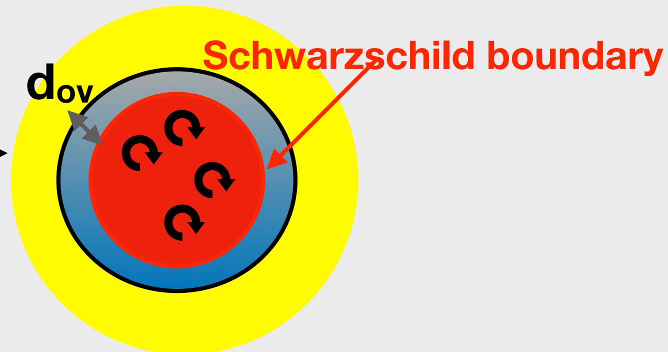
☛ Chemical mixing, transport of angular momentum, wave excitation, etc...



☛ Affects the Li depletion in solar type stars, core size, age, surface properties and abundances, last stages of evolution, etc...

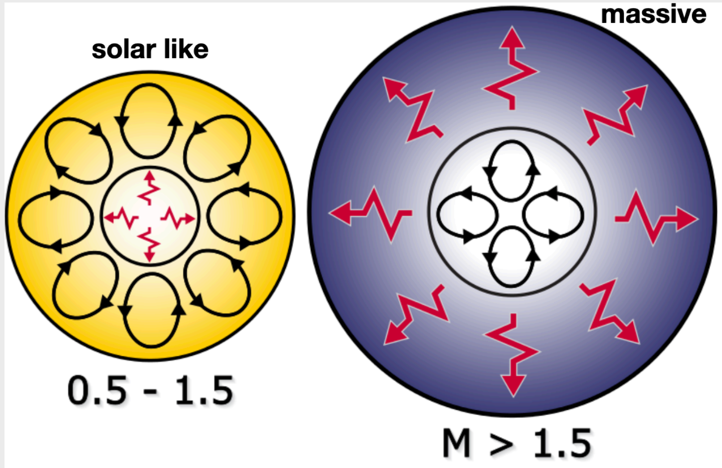
Standard treatment in 1D codes: instantaneous mixing over an arbitrary width $d_{ov} = \alpha_{ov} H_P$

(α_{ov} free parameter)



Goals & Questions we want to address with MUSIC:

1) Derive scaling laws d_{ov} (M_{star}, L_{star}, etc..) to implement in 1D codes for a range of stellar masses at various age (pre-MS, MS, post-MS)



2) Can we use the same numerical and statistical framework for envelope and core overshooting to derive d_{ov}?

- Envelopes (Mach ~ 10^{-4} - 1)   Cores (Mach < 10^{-4})
- Generalisation of statistical analysis based on **extreme events of penetrating flows** to convective envelopes (downward) and cores (upward)?

(Pratt et al. 2017)

3) Analysis of gravity and acoustic waves → build the link with asteroseismology *(see next talk of Arthur Le Saux)*

4) What is the impact of rotation and magnetic field on convective penetration?

1) Convective envelopes of solar-type stars

2D Experiment: Numerical simulations of a 1 $M_\odot$ solar-like model with enhanced luminosity: L x 1, 10, 10^2, 10^4

(Baraffe et al. 2021; Le Saux et al. 2022)

The problem of thermal relaxation of stellar hydrodynamical simulations

Achieving thermal relaxation is a well-known challenge for hydrodynamical simulations based on realistic stellar structures

- $\tau_{thermal} = GM^2/(RL) \gg \tau_{dyn}, \tau_{conv},$

→ computationally unreachable for major phases of evolution (MS, He burning)

☛ Common procedure:

Artificially increase the luminosity and/or the thermal diffusivity

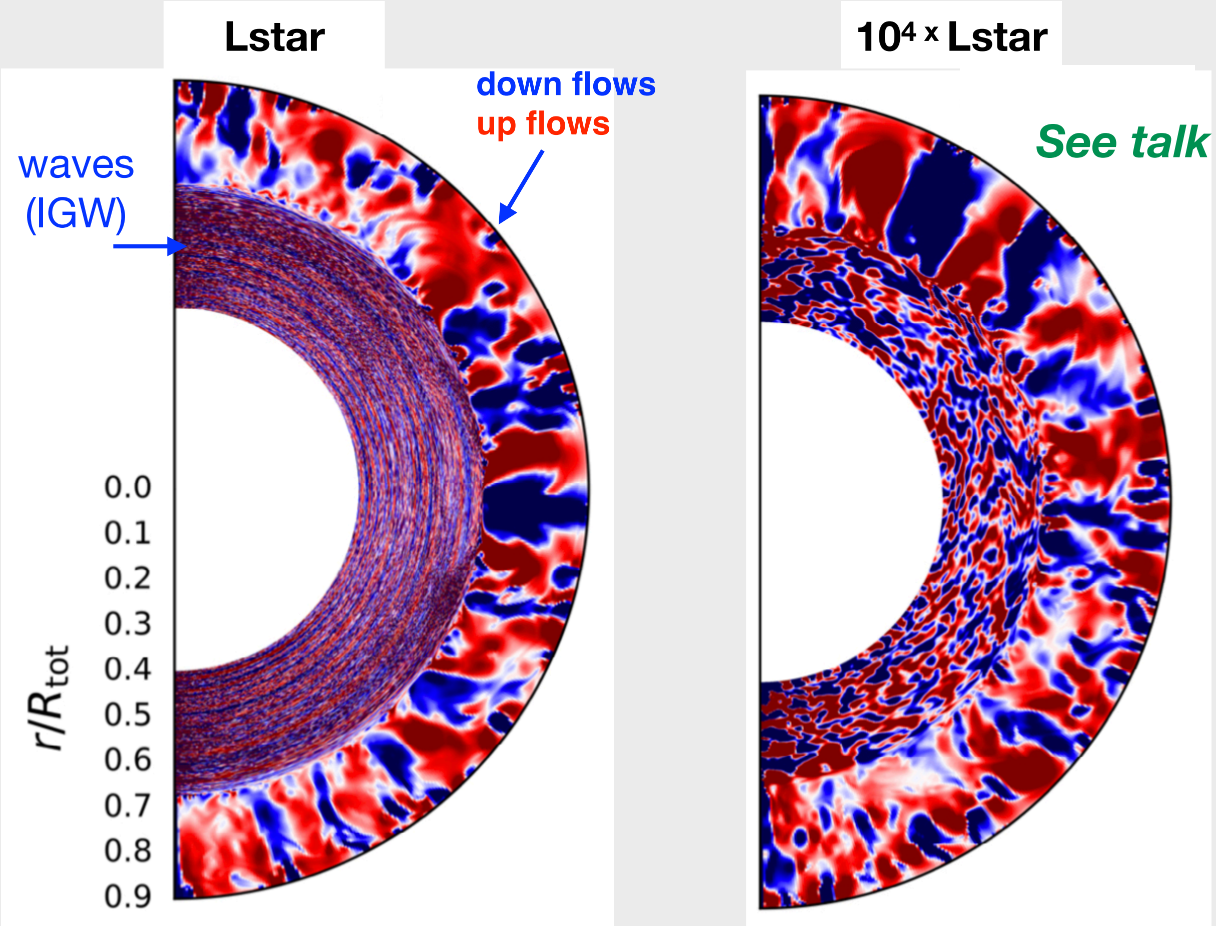
L can be increased by up to 10^7

⟹ decrease the thermal timescale $\tau_{thermal}$

⟹ enables to reach a thermally relaxed steady state ⟺ "accelerate" the simulation

(Meakin & Arnett 2007; Brun et al. 2011; Rogers et al. 2013, Cristini et al. 2017; Edelmann et al. 2019; Horst et al. 2020, etc...)

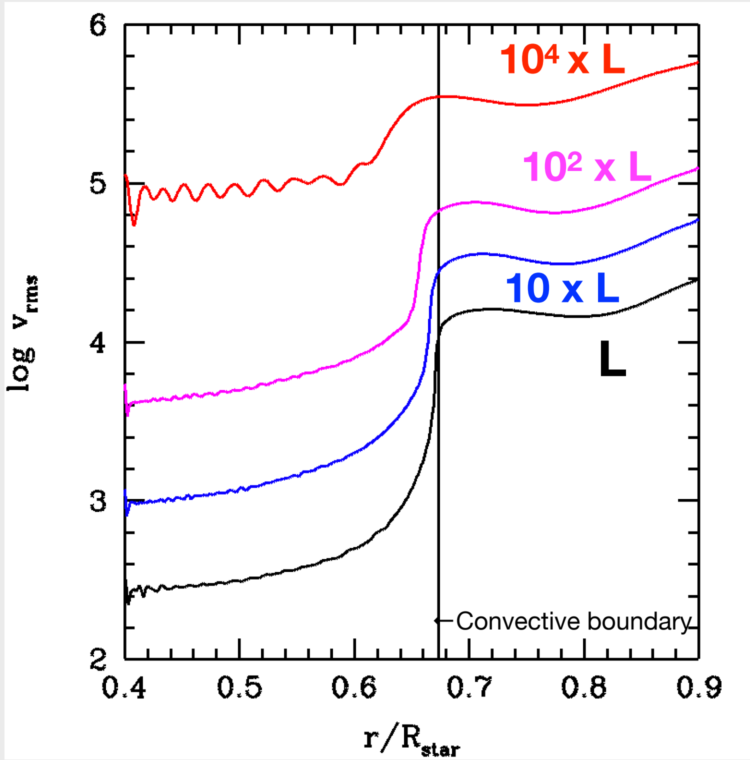
Radial velocity snapshot: convective envelope and radiative core



See talk of Arthur Le Saux for impact on waves

- **Increase of the velocities in the convective envelope with L:**

  $v_{rms} \propto L^{1/3}$



Scaling expected from simple dimensional arguments:

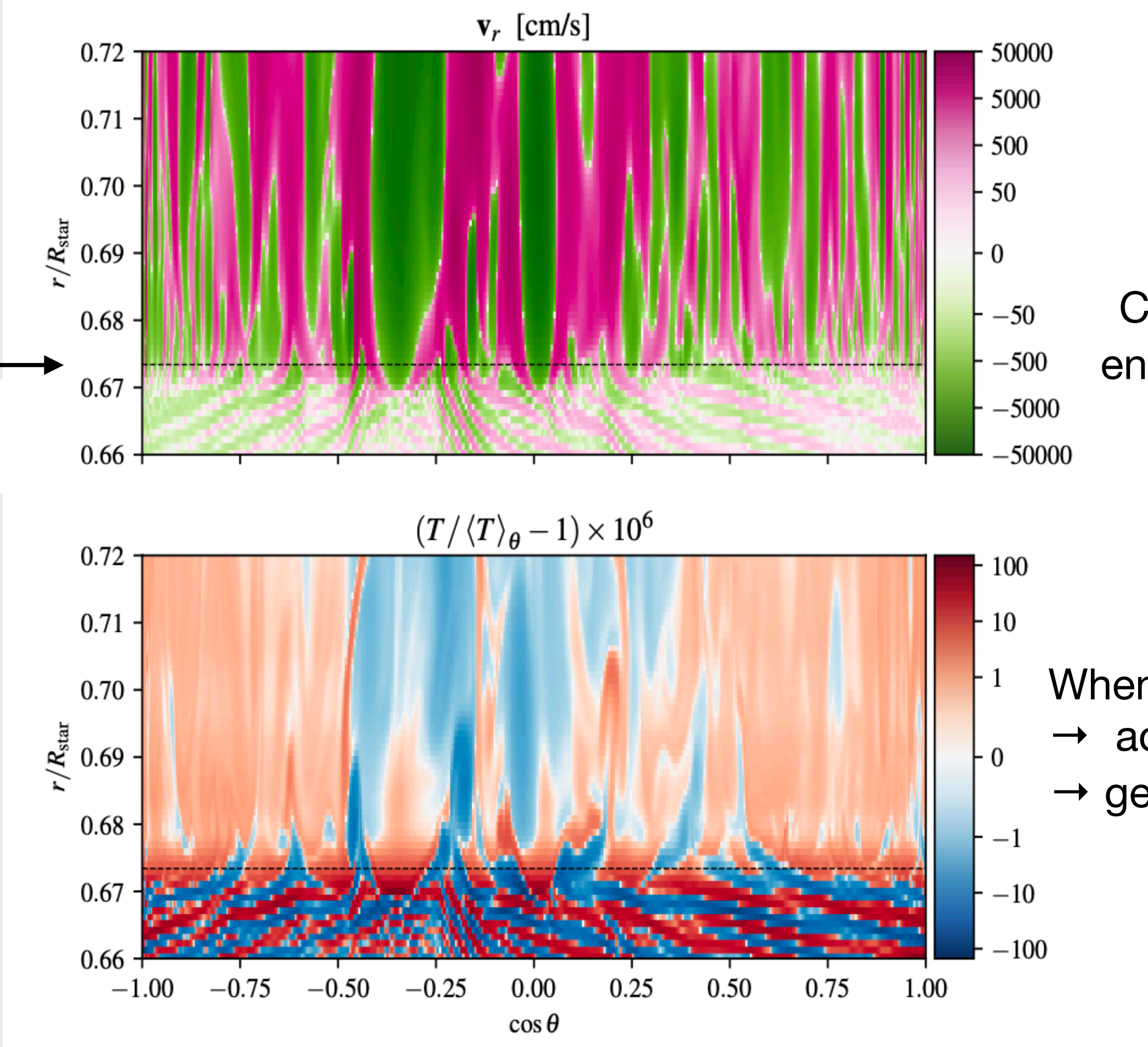
- $L = 4\pi r^2 F_{tot}$ and $F_{tot} = F_{kin} + F_{enth}$

  $F_{kin} \propto v^3$ $\quad F_{enth} \propto v\,\delta T \propto v^3$ $\quad (\delta T/T \sim \delta P/P \sim \rho v^2)$

- **Analysis of penetrative flows ("plumes") as a function of L_{star}**

Visualisation of radial velocity and temperature fluctuations

convective boundary



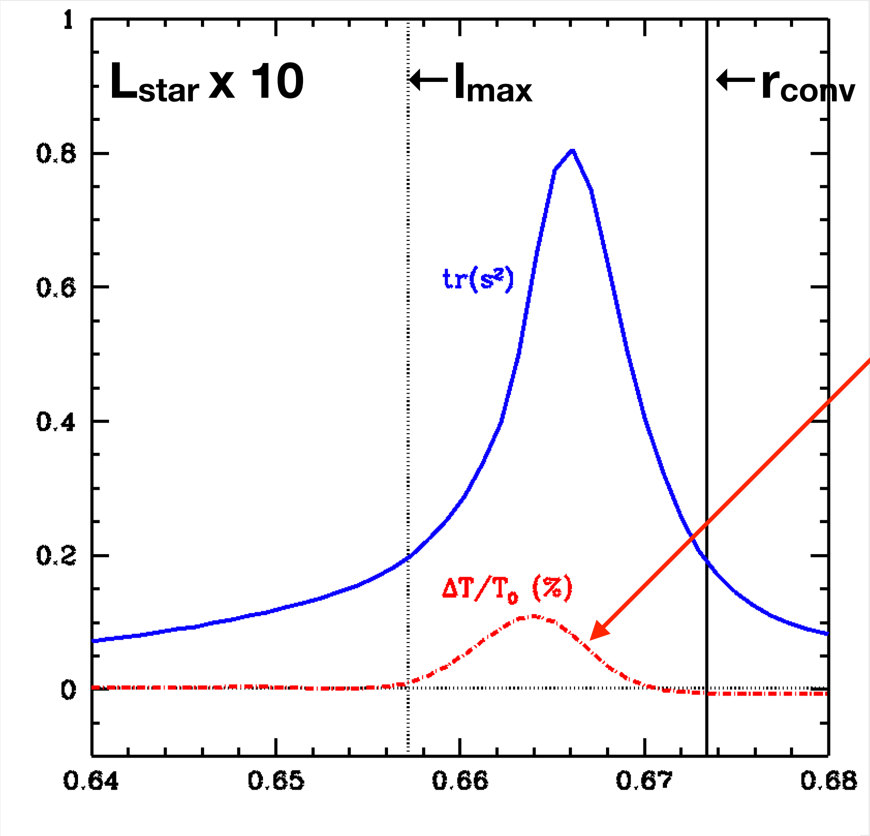
Convective downflows transport low entropy (cool) material (i.e negative δT)

When downflows cross the boundary
→ adiabatically compressed
→ get hotter (i.e positive δT) and less dense

Use the framework based on extreme events to infer an overshooting depth:

Extreme penetrating plumes (and not the average) characterise the relevant penetration depth in stars → **contribute to mixing on the long term** *(Pratt et al. 2017, 2020; Baraffe et al.2021)*

- **Local heating due to convective flow penetration**



local increase of the temperature in the region of penetration ($\Delta T = \langle T \rangle_{\theta, t} - T_{init}$)

→ peak in T corresponds to a peak of the rate-of-strain tensor tr(s^2)

⇒ compression and shear induce local heating and thermal mixing (through mixing of hot material)

☛ Modification of the local background is enhanced with increasing L

⇒ Reduce the braking of the penetrating plumes

⇒ strongest plumes progress deeper → broadening the penetration region

⟹ A "boosted" model is not only an "accelerated" version of a reference model

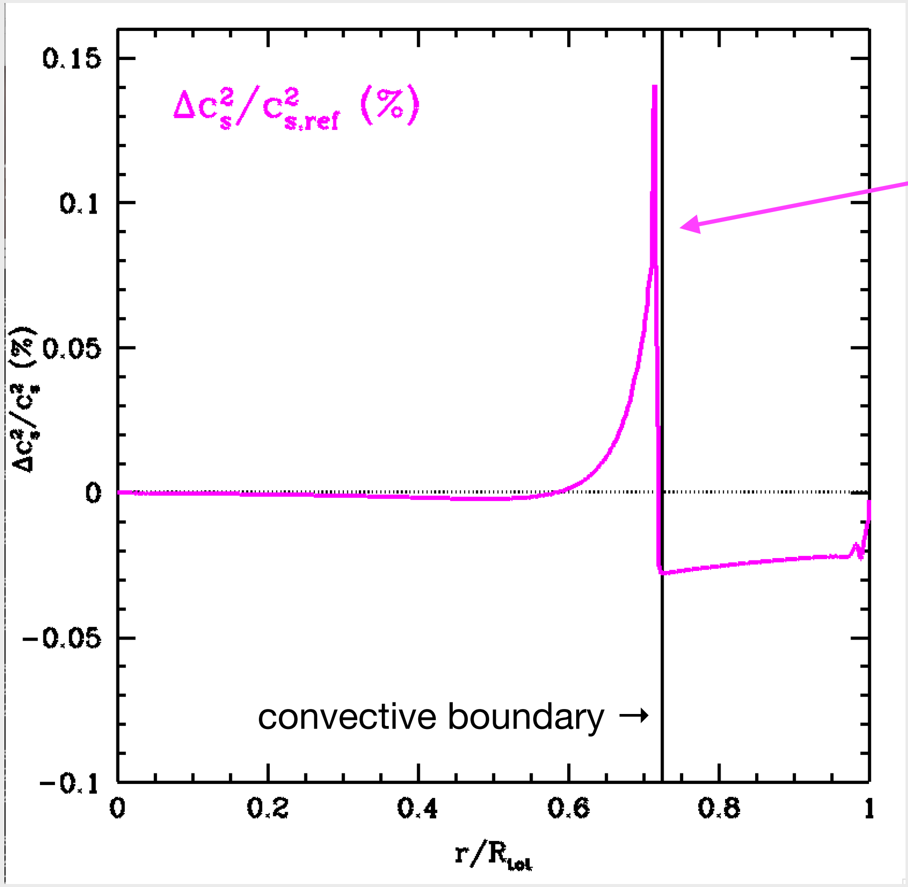
Increasing L can push the simulated conditions away from the original target star

⟹ These simulations may describe different physical conditions

• Impact of local heating on the solar structure and the "solar modelling" problem

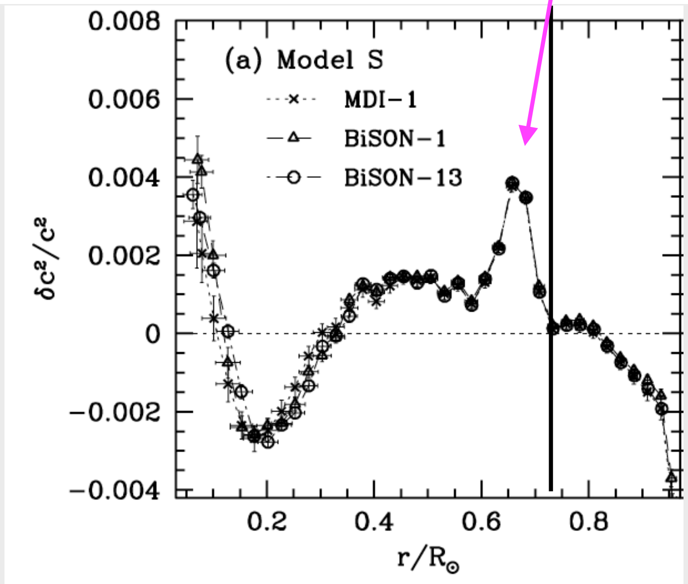
Test on a 1D model: Modification of the temperature profile just below the convective envelope, following the hydro simulations *(Baraffe et al. 2022)*

Difference between modified and non-modified Sun model



Peak in the speed of sound profile which could help reproducing helioseismology

Relative difference between the Sun and a standard model



(Basu et al. 2009)

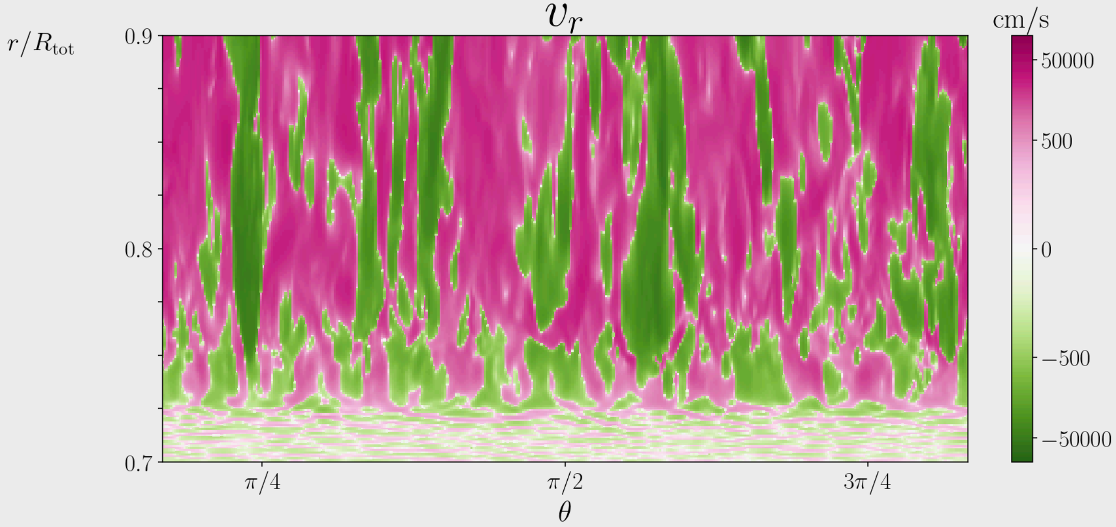
• Next steps

- Extension to 3D

Preliminary results for a solar model show similar structures with penetrating "plumes"

(Vlaykov, et al. in prep)

Visualisation of radial velocity for an arbitrary angle φ (513^3)



- Analysis of overshooting depths for pre-MS and MS solar-like models *(Vlaykov, et al. in prep)* and of Red Giant Branch stars *(Pratt et al. in prep)*

- Impact of rotation (and magnetic field) *(Vlaykov, Guillet, et al. in progress)*

2) Study of the convective core of massive stars

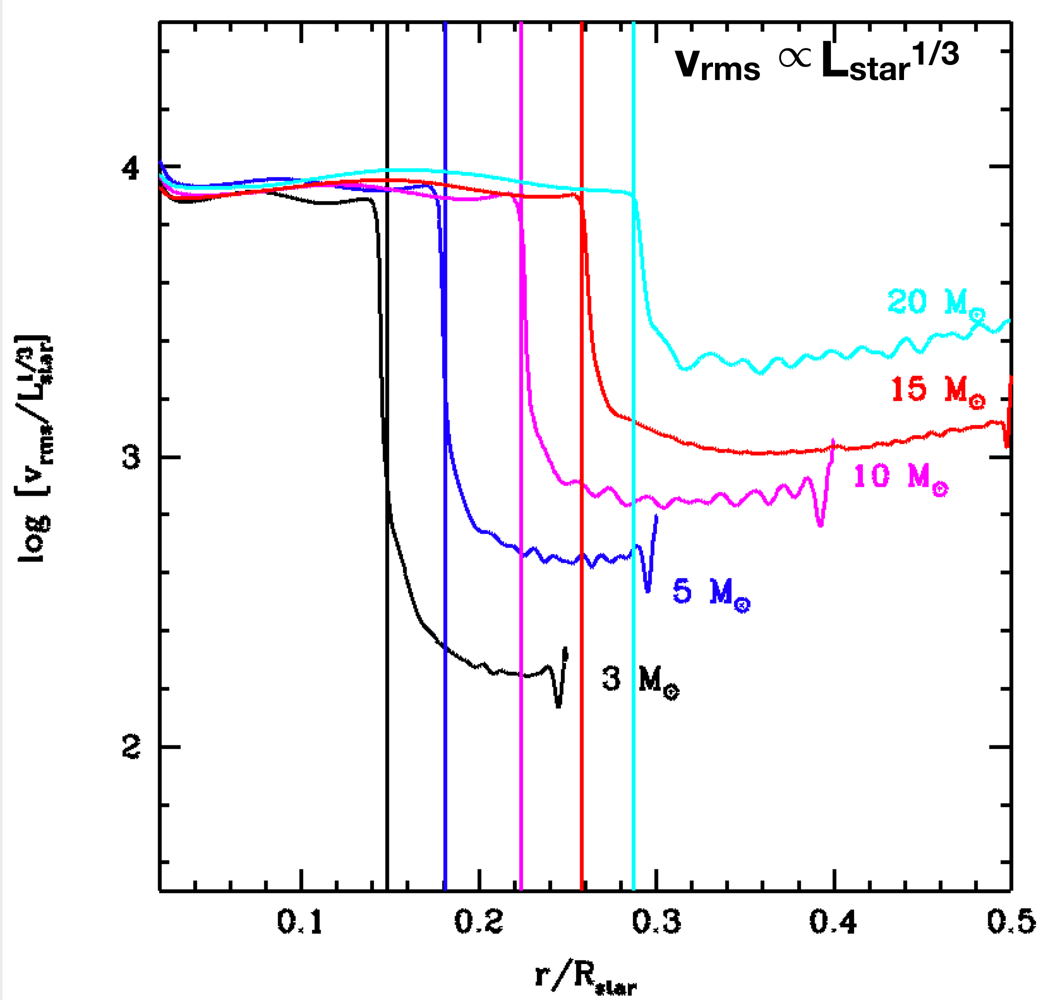
- 2D survey of convective penetration as a function of stellar mass: 3 $M_\odot$ - 20 $M_\odot$

Cores at the beginning of H burning:

R_{core}, M_{core} ↑ with M_{star}

$L_{star} \propto M_{star}^{3.3}$

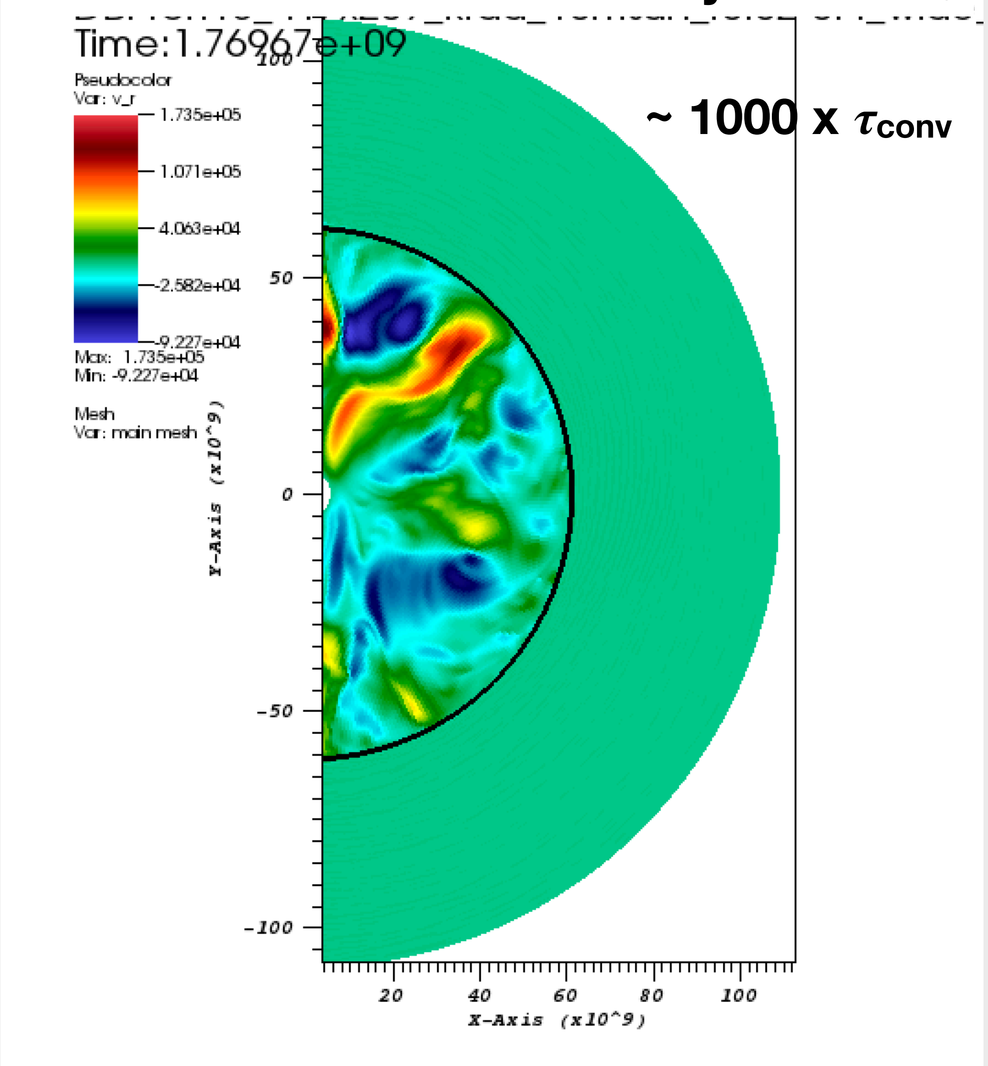
☛ **Scaling of v_{rms} with L:** $v_{rms} \propto L_{star}^{1/3}$



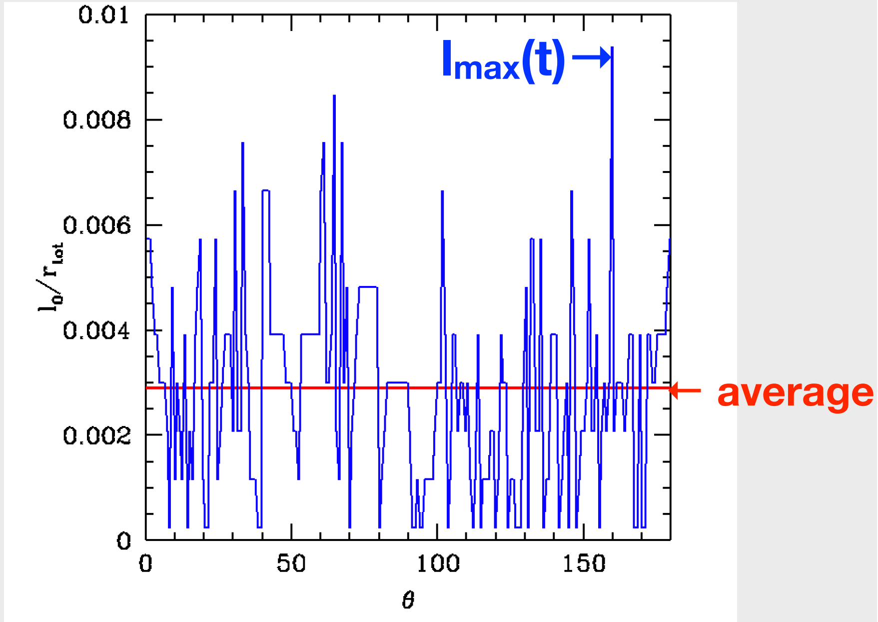
(Baraffe, Clarke, Mason et al. in prep)

- **Use of statistical approach of extreme penetrating flows to determine the extent of the overshooting layer as a function of stellar mass**

Visualisation of radial velocity for 10 $M_\odot$



Extent of penetration based on first zero of f_k and $f_{\delta T}$ at a given time t



vert. kinetic energy flux $\mathbf{f_k(r,\theta,t) = 1/2\, \rho\, v^2\, v_r}$

vertical heat flux $\mathbf{f_{\delta T}(r,\theta,t) = \rho\, c_P\, \delta T\, v_r}$

(Pratt et al. 2017)

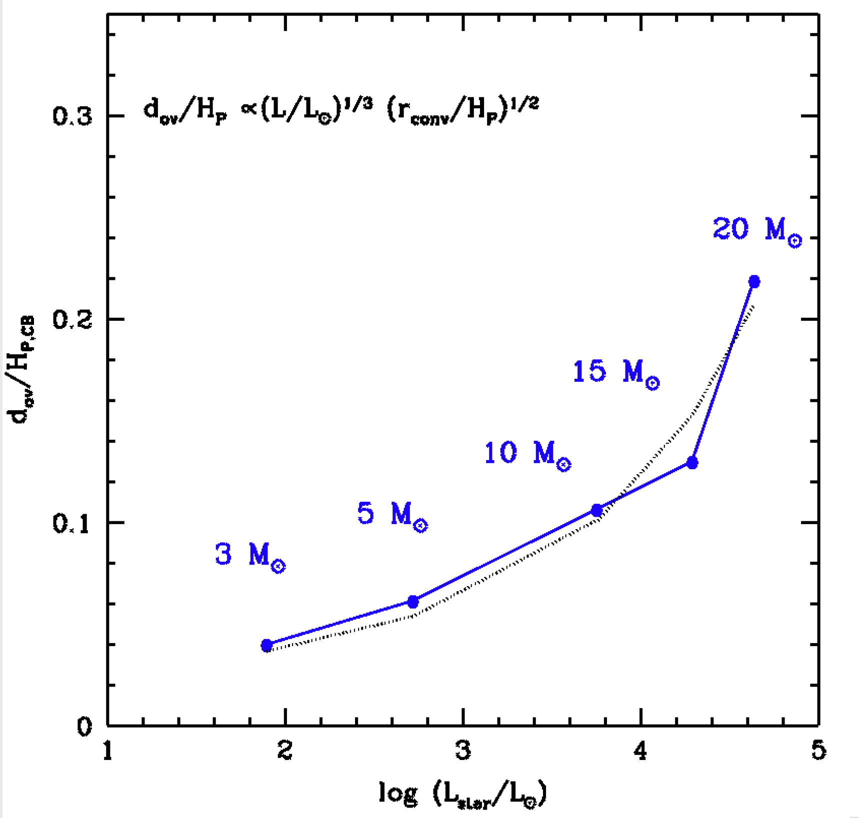
→ **Distribution of maximum depths of penetrating convective flows $l_{max}(t)$**

→ **Time average of l_{max} provides an effective width of the overshooting layer**

☛ **can be used to characterise the extent of mixing on the long term $d_{ov} = < l_{max}(t) >_t$**

Scaling with L and r_{conv}:

$d_{ov} \propto L^{1/3} (r_{conv}/H_P)^{1/2}$



☛ **Pioneering analytical model of Zahn (1991)**

First order estimate of the deceleration of a convective downdraft in a nearly adiabatically stratified penetration layer:

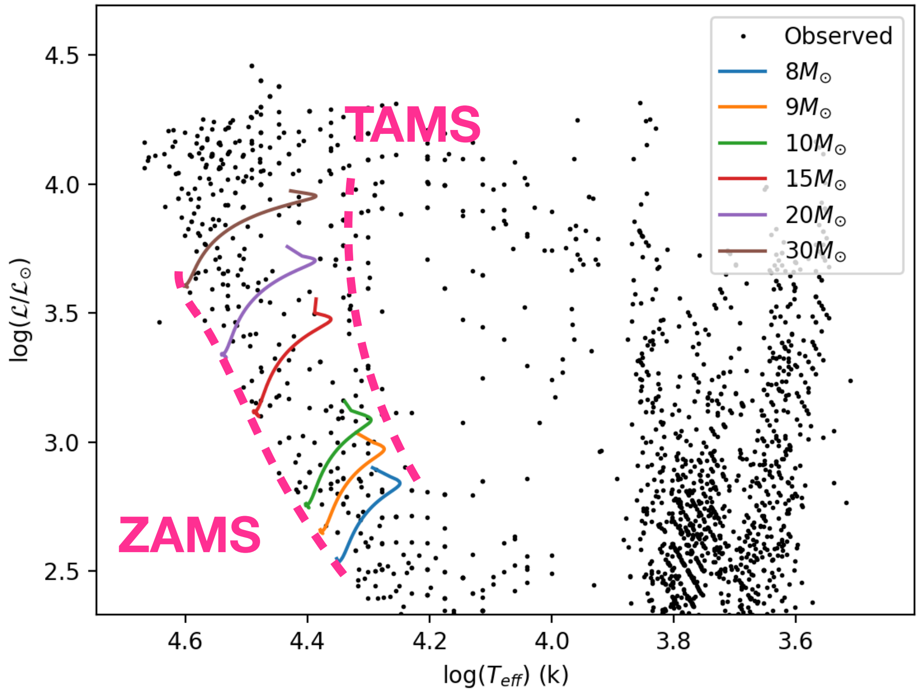
$$1/2\ dv^2/dz = g\ \delta\rho/\rho \propto g\ \delta T/T$$

→ Estimate of a penetration distance $L_P \propto L^{1/2} (r_{conv}/H_P)^{1/2}$

⇒ Application of our scaling relationship to stellar evolution models

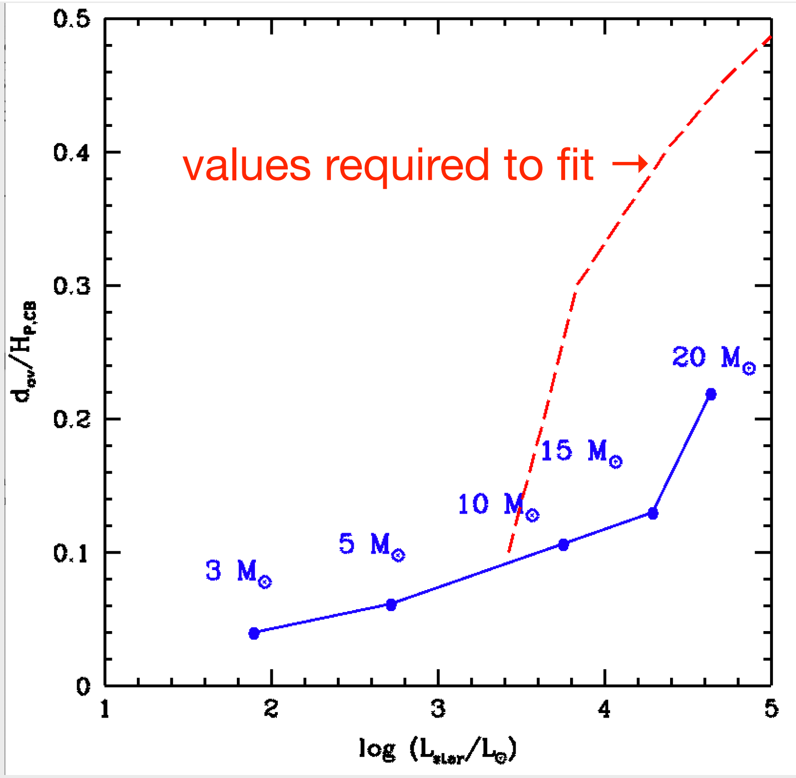
$$d_{ov}/H_{P,CB} = 3.05 \times 10^{-3} \times (L/L_\odot)^{1/3} \times (r_{conv}/H_{P,CB})^{1/2} + 0.02$$

Comparison of tracks with Milky Way stars from Castro et al. (2014)



→ First predictions going in the right direction with d_{ov} ↑ with M_{star}

→ But seems to need an increase of d_{ov} (up to ~factor 2) for M > 10 $M_\odot$



Conclusions

Generalised approach with MUSIC to address convective boundary mixing for envelopes (downward overshooting) and cores (upward overshooting)

✓ **Convective envelopes:**

📌 **Artificial enhancement of L_{star} should be taken with caution**

📌 **These experiments reveal a local heating in the penetration region**

↳ **solar modelling problem**

✓ **Convective cores:**

📌 **Preliminary scaling found: $d_{ov} \propto L^{1/3} (r_{conv}/H_P)^{1/2}$ consistent with observations suggesting $d_{ov} \uparrow M_{star}$**

📌 **But first predictions (based on ZAMS cores) seem to underestimate d_{ov}**

↳ **Effect of rotation? MHD??**

↳ ↳ **Development of double-diffusive instabilities?**