



From laboratories to astrophysics: the expanding universe of plasma physics  
Spring School, 1 – 12 May 2017

## Solar wind and space plasma turbulence

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# In this lecture

0. Solar wind : weakly collisional / fluid behaviour

1. Solar wind turbulence at MHD scales,  $f=[10^{-4},10^{-1}]$ Hz,  $l=[10^4,10^7]$ km

- spectra of different physical quantities ( $B, V, n_e$ )
- k-anisotropy
- intermittency

2. Turbulence around ion scales,  $f=[0.1-1]$ Hz,  $l=[10^3,10^4]$ km

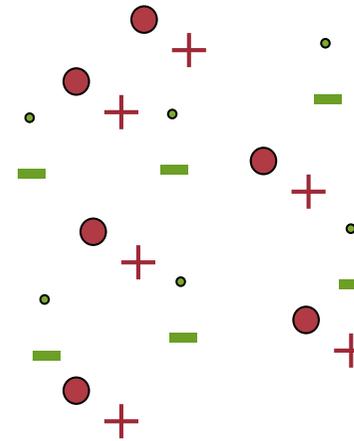
- spectral shape and transition scale (Larmor radius? Inertial length? ...?)
- coherent structures
- ion instabilities and waves

3. Turbulence at sub-ion & electron scales  $f > 3$ Hz,  $l < 200$  km  
(up to the observational limit of 400 Hz, 300 m)

- general spectrum?
- dissipation of e/m turbulence at electron scales ?
- nature of fluctuations: oblique waves + coherent structures...
- parallel whistlers waves and electron instabilities

# The solar wind

Closest stellar wind where we can do in-situ measurements with a number of space missions

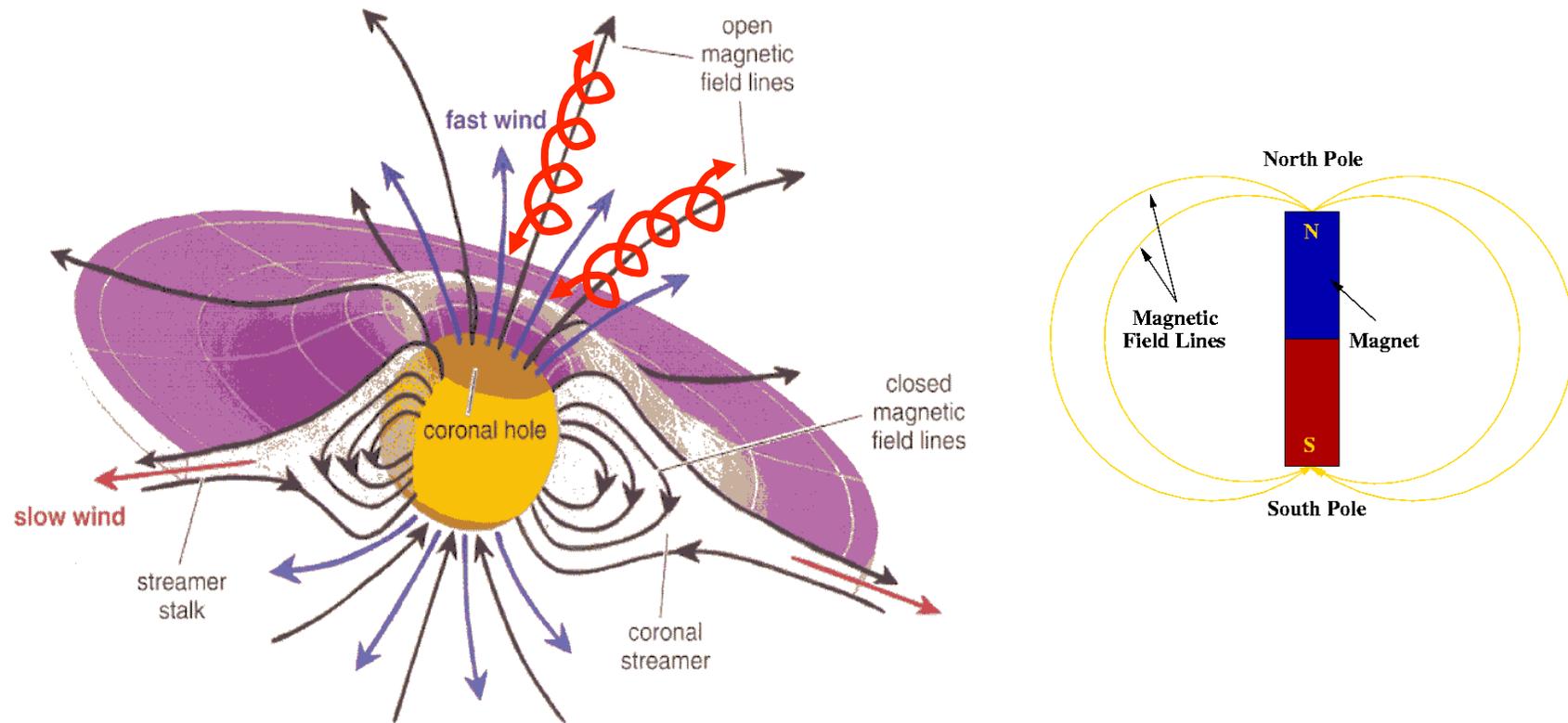


plasma : ionized  
gas (e<sup>-</sup> , p<sup>+</sup>, α<sup>+</sup>)

- Expansion of the solar corona in interplanetary space
- Plasma: ionized gas, essentially e<sup>-</sup> and p<sup>+</sup> (+5% of heavier ions)
- Mean speed ~500 km/s; density ~5cm<sup>-3</sup>, temperature (e<sup>-</sup>,p<sup>+</sup>) ~20 eV
- Few collisions (1 collisions/1AU) => conductivity ~ ∞ (viscosity η = 0) => magnetic field is frozen in plasma
- Solar wind transports coronal magnetic field

$$\partial_t B = \nabla \times (V \times B) + \eta \Delta B$$

# Magnetic field of the Sun: slow and fast streams and 11 years cycle



- Fast wind blows out from the coronal holes (open field lines).
- Slow wind – from the coronal streamers, above the closed field lines and along the heliospherical current sheet (c.f. purple zone).
- Every 11 years magnetic dipole of the Sun reverses.

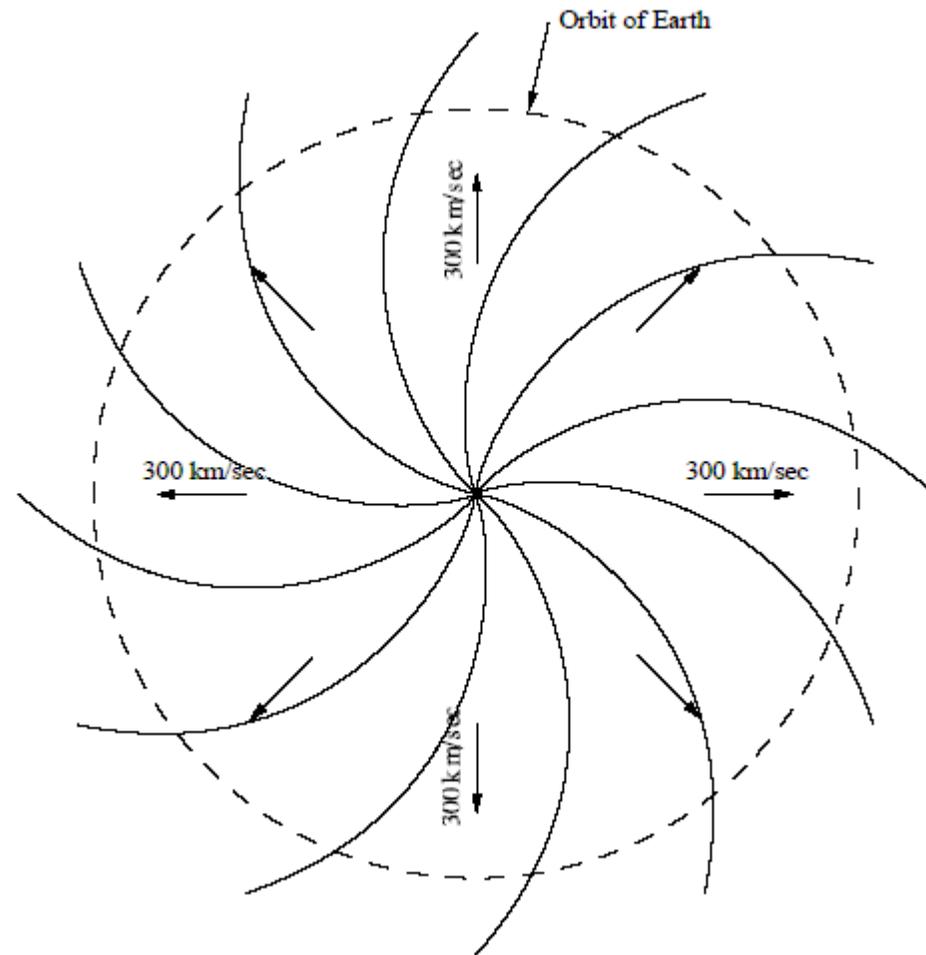
# Magnetic field of the Sun: Parker's spiral

Sun rotation =>

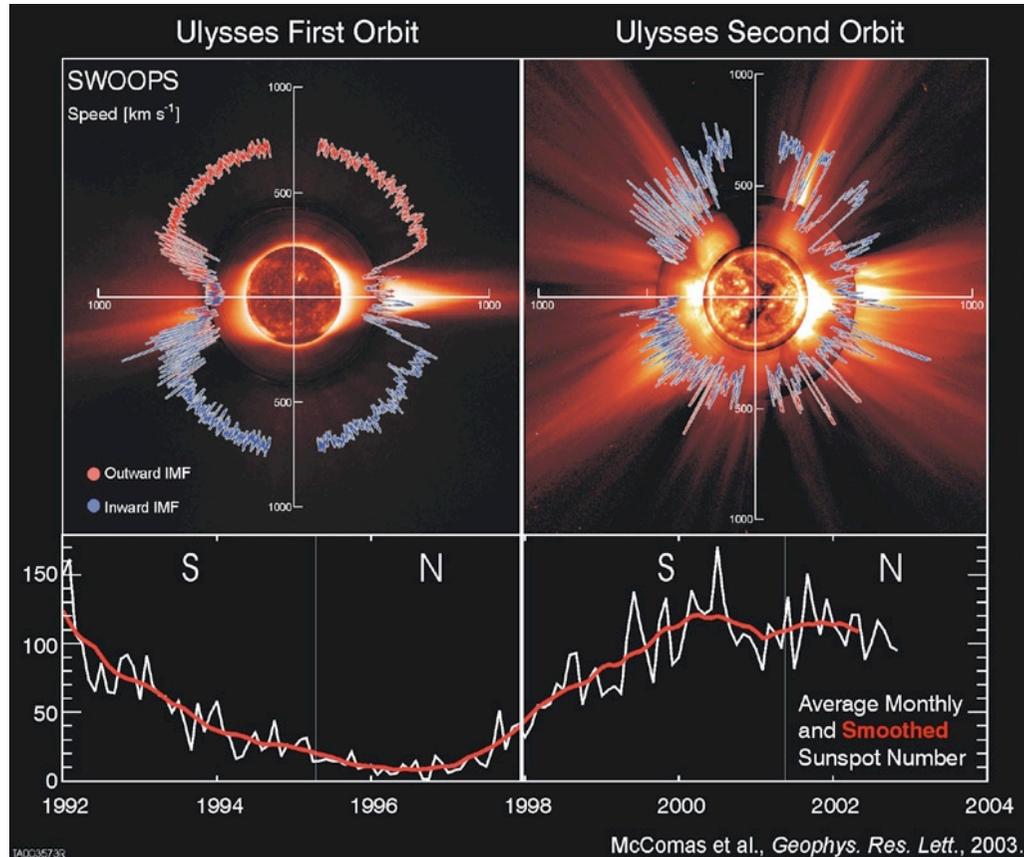
Interplanetary magnetic field  
form a spiral

(1 step = 6 AU = 25 days,  
for  $V_{sw}=400\text{km/s}$ )

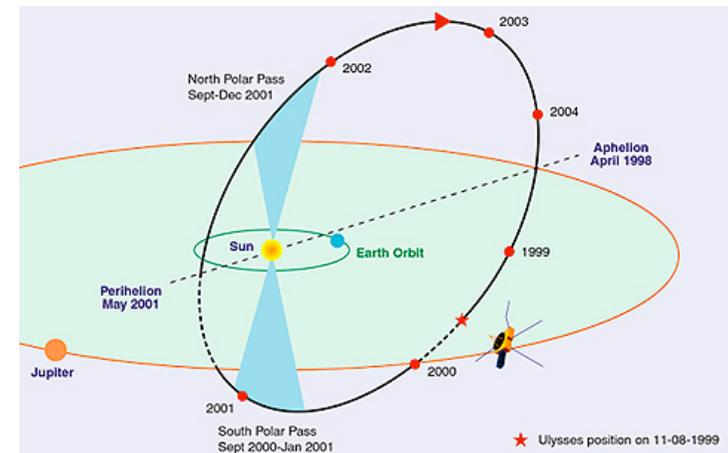
Predicted by Gene Parker



# Solar wind (slow and fast) and the 11 years solar cycle



[McComas et al. 2003]



$$\beta_p = \frac{nkT_p}{B^2 / 8\pi}$$

Two components, Slow and Fast streams :

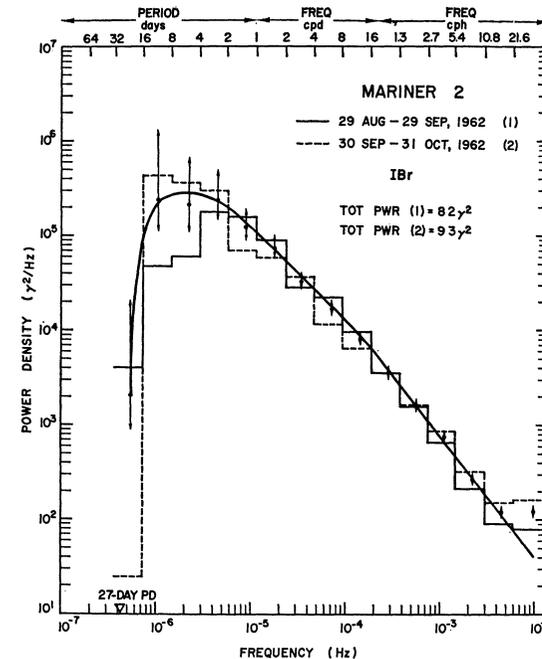
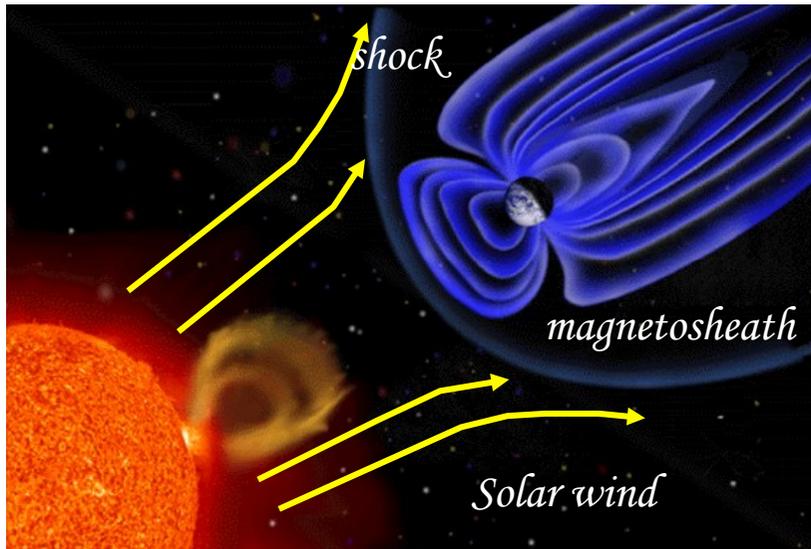
Slow :  $V = 300-400\text{km/s}$ ,  $n=5-25\text{cm}^{-3}$ ,  $T_p = 5-20\text{eV}$ ,  $T_e=5-20\text{eV}$ ,  $B\sim 5\text{nT}$ ,  $\beta\sim 1$

Fast :  $V = 500-800\text{km/s}$ ,  $n=1-10\text{cm}^{-3}$ ,  $T_p=10-30\text{eV}$ ,  $T_e=5-20\text{eV}$ ,  $B\sim 5\text{nT}$ ,  $\beta\sim 1$

Mean free path  $\sim 1$  AU (Sun-Earth distance) !

# Manifestations of a fluid behavior

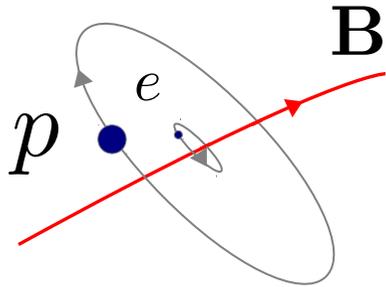
1. Interaction of the supersonic solar wind with a planetary magnetosphere => formation of the **shock wave in front of the magnetosphere**
2. Since the 1st satellite observations in the Heliosphere, we observe **turbulent spectra of fluctuations.**



Why is the fluid description is possible without collisions?  
– presence of plasma characteristic scales much smaller than the size of the system.

# Plasma scales (solar wind)

- Larmor radius ( $\rho_{i,e}$ ) and cyclotron frequency ( $\Omega_{ci,e}$ ) of a charged particle (electron or ion=proton) in a magnetic field B:



$$\rho_{i,e} = \frac{V_{\perp i,e}}{\Omega_{ci,e}} ; \Omega_{ci,e} = 2\pi f_{ci,e} = \frac{eB}{m_{i,e}c}$$

$$\rho_{i=p} \simeq 50 \text{ km}; f_{ci} \simeq 0.1 \text{ Hz}$$

$$\rho_e \simeq 1 \text{ km}; f_{ce} \simeq 300 \text{ Hz}$$

- Inertial length  $\lambda_{i,e}$  (scale of the demagnetization of the particles) and plasma frequency ( $\omega_p$ ):

$$\lambda_{i,e} = \frac{c}{\omega_{pi,e}} ; \omega_{pi,e}^2 = 2\pi f_{pi,e} = \frac{4\pi n e^2}{m_{i,e}}$$

$$\lambda_{i=p} \simeq 40 - 150 \text{ km}; f_{pi} \simeq 450 \text{ Hz}$$

$$\lambda_e \simeq 1 - 3 \text{ km}; f_{pe} \simeq 20 \text{ kHz}$$

- Debye length  $\lambda_D$  (sphere of influence of a given test charge in a plasma); at  $L > \lambda_D$  plasma is quasi-neutral:

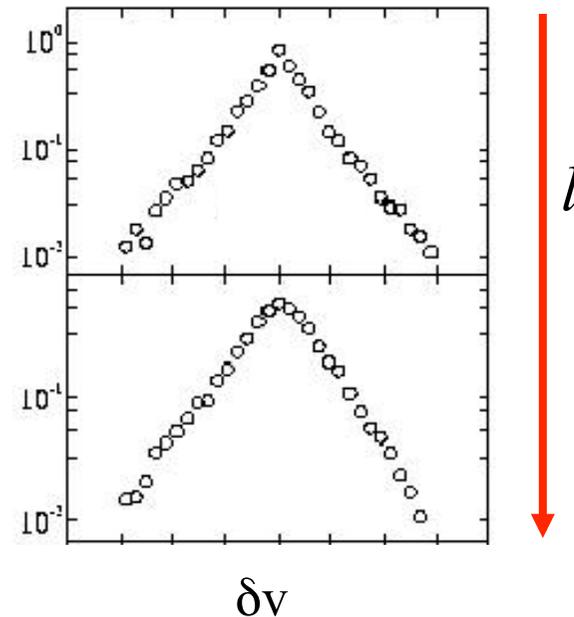
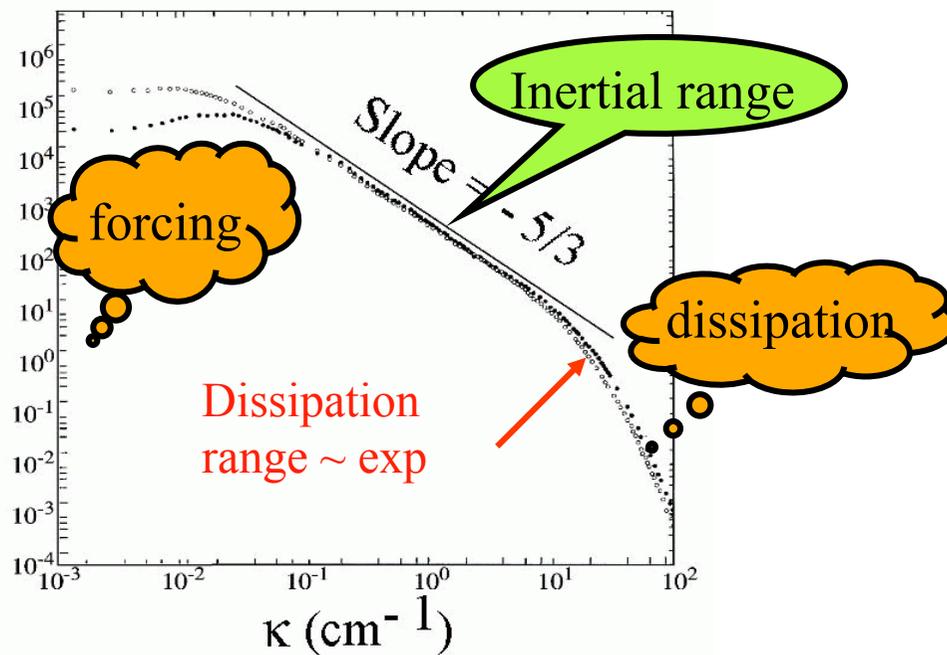
$$\lambda_D = \sqrt{\frac{k_B T}{8\pi n e^2}} \simeq 10 \text{ m}$$

# Turbulence

Locally unpredictable, but **statistical properties are predictable and universal**

1) velocity field energy  $\sim k^{-5/3}$  (scale invariance, same physics at all scales  $l$ ) [Kolmogorov'41]

2) intermittency : deviation from the Gaussianity at small  $l$



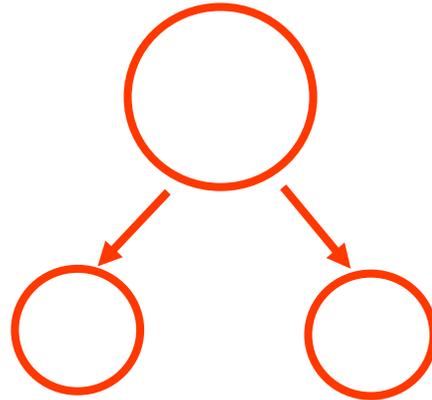
The Kolmogorov spectrum can be observed almost in all turbulent flows.

## **And what about space plasma turbulence?**

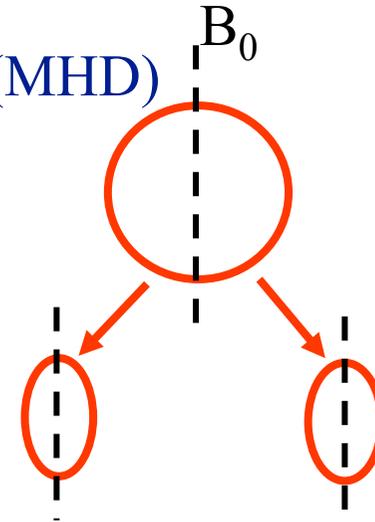
- How is it different from HD turbulence?
- Does it share the above universal characteristics, as power-law spectra and intermittency ?
- What about dissipation ?

# Turbulence in space plasmas

hydrodynamics



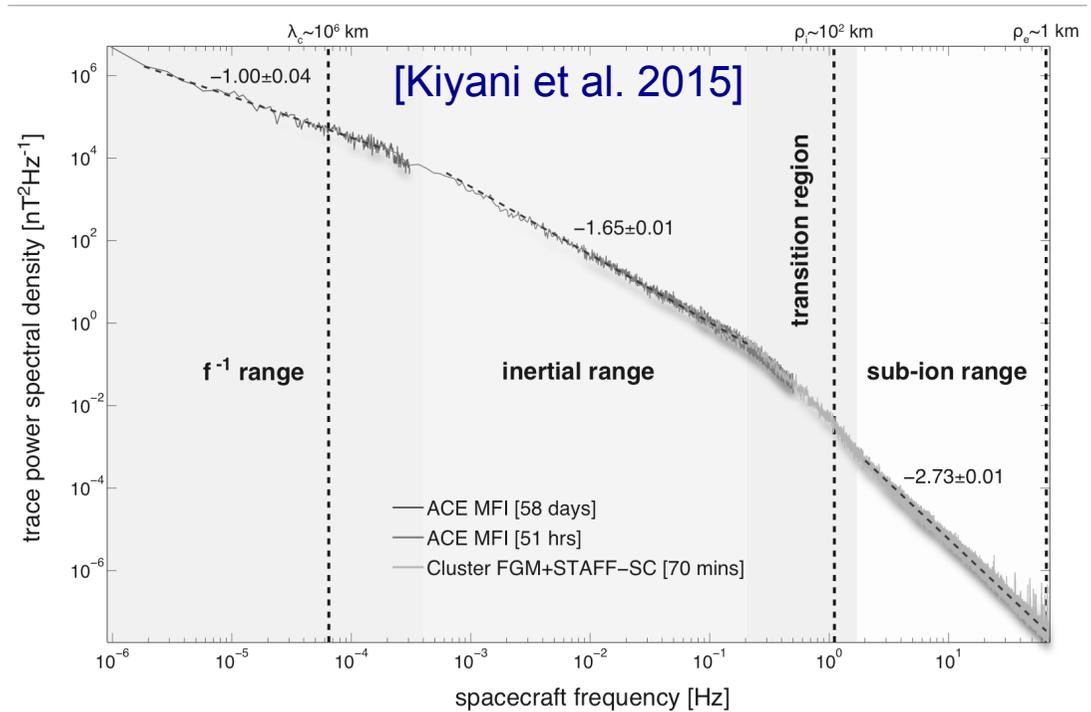
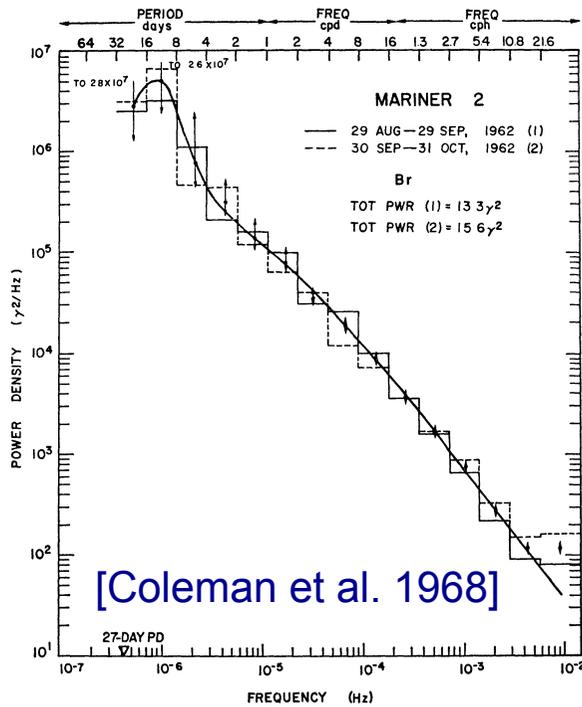
plasma (MHD)



1. Presence of a mean magnetic field  $B_0$  leads to an anisotropy of turbulent fluctuations.
2. Plasma waves: Alfvén, magnetosonic, mirror, whistlers, kinetic Alfvén waves (KAW), etc... (wave turbulence).
3. No collisions : m.f.p.  $\sim 1$  AU.
4. In plasmas there is a number of characteristic space and temporal scales.

# Magnetic turbulence in the solar wind

Typical spectral density of solar wind turbulence



~ 8 (and more) decades in scales (frequencies)  
 ~ 14 decades in energy density !

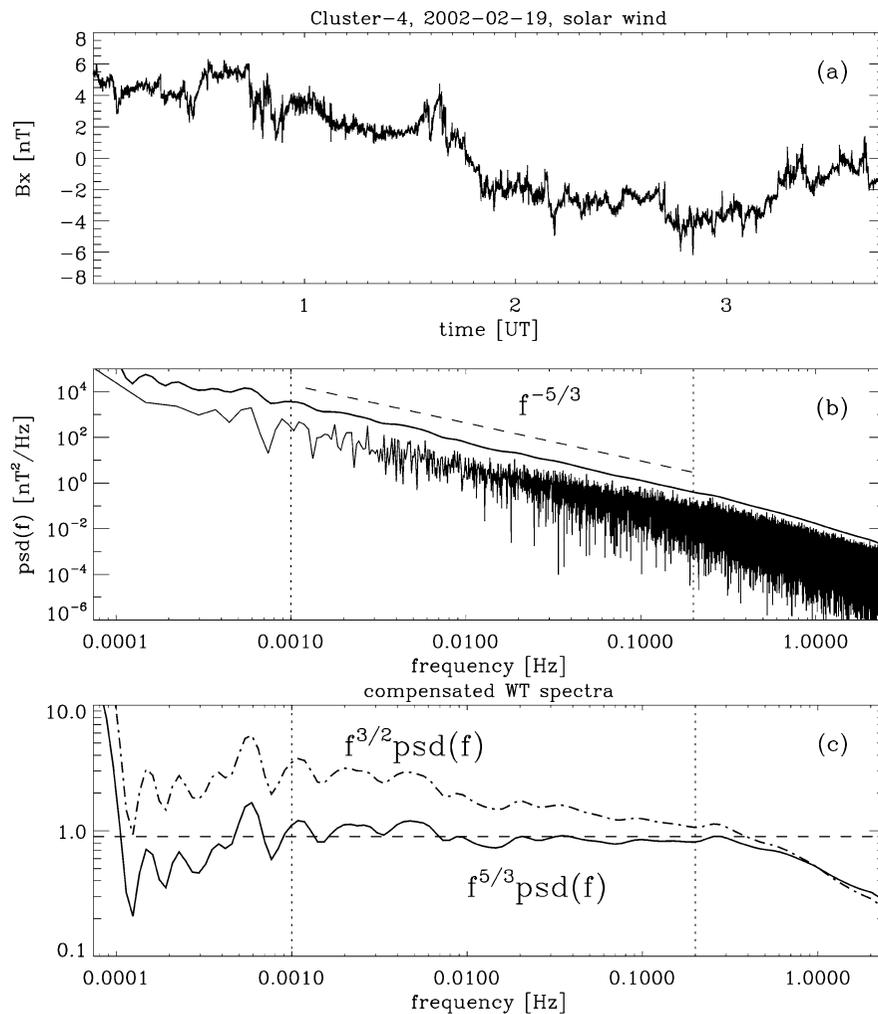
**How do we measure turbulent spectra  
in space?**

**Frequency/k-spectra?**

# Satellites in-situ measurements are time series

=> Fourier Transform (signal) gives frequency spectra:

Methods for Characterising Microphysical Processes in Plasmas



- example of Cluster/FGM  
(5 vectors/sec measurements)

How do we get  $k$ -  
spectra?

Taylor hypothesis:

$$\ell = V_{sw}\tau = V_{sw}/f$$
$$k = 2\pi/\ell = 2\pi f/V_{sw}$$

[Dudok de Wit et al. 2013, SSR]

## Taylor hypothesis

$$\omega_{obs} = \omega_0 + \mathbf{k} \cdot \mathbf{V}$$

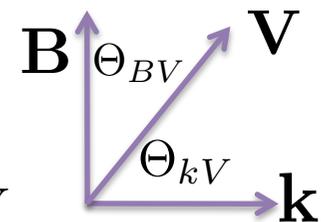
Supposing that  $\omega_0 \ll k.V$ , ( $V_\varphi \ll V$ ):

$$\omega_{obs} = \mathbf{k} \cdot \mathbf{V} = kV \cos(\Theta_{kV})$$

We don't know the angle between  $\mathbf{k}$  and  $\mathbf{V} \Rightarrow$  assumption of  $\mathbf{k} \perp \mathbf{V}$ :

$$\omega_{obs} = kV \rightarrow k = 2\pi f / V$$

For 2D turbulence:  $\mathbf{k} \perp \mathbf{B} \rightarrow \cos \Theta_{kV} = \sin \Theta_{BV}$

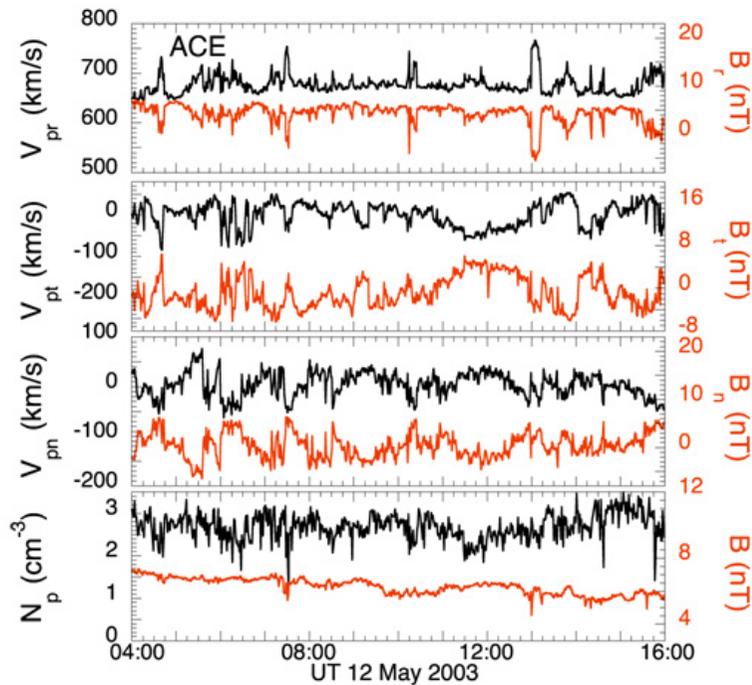


$$\omega_{obs} = \mathbf{k}_\perp \cdot \mathbf{V} = k_\perp V \cos(\Theta_{kV}) = k_\perp V \sin(\Theta_{VB})$$

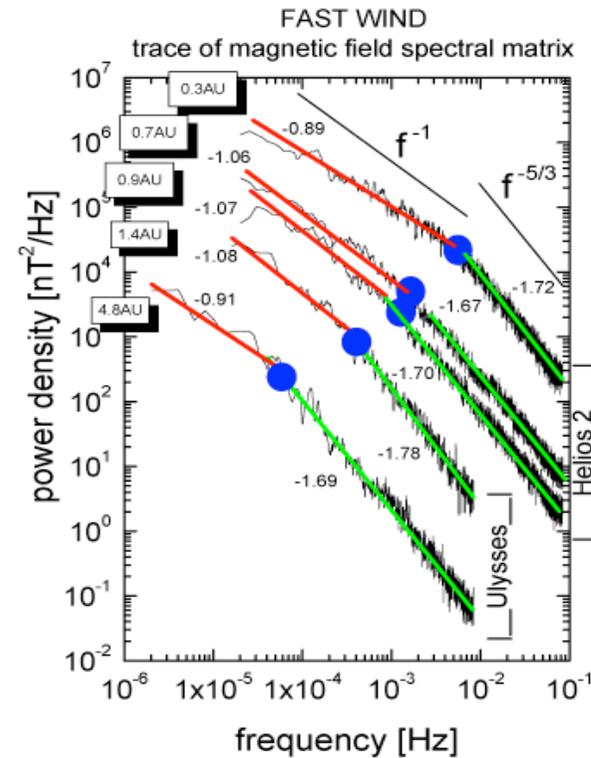
[Leamon+2000, Mangeney+2006, Bourouain+2012]

# Solar wind Turbulence and Alfvén waves

[Gosling et al., 2009; Belcher & Davis 1971]

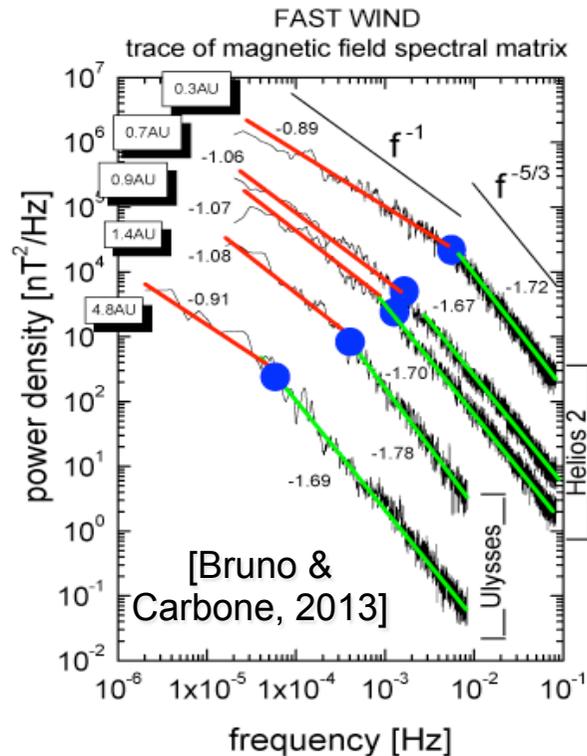


[Bruno & Carbone, 2013]



- Strong correlation between  $V$  and  $B$  fluctuations at 1 AU (Alfvén waves)
- These waves belong to  $f^{-1}$  spectral range.
- Kolmogorov turbulence is observed at smaller scales (MHD).

# Starting point of the Kolmogorov spectrum

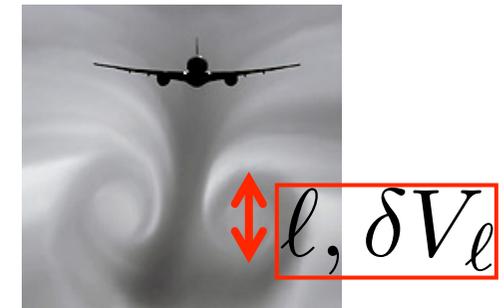
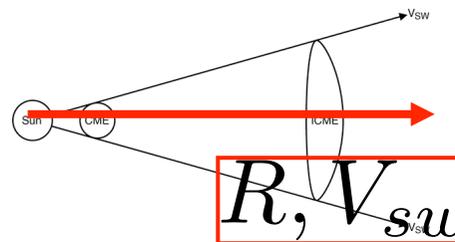


- The solar wind expansion time:

$$\tau_{exp} = R/V_{sw}$$

- The eddy-turnover time:

$$\tau_{NL} = \ell/\delta V_{\ell}$$

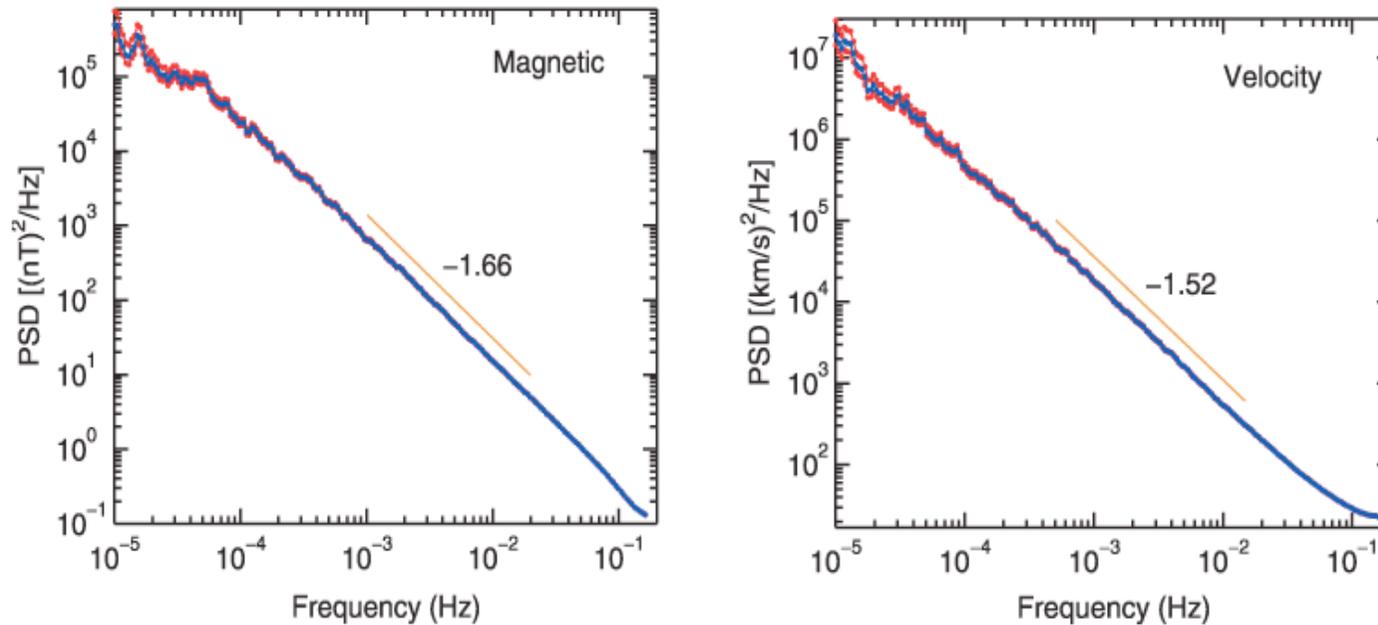


- Transition between  $f^1$  and  $f^{-5/3}$  spectrum corresponds to a scale where these 2 characteristic times are of the same order [Mangeney et al. 1991; Meyer-Vernet 2007]:

$$\tau_{exp} \simeq \tau_{NL}$$

# Solar wind Turbulence and Alfvén waves

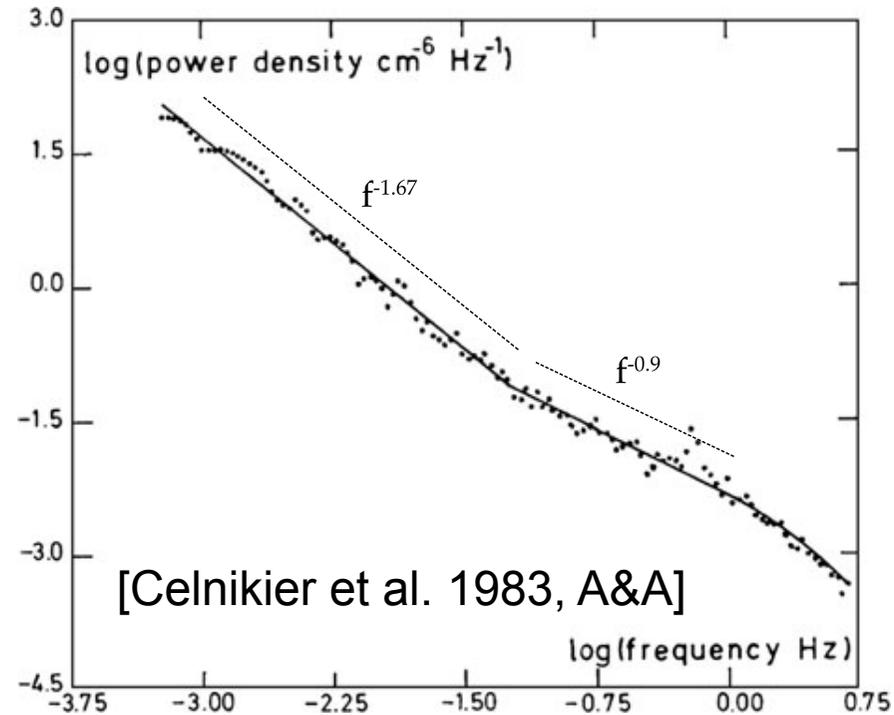
[Podesta et al., 2007; Salem 2000, PHD]



- Large scales fluctuations have Alfvénic nature  $dV \sim dB$
- However, turbulent spectra for B and V are different...
- Why?
  - Local dynamo process [Grappin et al., 1983] ?
  - B-V alignment [Boldyrev et al. 2005]
  - Compressibility ?

# Solar wind turbulence is compressible

Spectrum of electron density fluctuations in the solar wind as measured by ISEE 1 & 2  
See as well Chen et al. 13



Can the compressibility be the source of the non-alfvenicity of the inertial range in the solar wind turbulence?

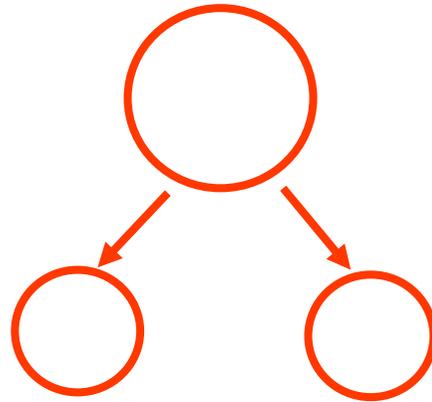
see S. Galtier, Les Houches 2015 cours, page 48:

$$\boxed{V = u\rho^{1/3} \rightarrow E_V \sim k^{-5/3}} \longrightarrow u^2/k \sim k^{-5/3} \rho^{-2/3} \sim k^{-3/2},$$

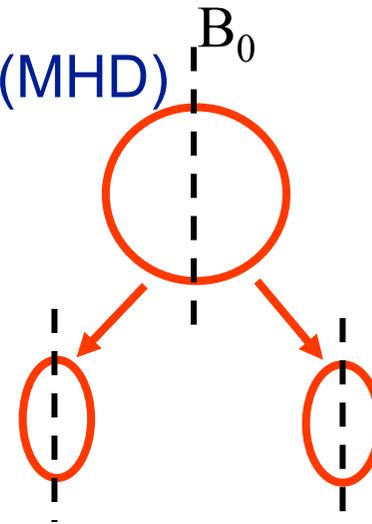
for  $\rho^2/k \sim k^{-5/3}$

# Anisotropy

hydrodynamics



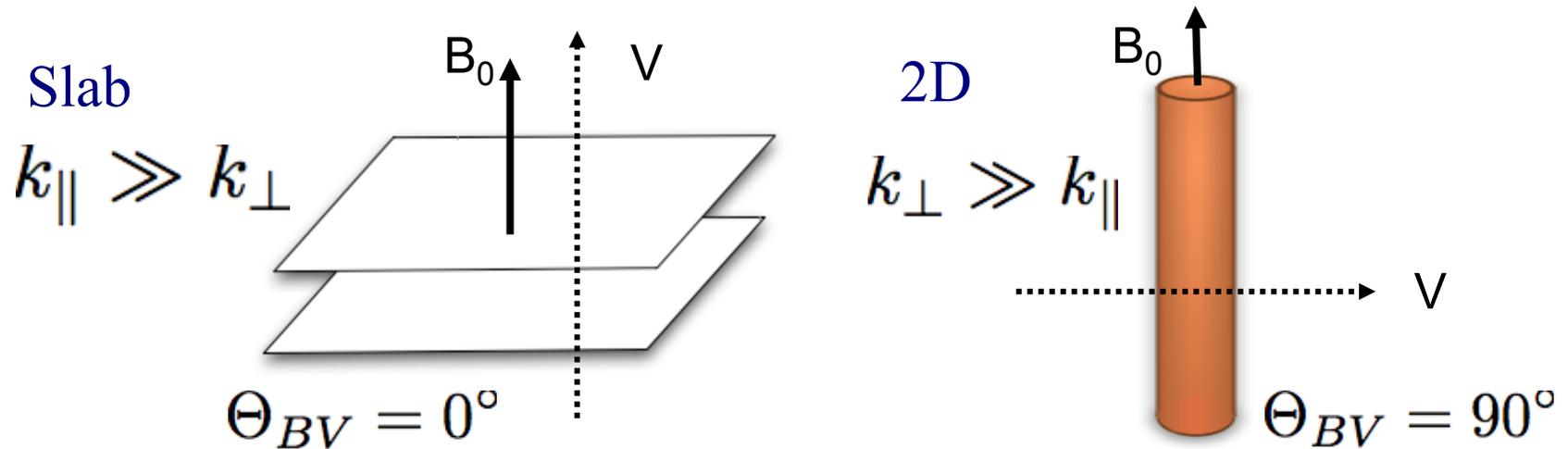
plasma (MHD)  $B_0$



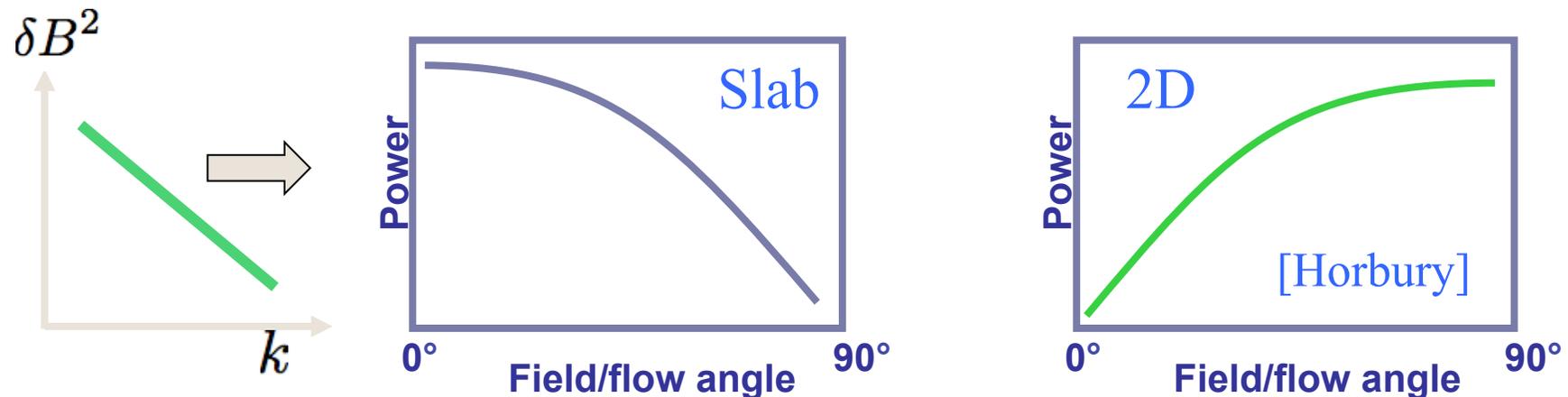
Presence of a mean magnetic field  $B_0$  leads to an anisotropy of turbulent fluctuations:

- anisotropy in amplitudes of fluctuations;
- anisotropy in topology (wave vectors).

# k-anisotropy of turbulent fluctuations

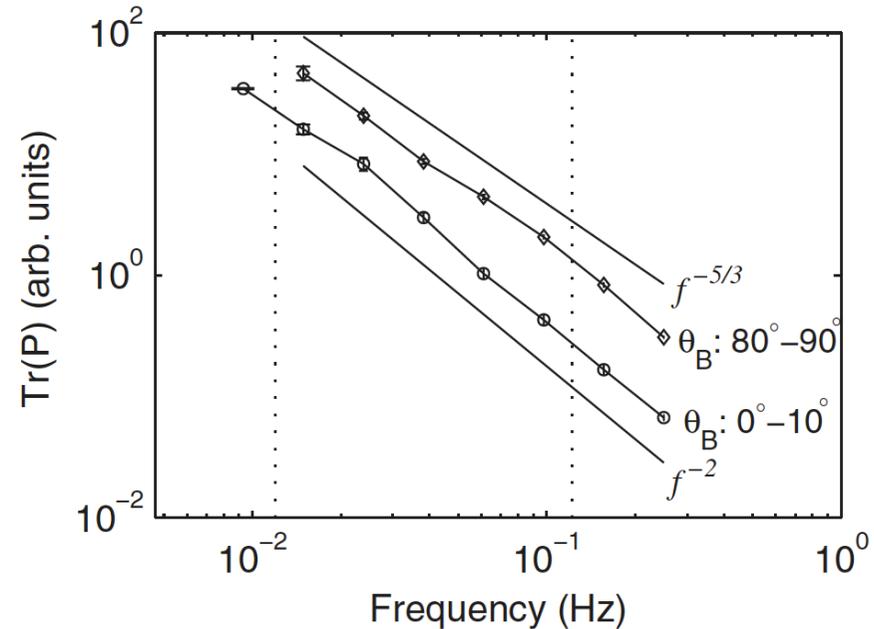
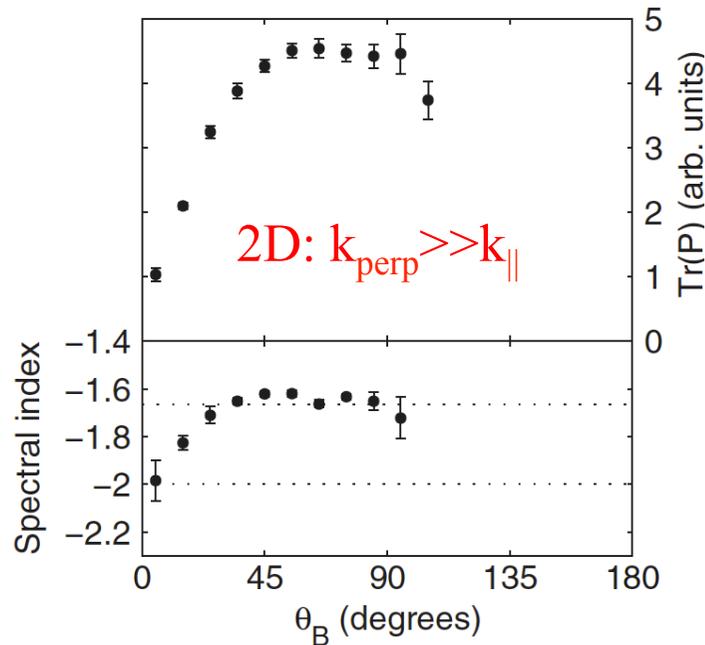


If Taylor hypothesis ( $V_{\varphi} \ll V$ ) is verified  $\Rightarrow$  variation of field-flow angle allows to resolve slab fluctuations while  $V$  is  $\parallel$  to  $B$  and 2D fluctuations while  $V$  is  $\perp$  to  $B$ . [Bieber et al., 1996; Mangeney et al., 2006; Horbury et al., 2008; Alexandrova et al. 2008, ...]



# Anisotropy of turbulent fluctuations at MHD scales

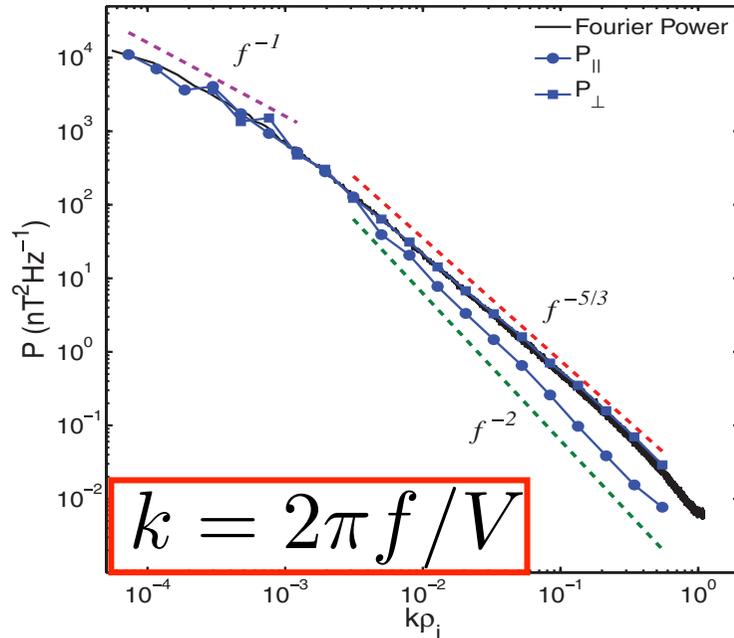
[Matthaeus et al. 1990, Bieber et al. 1996]: 2D + slab  
[Horbury et al., 2008, PRL]: **anisotropic cascade**  $k_{\text{perp}}^{-5/3}$ ,  $k_{\parallel}^{-2}$



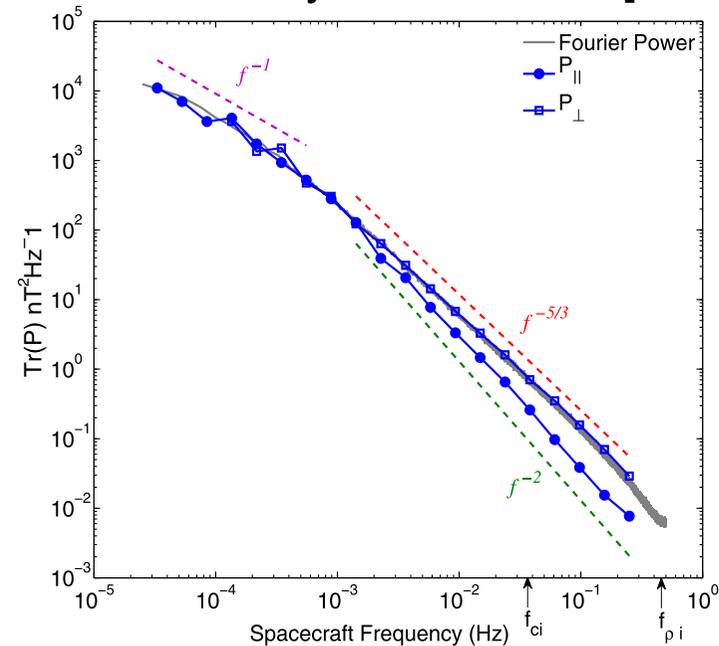
Results obtained using a large statistical sample (30 days of data) ;  
confirmed by Podesta et al. [2009] and Wicks et al. [2010]  
=> general result.

# Anisotropy of turbulent fluctuations at MHD scales

Wicks et al. [2010]



[Alexandrova et al. 2013, courtesy of Rob Wicks]

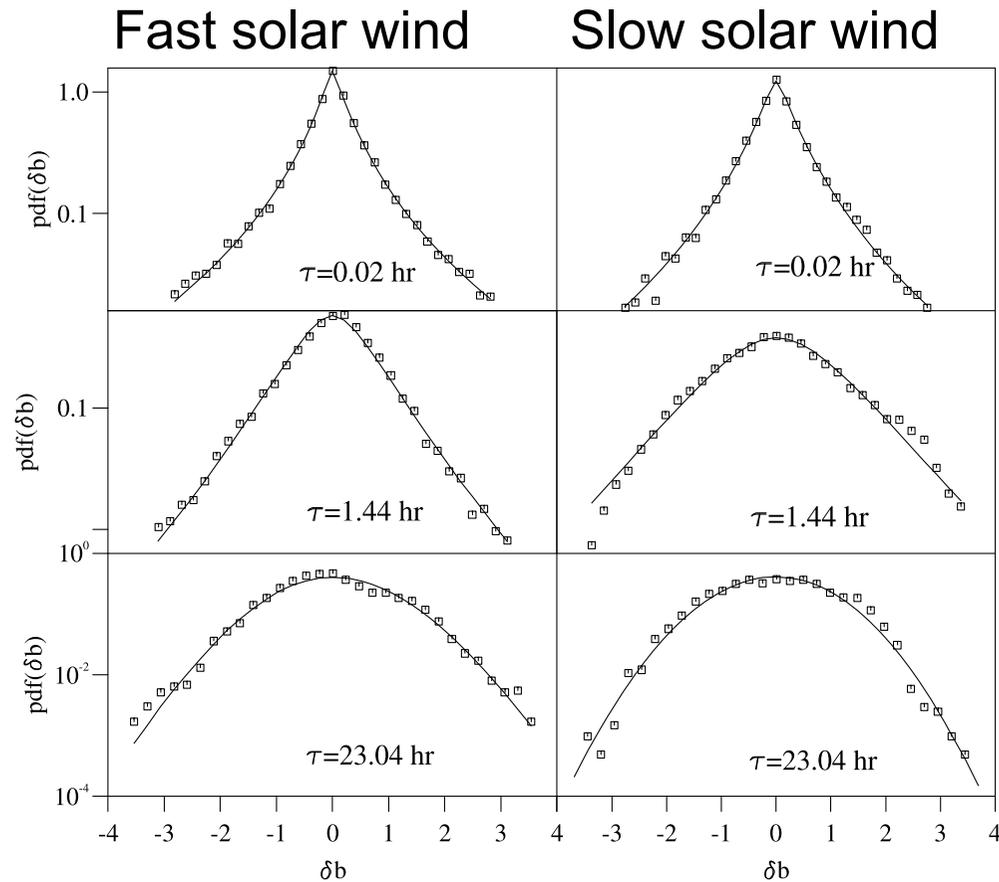


- In agreement with “Critical balance” model [Goldreich & Sridhar, 1995]
- Alfvénic strong turbulence with a “balance” between linear Alfvén time (along  $B_0$ ) and non-linear time (in plane perp. to  $B_0$ ), see Stas Boldyrev’s lecture.

$$\tau_A = \frac{\ell_{\parallel}}{V_A} \sim \tau_{NL} = \frac{\ell_{\perp}}{\delta V_{\perp}}$$

$$P(k_{\perp}) \sim k_{\perp}^{-5/3}; P(k_{\parallel}) \sim k_{\parallel}^{-2}$$

# Intermittency of turbulent fluctuations (inertial range)



[Sorriso-Valvo et al. 1999; Greco, Servidio et al.]

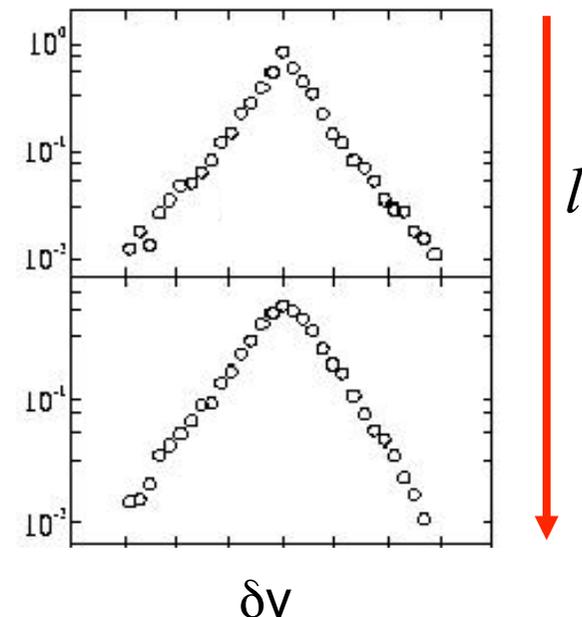
# Non-Gaussianity of turbulent fluctuations

In the inertial range of HD turbulence (K4/5 law):

$$\langle \Delta u_\ell^3 \rangle = -\frac{4}{5} \langle \varepsilon \rangle \ell$$

$\langle \varepsilon \rangle$  averaged energy dissipation rate;  
(see Politano & Pouquet, 98, for incompressible MHD)

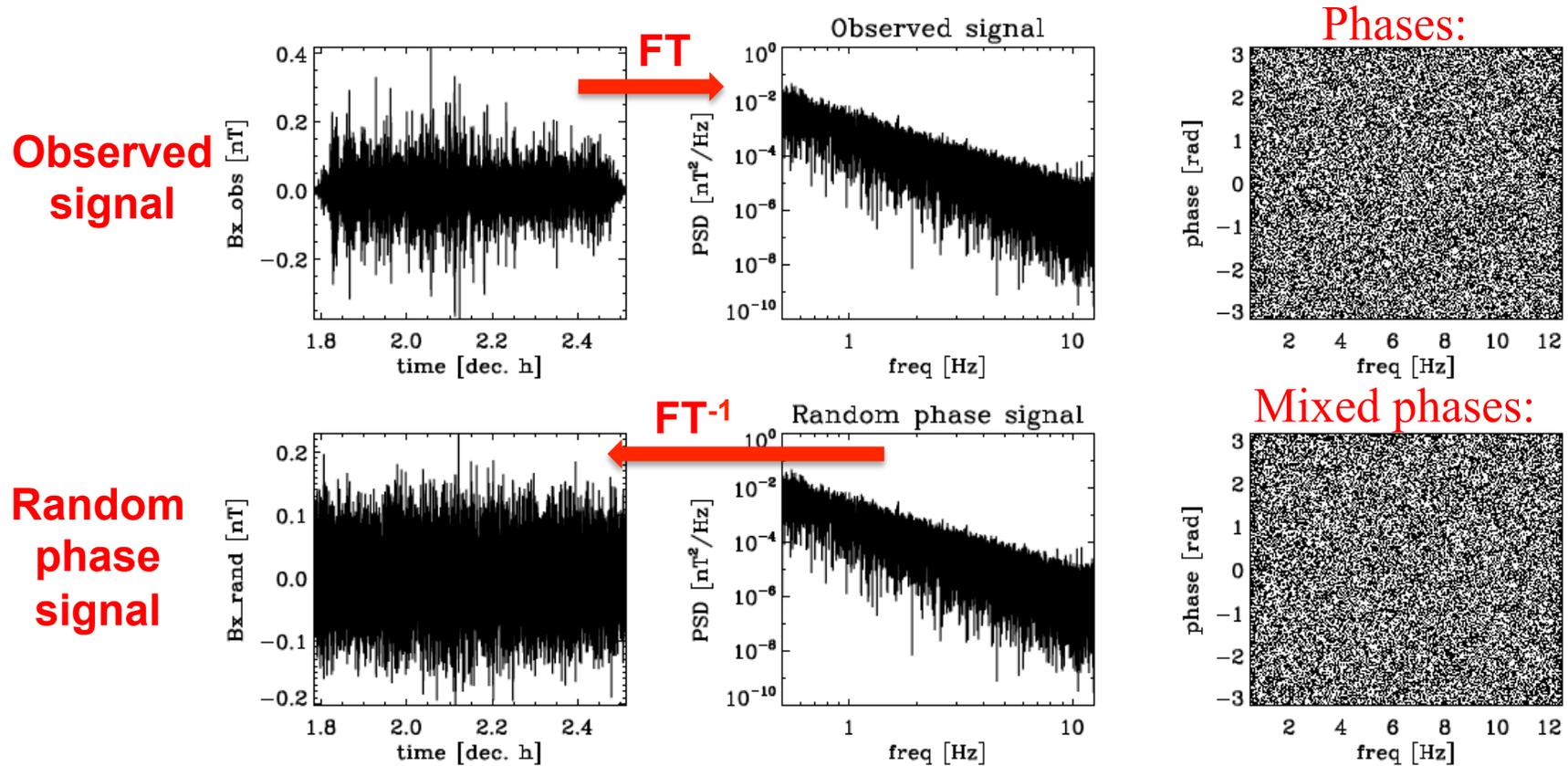
- For Gaussian fluctuations the 3d moment is zero;
- Here, the third-order moment of fluctuations, which is related to the energy dissipation rate, is different from zero  $\rightarrow$
- Turbulence MUST show some nongaussian features within the inertial range.



**Def. of Intermittency:** scale dependent non-Gaussianity of turbulent fluctuations.

# Non-Gaussianity: what does it mean ?

Cluster-1/STAFF-SC measurements, 2002-02-19



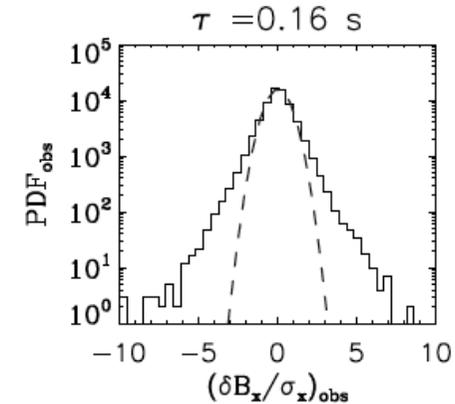
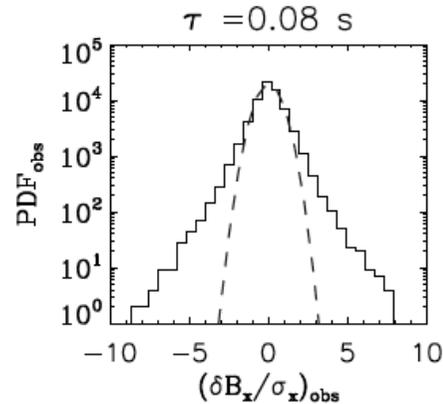
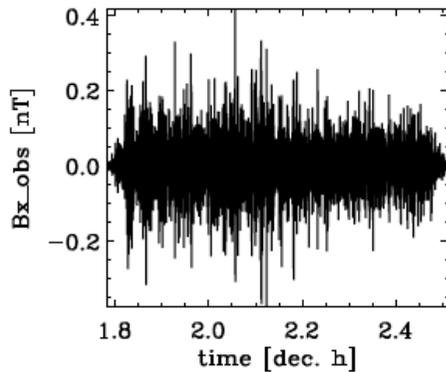
From the observed signal we construct a signal with random phases but with the same spectrum.

[Rossi, Tesi di Lauria, 2011; Hada et al. 2003; Koga & Hada, 2003; Sahraoui, 2008]

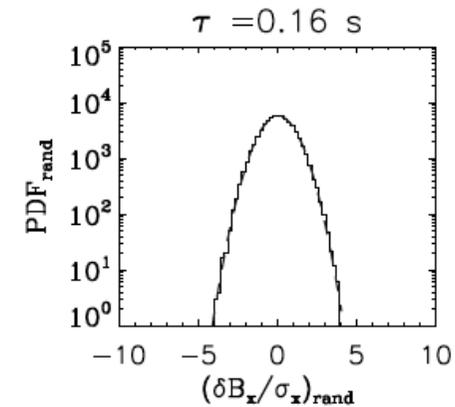
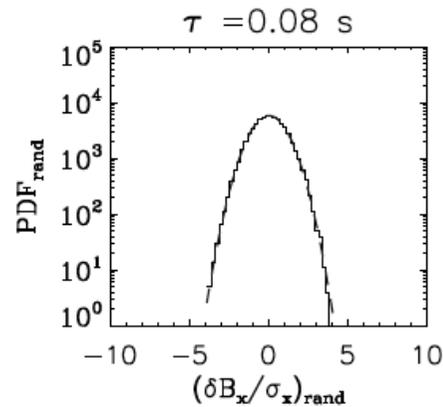
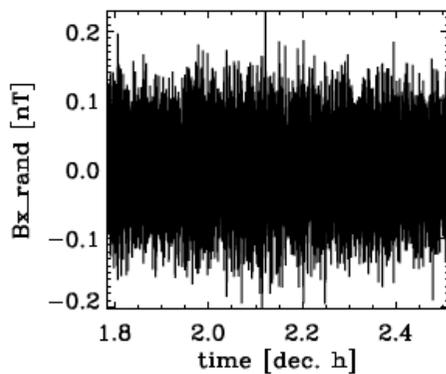
# Non-Gaussianity: what does it mean ?

[Claudia Rossi, Tesi di Laurea, 2011]

Observed  
signal



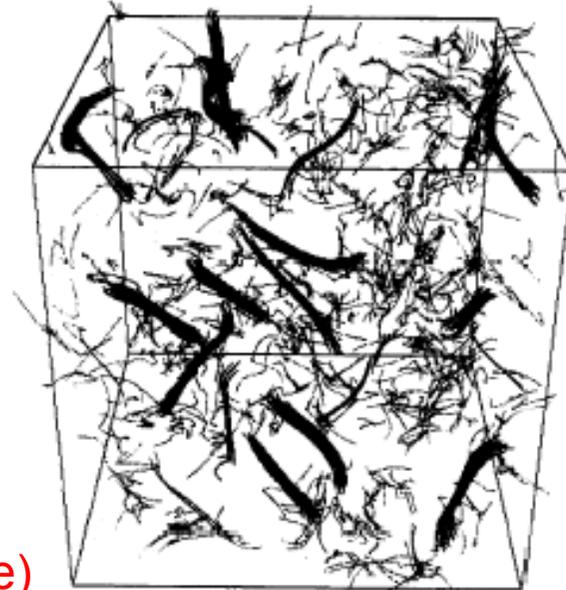
Random  
phase  
signal



Non-Gaussian tails  $\Leftrightarrow$  coupled phases !

# Intermittency = coherent structures

In HD turbulence, intermittency corresponds to appearance of coherent structures: filaments of vorticity with length  $\sim L_{\text{injection}}$  and cross-section  $\sim L_{\text{dissipation}}$



[She et al., 1991]

## Definition of a coherent structure (see as well T. Dudok de Wit's lecture)

- Localisation in space (time).
- Delocalisation in Fourier space (phase coupling over a large range of scales).
- Particular geometry (coupling between components).

Coherent structures in the solar wind at  $L_{\text{dissipation}}$  ?

$L_{\text{dissipation}}$  = plasma kinetic scales ?

First plasma scales encountered by the cascade?

# **Turbulence at kinetic scales**

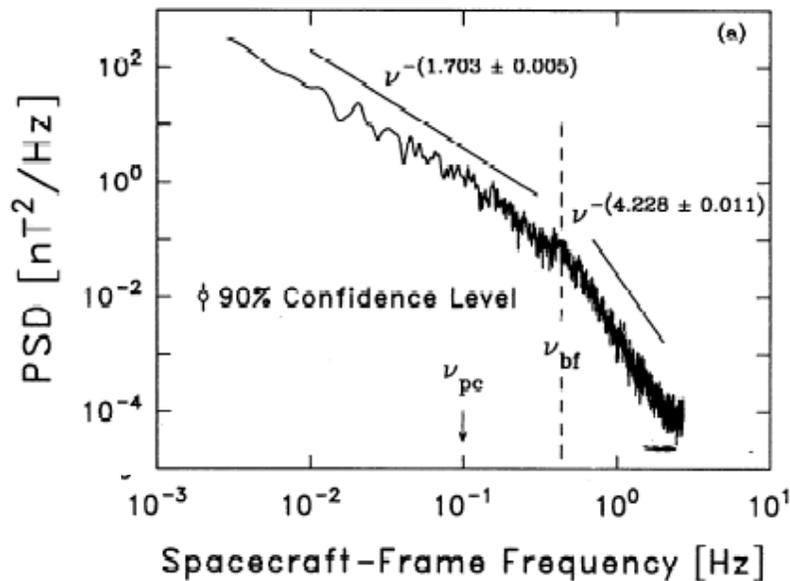
## **1. Ion scales : spectra and coherent structures**

$$f_{ci} = \frac{eB_0}{2\pi m_i c}, \quad k\rho_i \sim 1, \quad kc/\omega_{pi} \sim 1$$

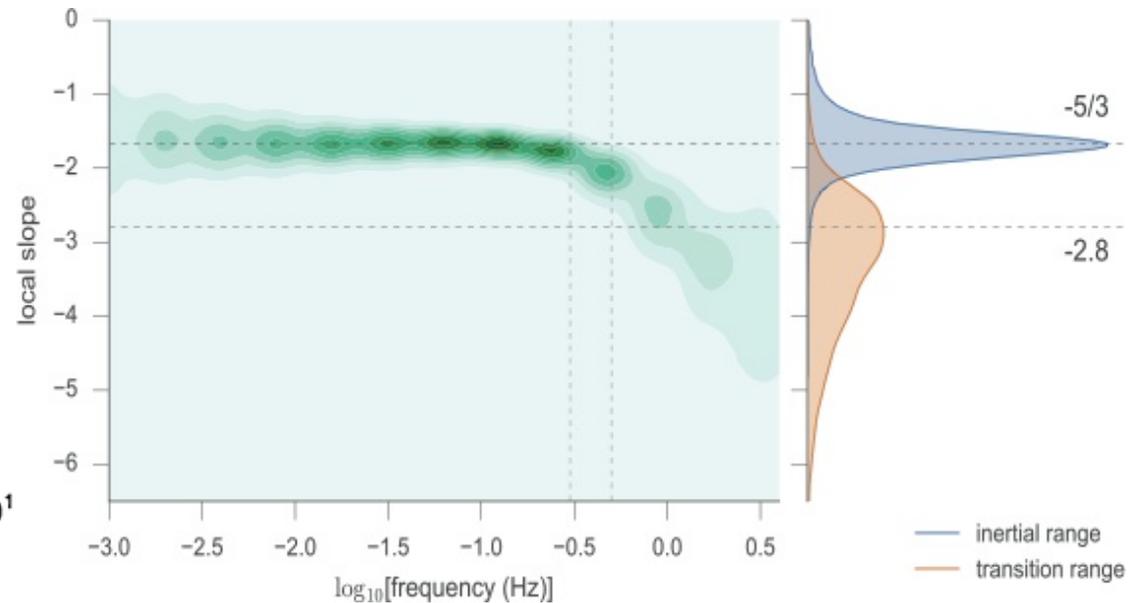
# Turbulence around ion scales

## Magnetic field spectrum

[Leamon et al, 1998]



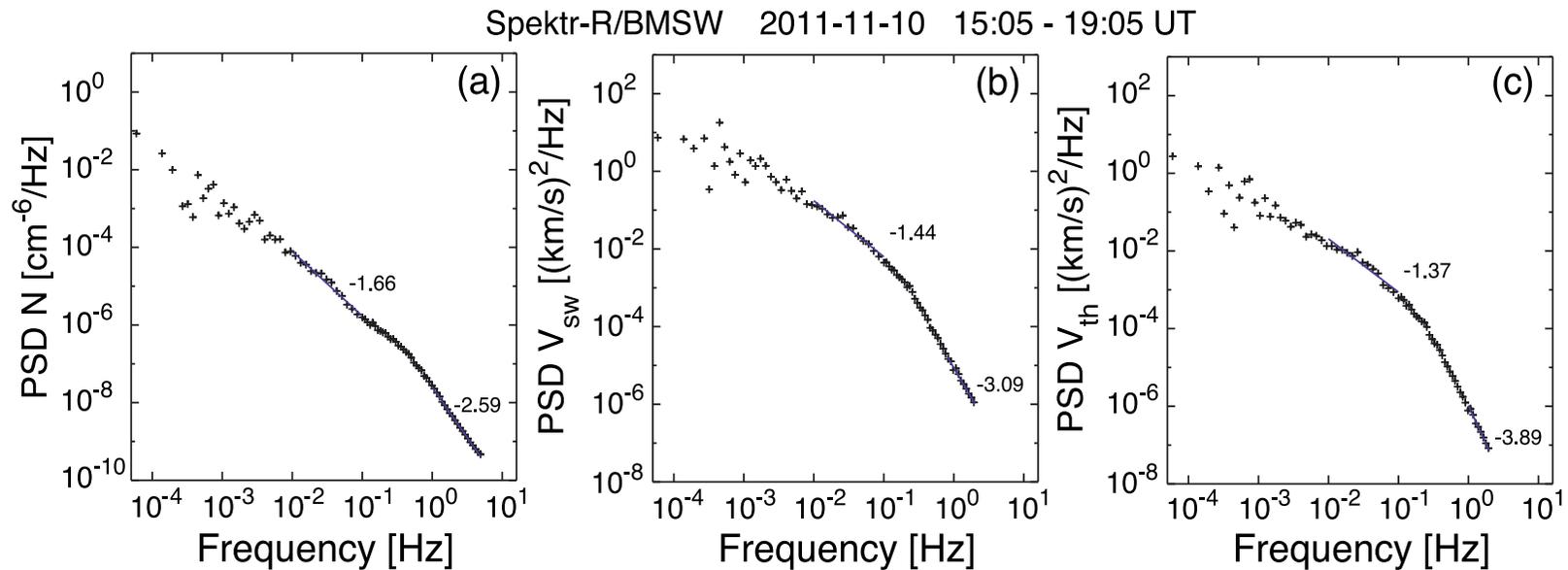
[Lion, 2016, PHD thesis]



- There exist a spectral “break” close to ion scales
- Spectral variability at ion scales [0.3,3]Hz (no universal behavior).
- Attention: less than 1 decade is measured..., e.g. [Smith et al., 2006].

# Turbulence around ion scales

## Ion moments spectra

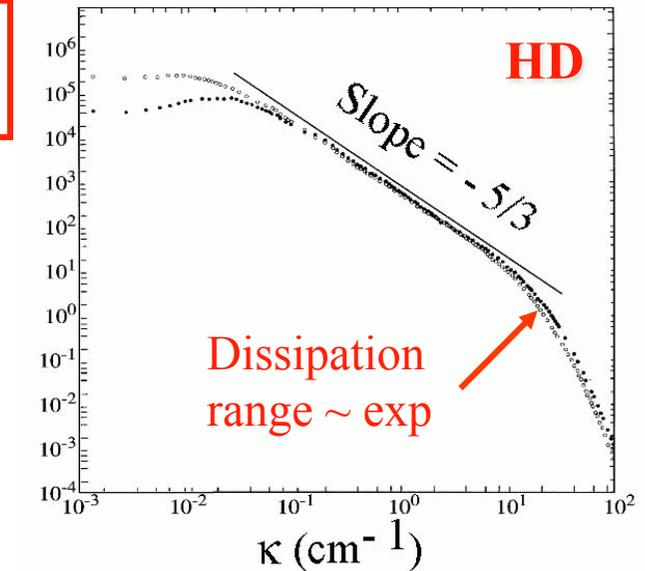
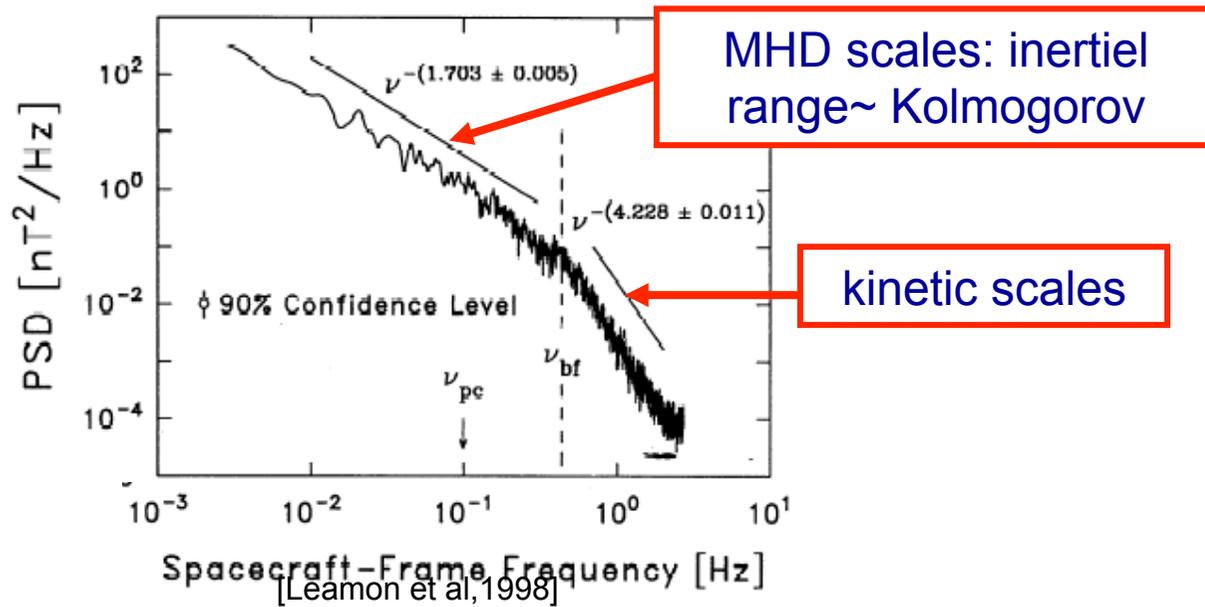


**Fig. 10** Spectra of ion moments, (a) density, (b) velocity, (c) ion thermal speed, up to  $\sim 3$  Hz as measured by *Spektr-R/BMSW* (Bright Monitor of Solar Wind) in the slow solar wind with  $V_{sw} = 365$  km/s and  $\beta_p \simeq 0.2$ . Figure from Šafránková et al. (2013)

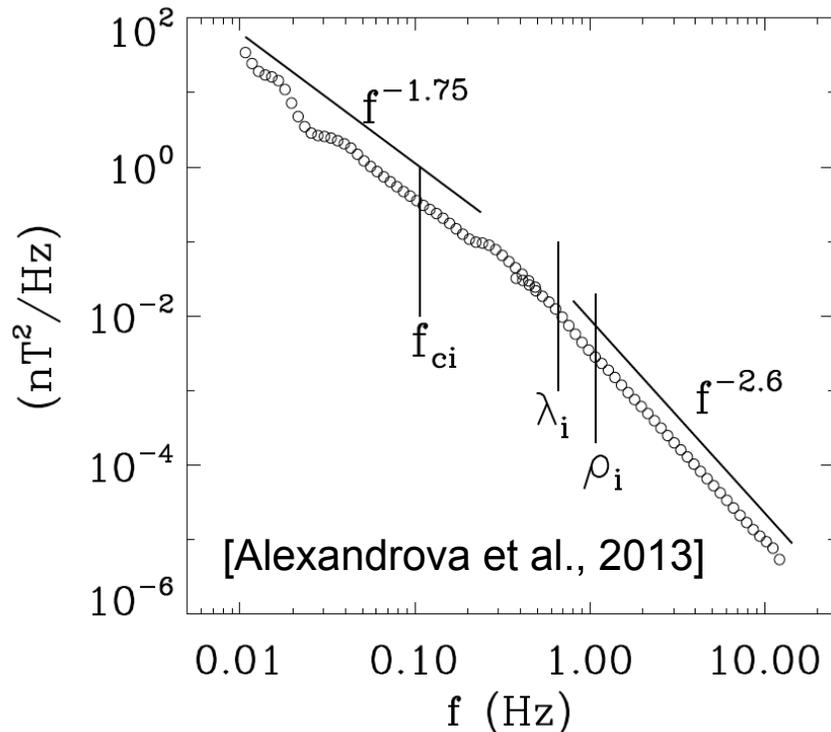
# Turbulence around ion scales

## Open Questions:

- onset of dissipation range [e.g. Leamon+'98,99,00; Smith'06,...] ?
- starting point of another cascade [e.g. Biskamp+'96; Galtier'06; Alexandrova+'08,'13] ?
- or combination of both ?
- which ion scale is responsible for the break ?
- Intermittency / presence of coherent structures ?



# Which ion scale is responsible for the break?



Time scale ( $\sim 0.1$  Hz)

$$f_{ci} = \Omega_{ci}/2\pi ; \Omega_{ci} = eB/m_i c$$

Spatial scales ( $\sim 100$  km)

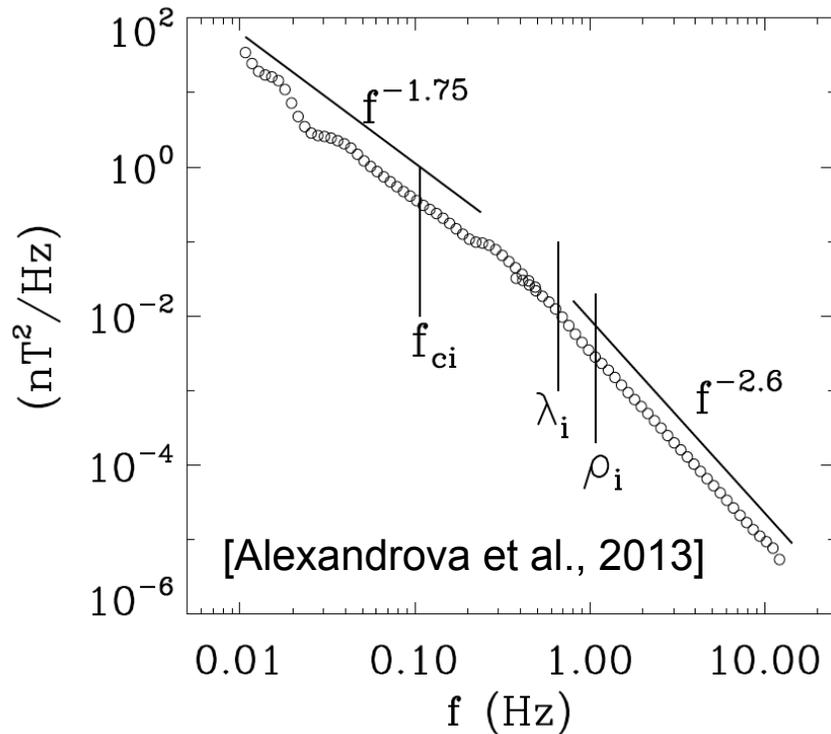
$$\rho_i = \frac{V_{\perp i}}{\Omega_{ci}} ; \lambda_i = \frac{c}{\omega_{pi}} = \frac{V_A}{\Omega_{ci}}$$

In frequency spectrum, these scales appear at Doppler shifted frequencies:

$$f_{\rho_i} \simeq \frac{V_{solar\ wind}}{\rho_i} ; f_{\lambda_i} \simeq \frac{V_{solar\ wind}}{\lambda_i}$$

- All characteristic time and spatial ion scales are observed close to the spectral break frequency...
- How can we distinguish between different scales?
- Important in order to understand which physical mechanisms “break the spectrum” (e.g., if it is  $f_{ci} \Rightarrow$  damping of Alfvén waves).

# Which ion scale is responsible for the break?



- Leamon et al. 2000 :  $\lambda_i$
- Schekochihin et al. 2009:  $\rho_i$
- Perri et al. 2010 : any of the scale/ combination of scales
- Bourouaine et al. 2012:  $\lambda_i$
- Bruno et al. 2014: resonant  $|\mathbf{k}|$  of parallel Alfvén waves
- Chen et al. 2014: beta dependent.

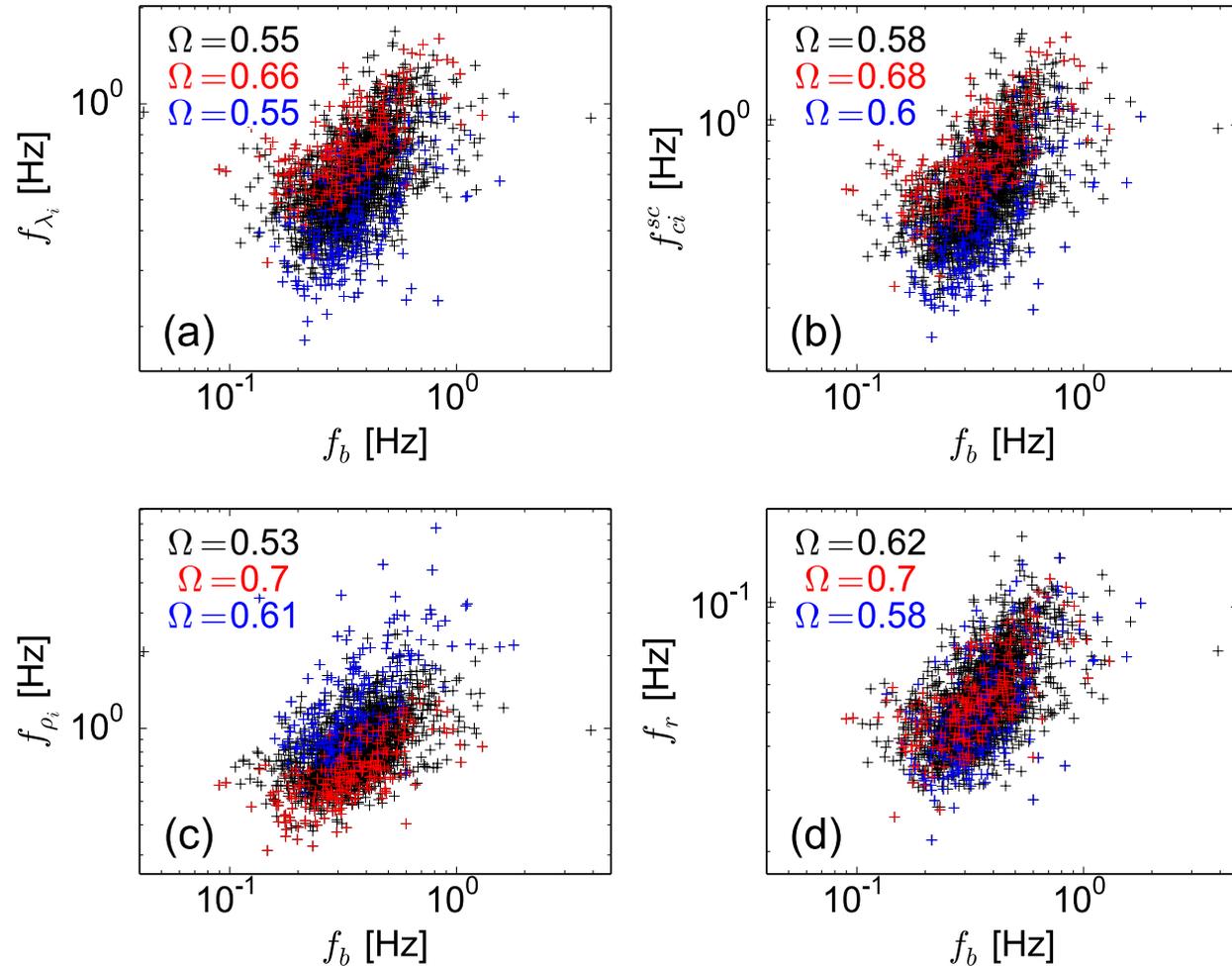
$$\beta_i = 2\mu_0 n k_B T_i / B^2 = \rho_i^2 / \lambda_i^2.$$

⇒ The largest characteristic ion scale “breaks” turbulent spectrum [Chen et al. 2014].

# Ion scales and spectral break (statistical study)

[Sonny Lion, PHD,  
2016, 6 years of  
STEREO/MAG data]

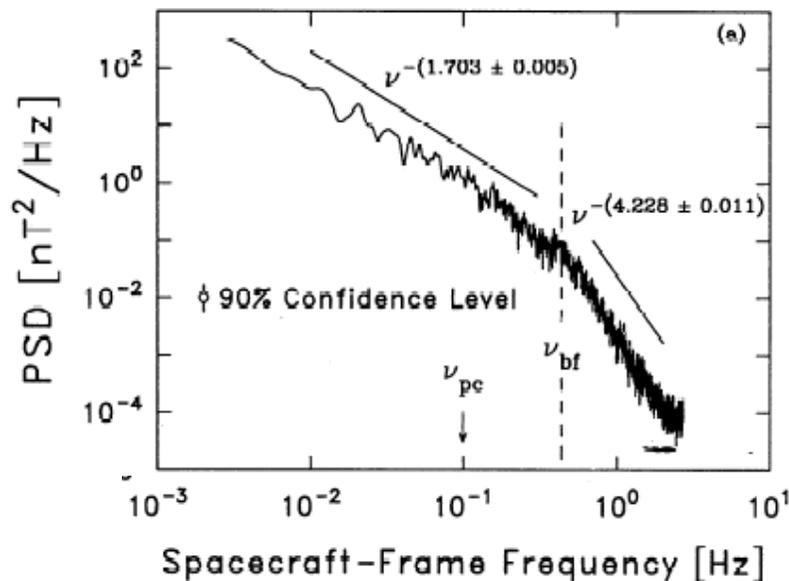
Red: high  $\beta_p > 1$   
Blue: low  $\beta_p < 0.2$   
Black: all data



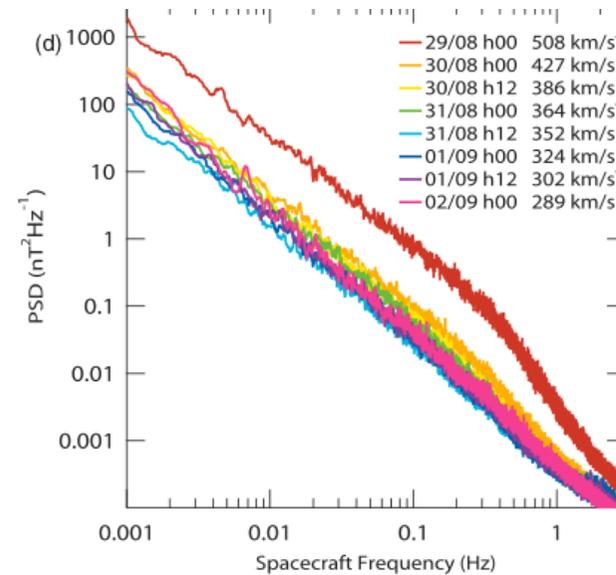
- Spectral break  $f_b$  is not always well-defined  $\Rightarrow f_b =$  intersection of two power-laws.
- All ion scales correlate well with  $f_b$  !  $\Rightarrow$  Not one scale (or physical mechanism) which is responsible for the spectral break?

# Break at ion scales: permanent feature ?

- Sometimes: we observe a clear spectral “break” [Leamon et al. 1998].
- However, usually the break is not visible and we define it as an intersection of 2 power-laws.



[Leamon et al, 1998]

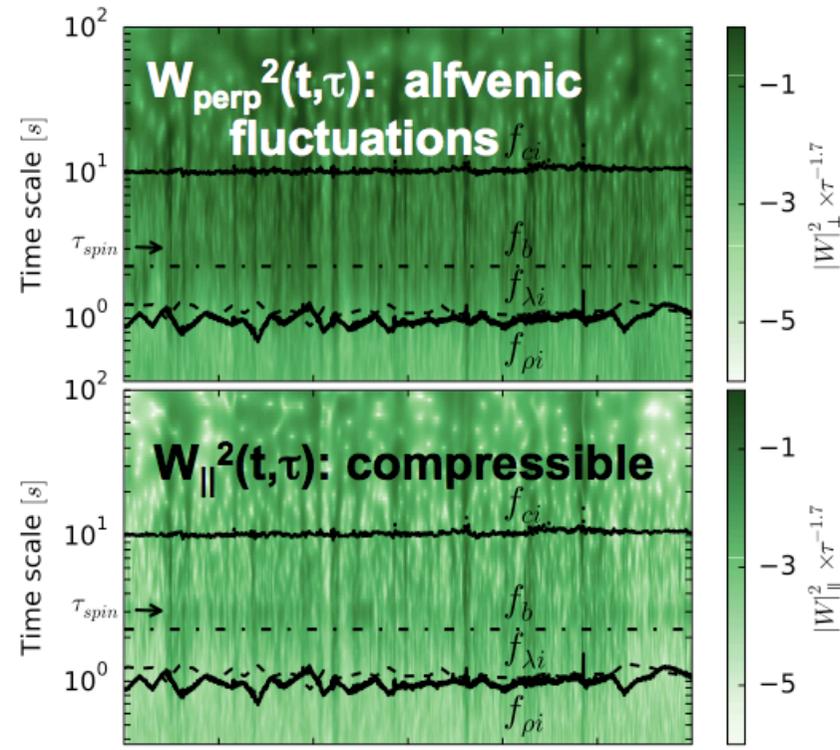
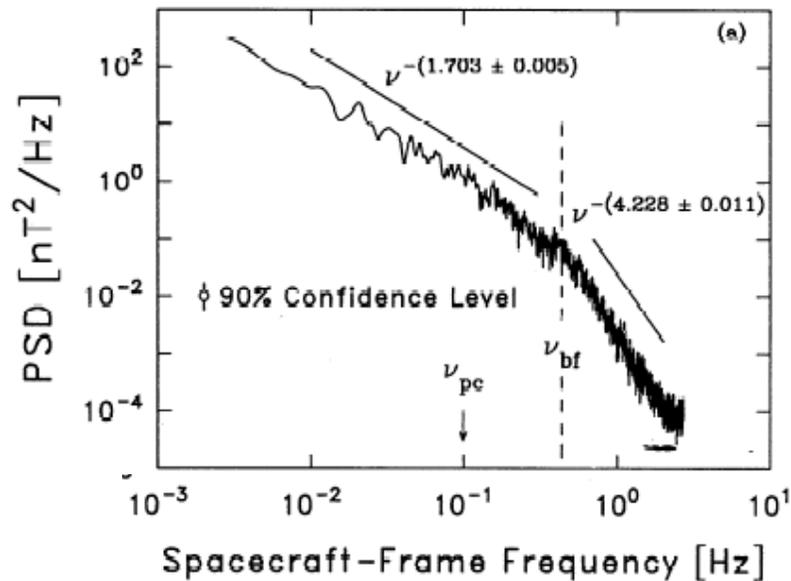


[Bruno et al. 2014]

- There exist spectra without a break [Bruno et al. 2014; Smith et al. 2006]...
- What is particular with the fast solar wind and Leamon's 1998 spectrum?

# Let's re-visit Leamon's spectrum

The time interval studied in [Leamon et al, 1998], with  $V=690\text{km/s}$ ,  $T_{\text{perp}}/T_{\parallel}=1.8$ ,  $\beta=0.5$



- Fourier (wavelet) spectra: mean characteristic of turbulence, no information on homogeneity of fluctuations.
- Time-frequency (time-scale) analysis with wavelets allows us to see the 'texture' of turbulence (see the lecture of T. Dudok de Wit).

# Fourier vs wavelet transforms

$$B_x[j] = B_x(t_j) = B_x(t_0 + j\Delta t)$$

$$\Delta t = T/N, \quad t_j = j\Delta t, \quad j = 0, 1, \dots, N - 1$$

Fourier Transform:

$$\hat{B}_x(f) = \frac{1}{N} \sum_{j=0}^{N-1} B_x(t_j) e^{-2\pi i f t}$$

$$f_n = n/T, \quad n = 0, \dots, N - 1$$

**Fourier:** Time dependence is lost, best frequency localization.

**Wavelets:** time-frequency dependence, frequency (or time scale  $\tau=1/f$ ) resolution verifies the uncertainty principle:

$$\Delta\tau^{-1} \Delta T \sim \text{const}$$

Wavelet Transform:

$$\mathcal{W}_x(\tau, t) = \sum_{j=0}^{N-1} B_x(t_j) \psi^*[(t_j - t)/\tau]$$

Ex.: Morlet mother function:

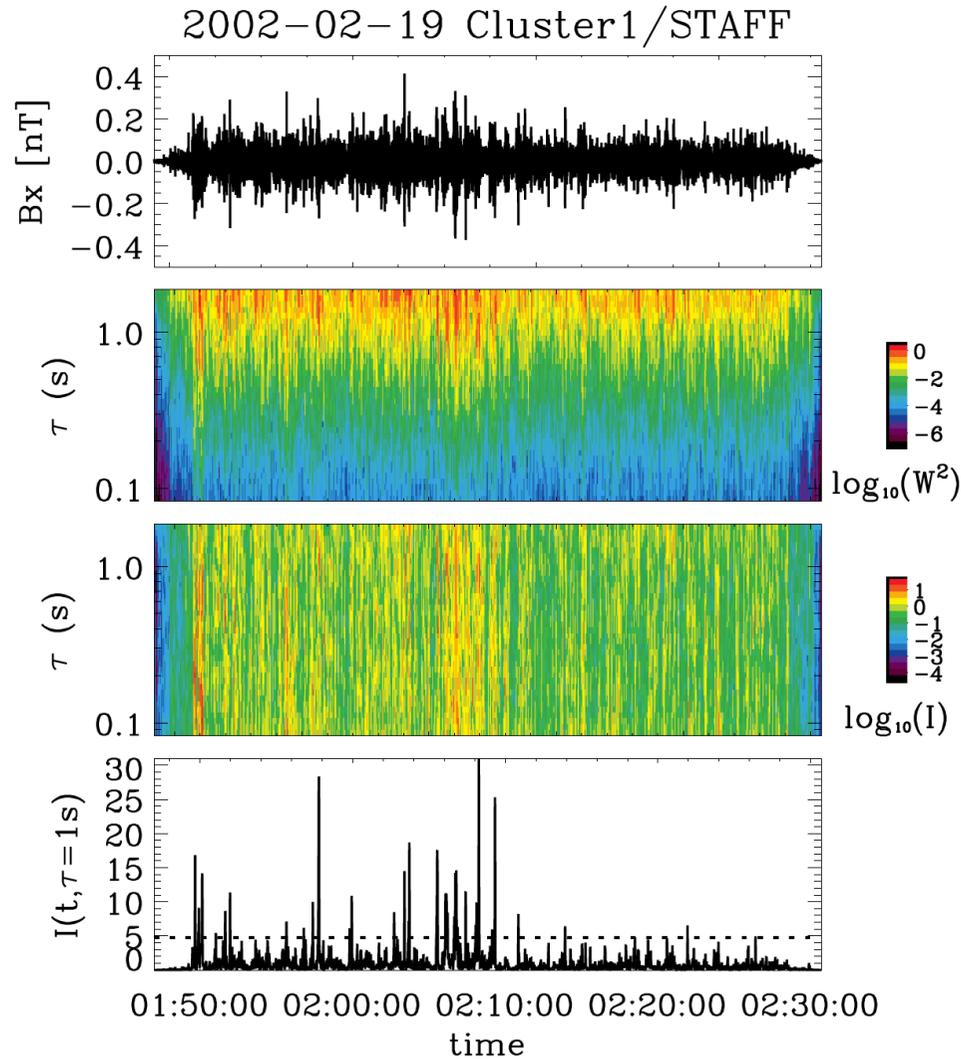
$$\psi_0(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2}$$

$$\tau_m = \tau_0 2^m, \quad m = 0, 1, \dots, M$$

$$M = \log_2(N\delta t/\tau_0)$$

[Farge, 1992; Torrence & Compo 1998]

# Looking for coherent structures with Morlet wavelets



## Morlet Wavelet Transform

$$\mathcal{W}_x(\tau, t) = \sum_{j=0}^{N-1} B_x(t_j) \psi^*[(t_j - t)/\tau]$$

$$\psi_0(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2}$$

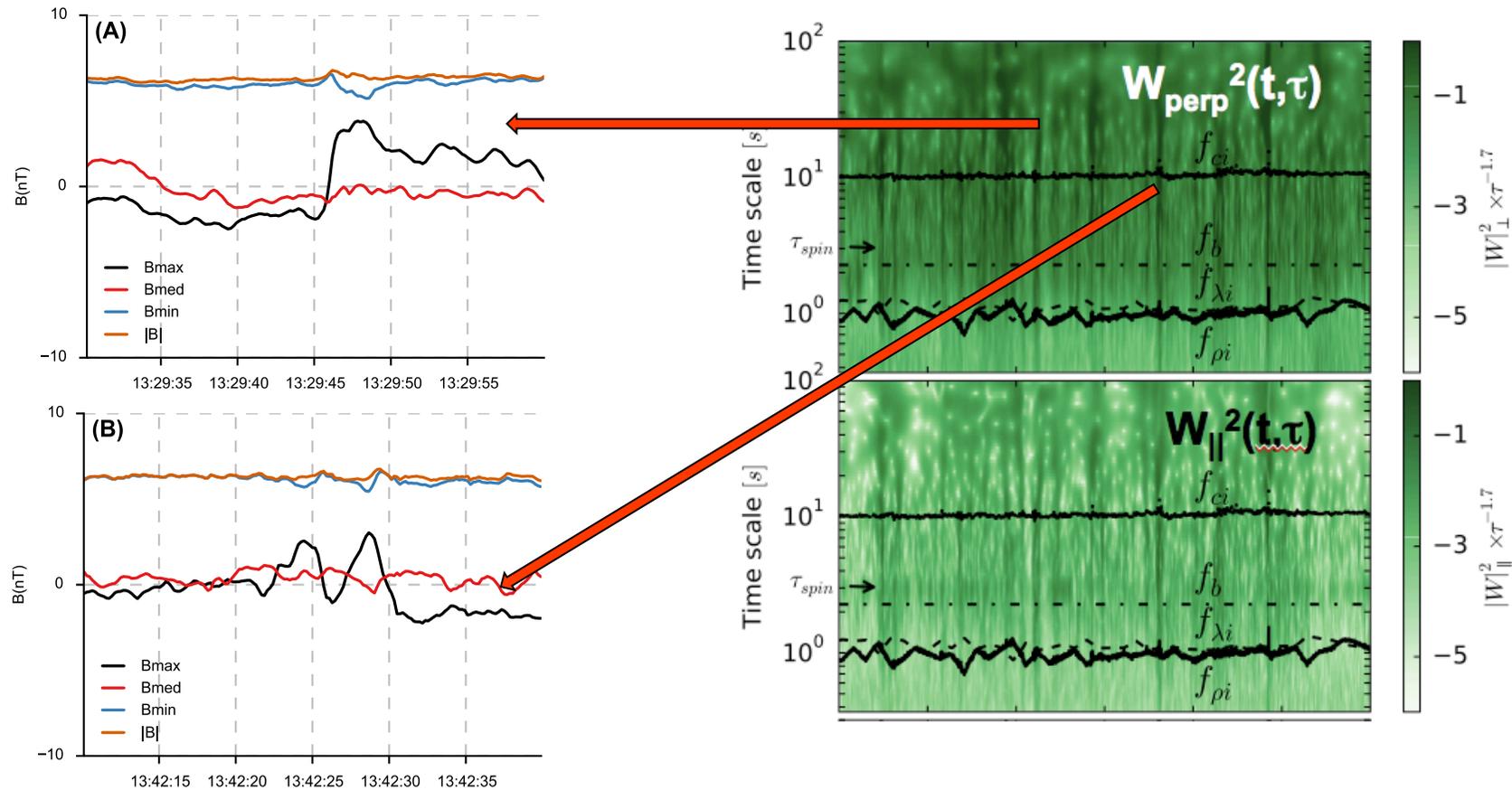
Intermittency measure [Farge 1992] :

$$I(t, \tau) = \frac{|W(t, \tau)|^2}{\langle |W(t, \tau)|^2 \rangle_\tau}$$

=> we see localised events covering all scales > mean

# Coherent current sheets and vortices

- Distributions of energy in time and scales for Alfvénic and compressible fluctuations (wavelet scalogrammes  $W_{\text{perp}}^2$  and  $W_{\parallel}^2$ ) => presence of coherent events simultaneously in  $W_{\text{perp}}^2$  and  $W_{\parallel}^2$ .

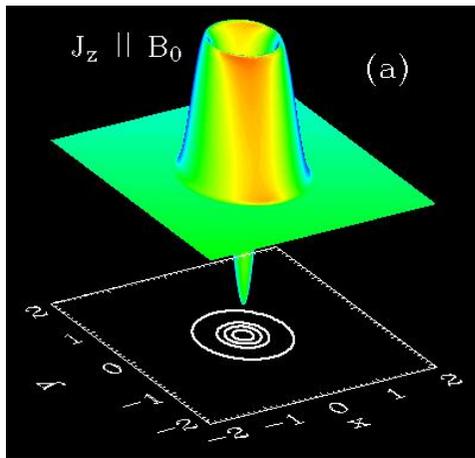


- Large amplitude current sheets and Alfvén vortices ( $\delta B/B_0=0.7$ )

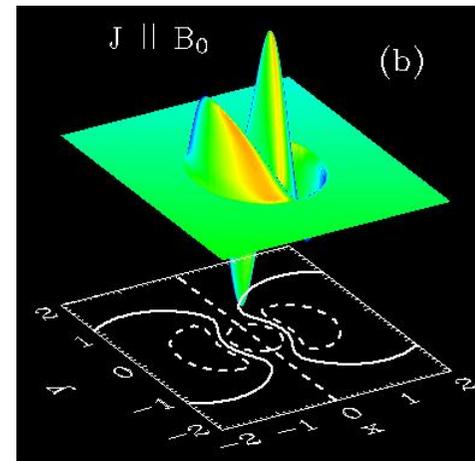
# Alfven vortices ~ 2D HD vortices

$$\Psi = \xi A; \quad \xi = \frac{u}{B_{0y}} \quad = \quad \delta V_{\perp} / V_A = \xi \delta B_{\perp} / B_0$$

Vector potential, A, ~ to stream function  $\Rightarrow$   
field lines || stream lines & current || vorticity



**Monopole ~ force free  
current, standing structure**



**Dipole ~ two inversed  
currents, propagates**

$$\frac{\partial_z}{\nabla_{\perp}} \sim \frac{\partial_t}{V_A \nabla_{\perp}} \sim \frac{\delta B_z}{\delta B_{\perp}} \sim \frac{\delta V_z}{\delta V_{\perp}} \sim \frac{\delta B_{\perp}}{B_0} \sim \frac{\delta V_{\perp}}{V_A} \sim \varepsilon.$$

[Petviashvili & Pokhotelov, 1992]

# Localized solutions of 2D incompressible Navier-Stokes equation

$$\partial_t \omega + (\delta \mathbf{V}_\perp \cdot \nabla) \omega = 0$$

$\omega$  - vorticity &  $\Psi$  - stream function

$$\omega = \nabla \times \delta \mathbf{V}_\perp = -\Delta \Psi ; \quad \delta \mathbf{V}_\perp = -\nabla \times \Psi$$

**Particular case:**

slow variations & **vorticity is localized** in a circle of the radius  $a$

$$\begin{cases} \Delta \Psi + k^2 \Psi + c = 0, & r < a & \text{- Helmholtz's equation} \\ \Delta \Psi = 0, & r \geq a & \text{- Laplace's equation} \end{cases}$$

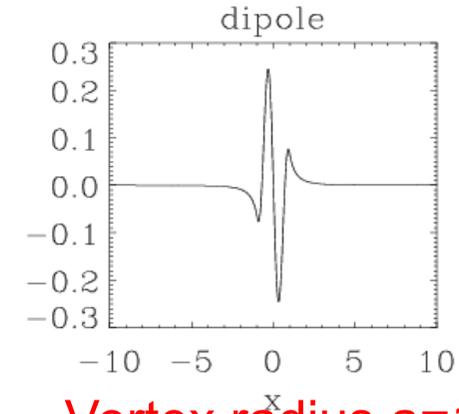
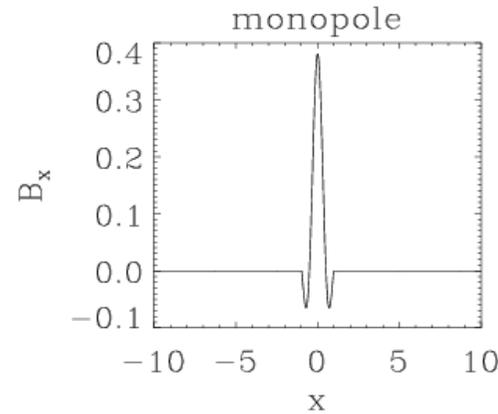
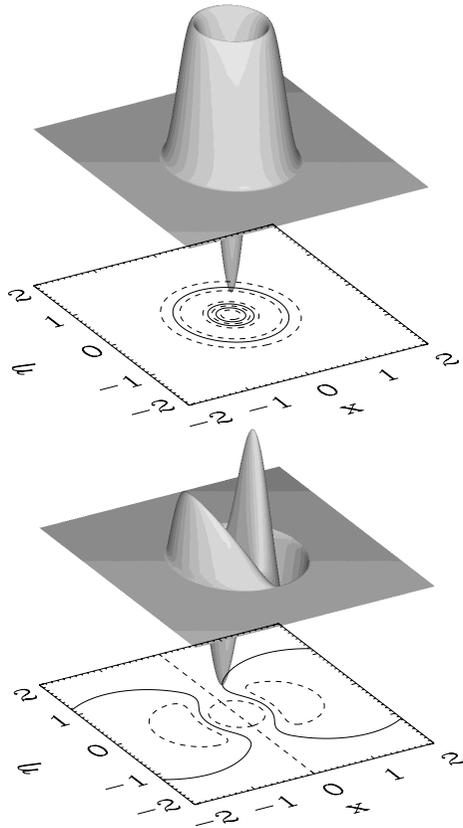
**monopole**

$$\rightarrow \begin{cases} \Psi = \Psi_0 (J_0(kr) - J_0(ka)) + ux \left( 1 - \frac{2}{kr} \frac{J_1(kr)}{J_0(ka)} \right), & r < a \\ \Psi = a^2 u \frac{x}{r^2}, & r \geq a. \end{cases}$$

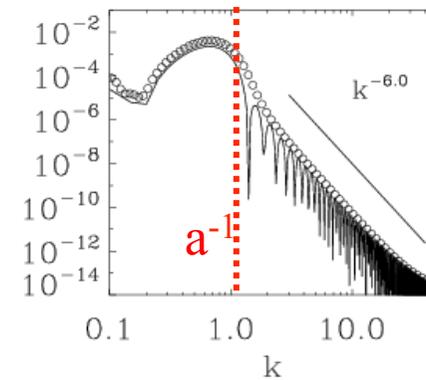
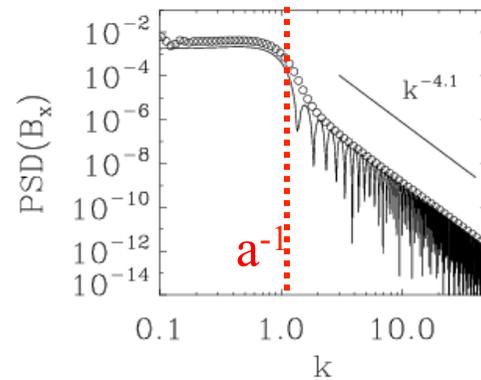
**dipole**

# Spectral properties of Alfvén vortices

$$\frac{\partial_z}{\nabla_\perp} \sim \frac{\partial_t}{V_A \nabla_\perp} \sim \frac{\delta B_z}{\delta B_\perp} \sim \frac{\delta V_z}{\delta V_\perp} \sim \frac{\delta B_\perp}{B_0} \sim \frac{\delta V_\perp}{V_A} \sim \varepsilon. \quad [\text{Petviashvili \& Pokhotelov, 1992}]$$

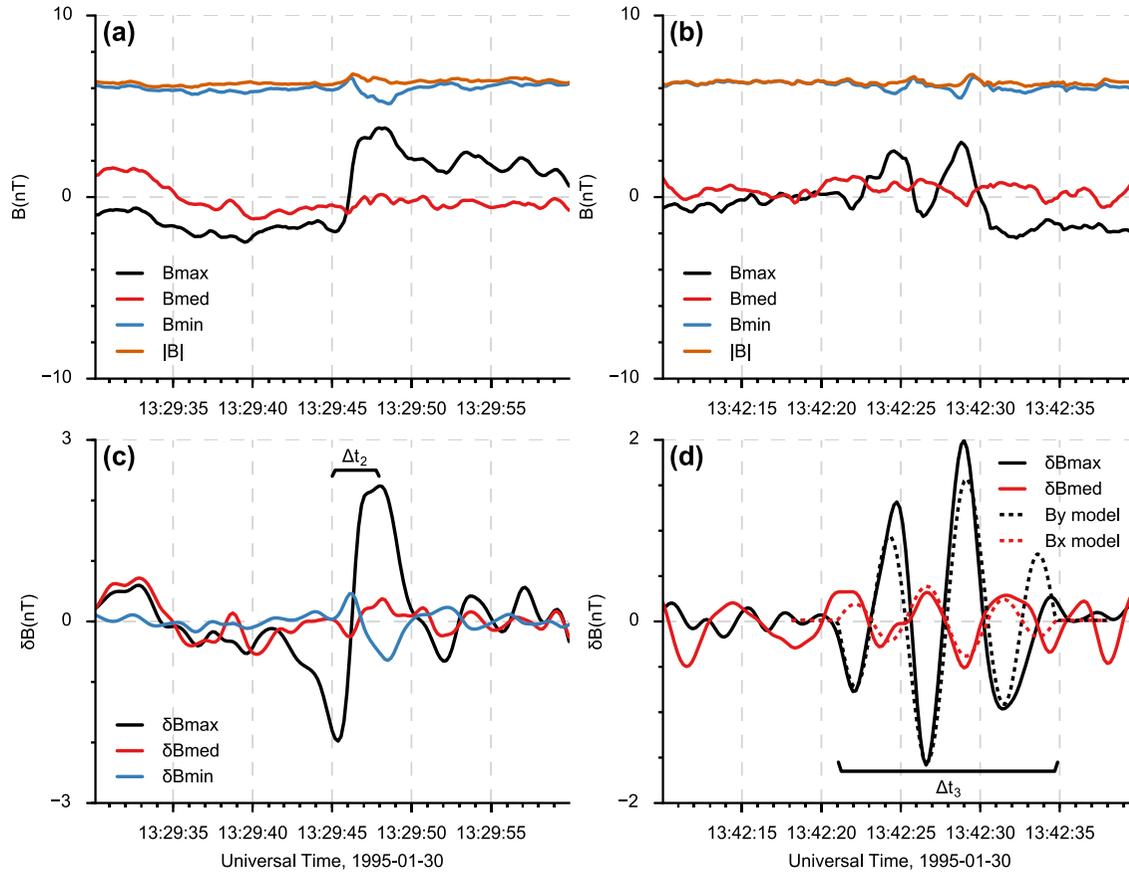


Vortex radius  $a=1$



- Spectral knee at  $k=a^{-1}$  ; power law spectra above it
- Monopole  $\Rightarrow \delta B^2 \sim k^{-4}$  (due to discontinuity of the current)
- Dipole  $\Rightarrow \delta B^2 \sim k^{-6}$  (due to discont. of the current derivative)

[Alexandrova 2008, NPG]



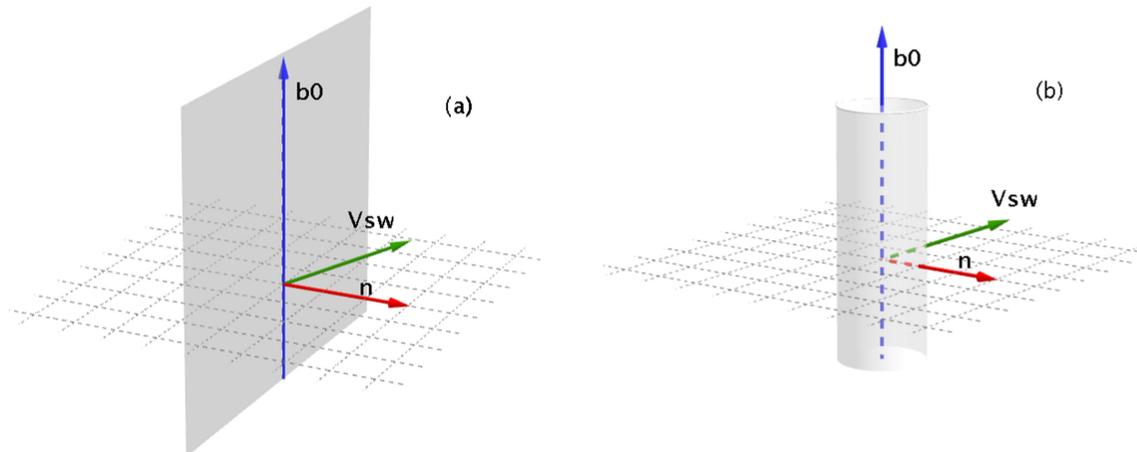
Raw magnetic data

Magnetic fluctuations

$$\delta B_i = B_i - \langle B_i \rangle_{10s}$$

$L_{\text{current sheet}} \sim 5$  ion scales

$L_{\text{vortex}} \sim (5-30)$  ion scales

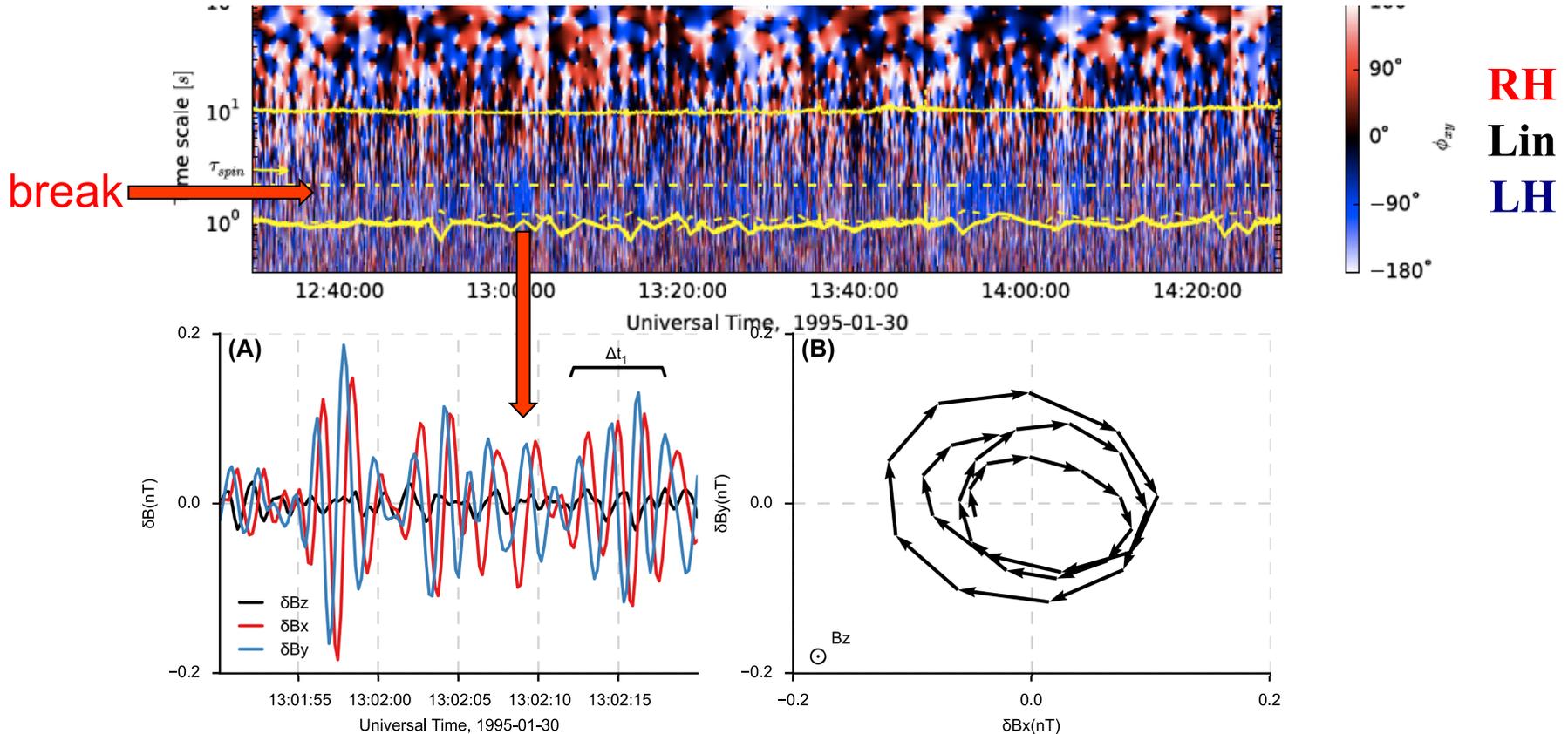


[Lion et al, 2016, APJ]

# Polarization in the plane perpendicular to **B**

Phase difference between Bx and By:

$$\Delta\phi_{xy}(t, \tau) = \phi_x(t, \tau) - \phi_y(t, \tau)$$



**Alfven-Ion-Cyclotron waves  
at ion break frequency**

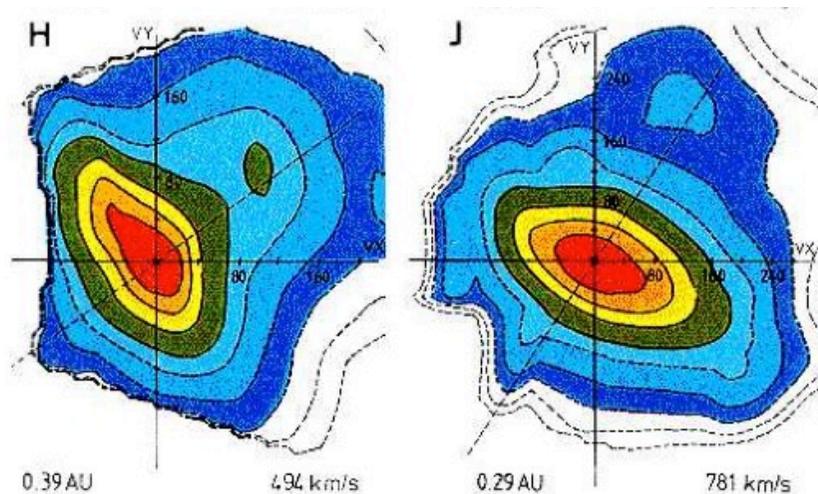
$$\delta B / B_0 = 0.03, \quad \Theta_{BV} = 160^\circ$$

$$T_{\perp} / T_{\parallel} = 3.5, \quad \beta_{\parallel} = 0.2$$

# Ion scale instabilities in the solar wind

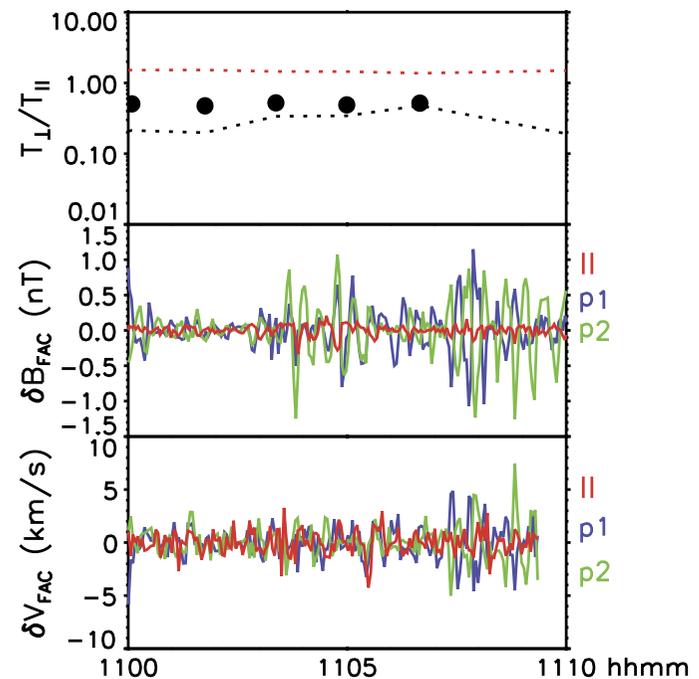
- In the solar wind ion distribution functions  $f(V_i)$  are anisotropic =>
- ion temperature anisotropy instabilities develop to isotropy  $f(V_i)$
- => quasi-monochromatic waves at a frequency/scale close to ion scales

[Marsch et al. 1983]

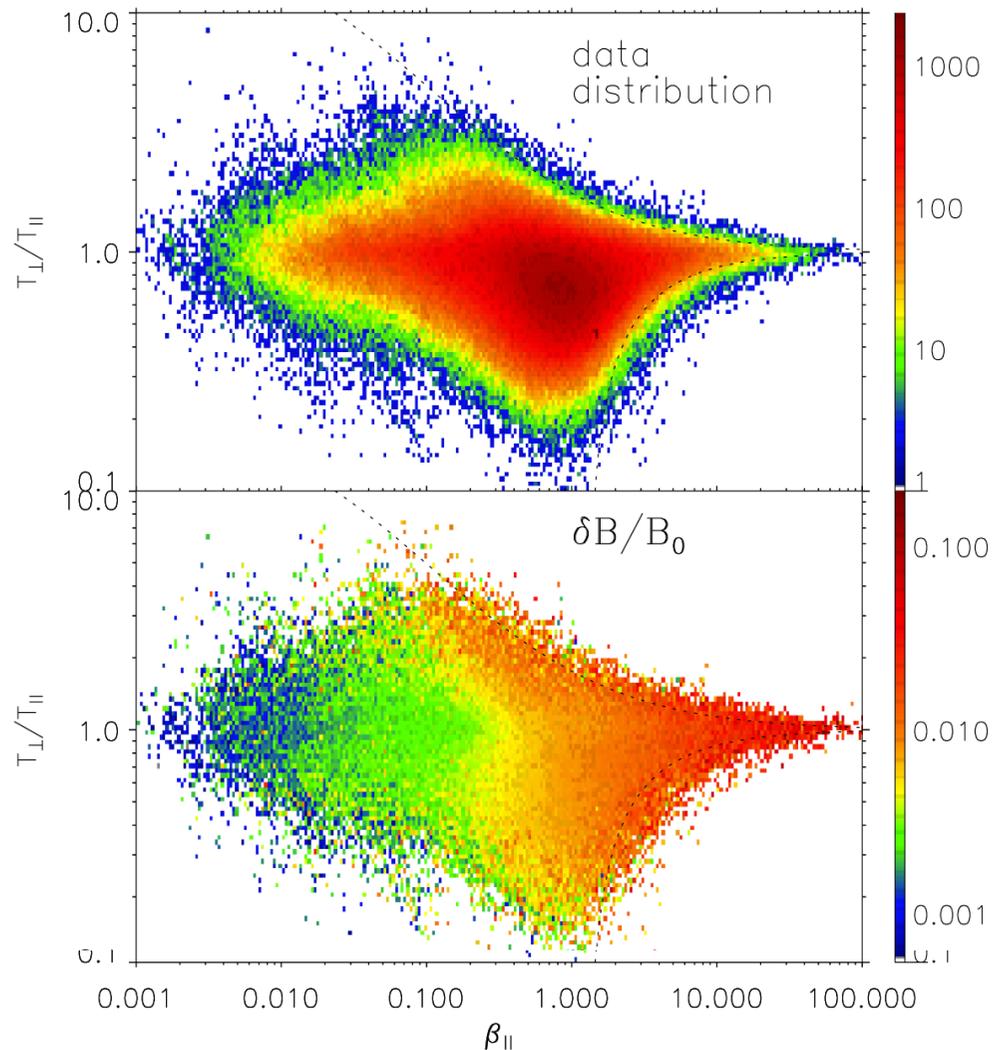


$$T_{i\perp} \neq T_{i\parallel}$$

[Alexandrova et al. 2013, SSR, courtesy of Stuart Bale]



# Ion temperature anisotropy as a function of plasma beta and 10 years of *Wind* data



$$\beta_{\parallel} = \frac{nkT_{\parallel}}{B^2 / 8\pi}$$

[Bale et al. 2009, PRL]

See as well:

[Matteini et al. 2007]

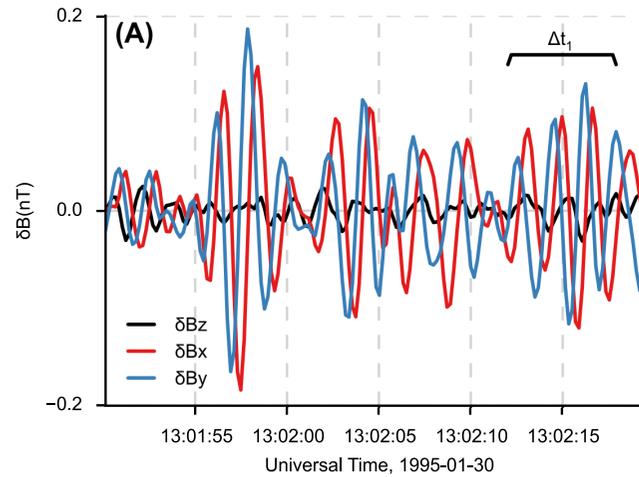
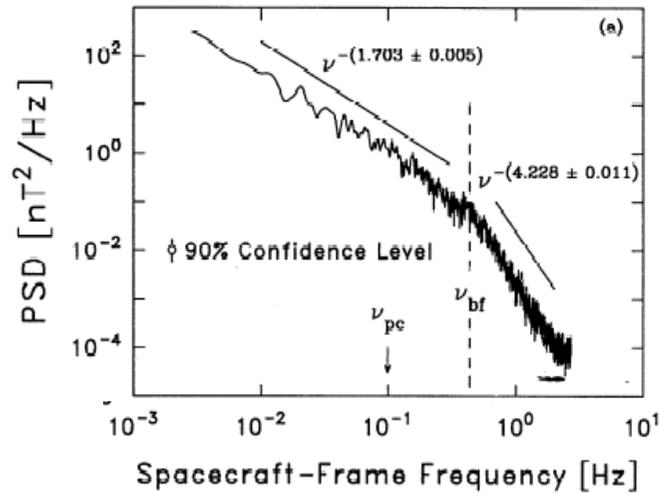
[Hellinger et al. 2006]

[Kasper & Lazarus 2002]

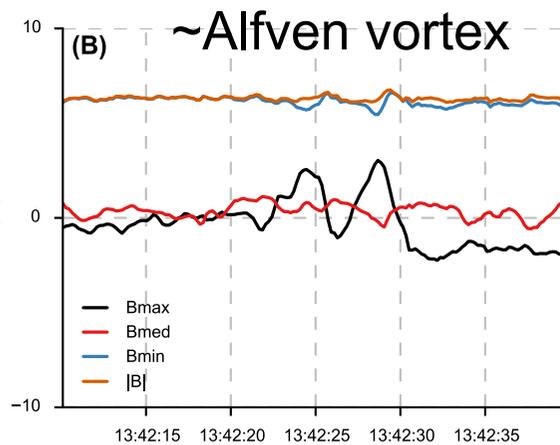
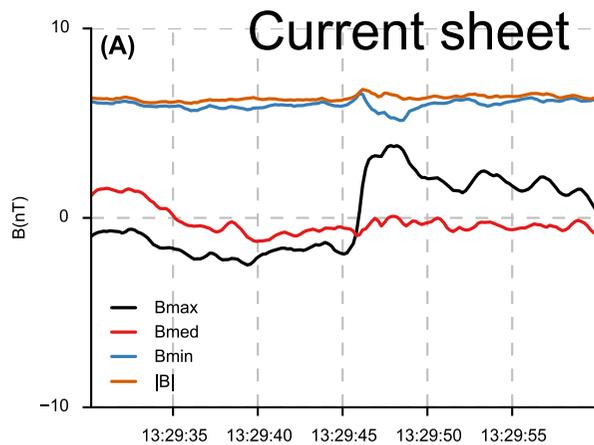
=> At ion scales one expects to have a superposition of background turbulence + waves/instabilities.

# Ion scales: superposition of different phenomena

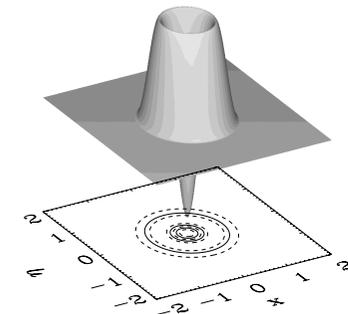
[Leamon et al. 1998]



Monochromatic Alfvén waves at  $\text{freq} \sim f_{ci}$  with  $k_{||}$  (generated by AIC instability).

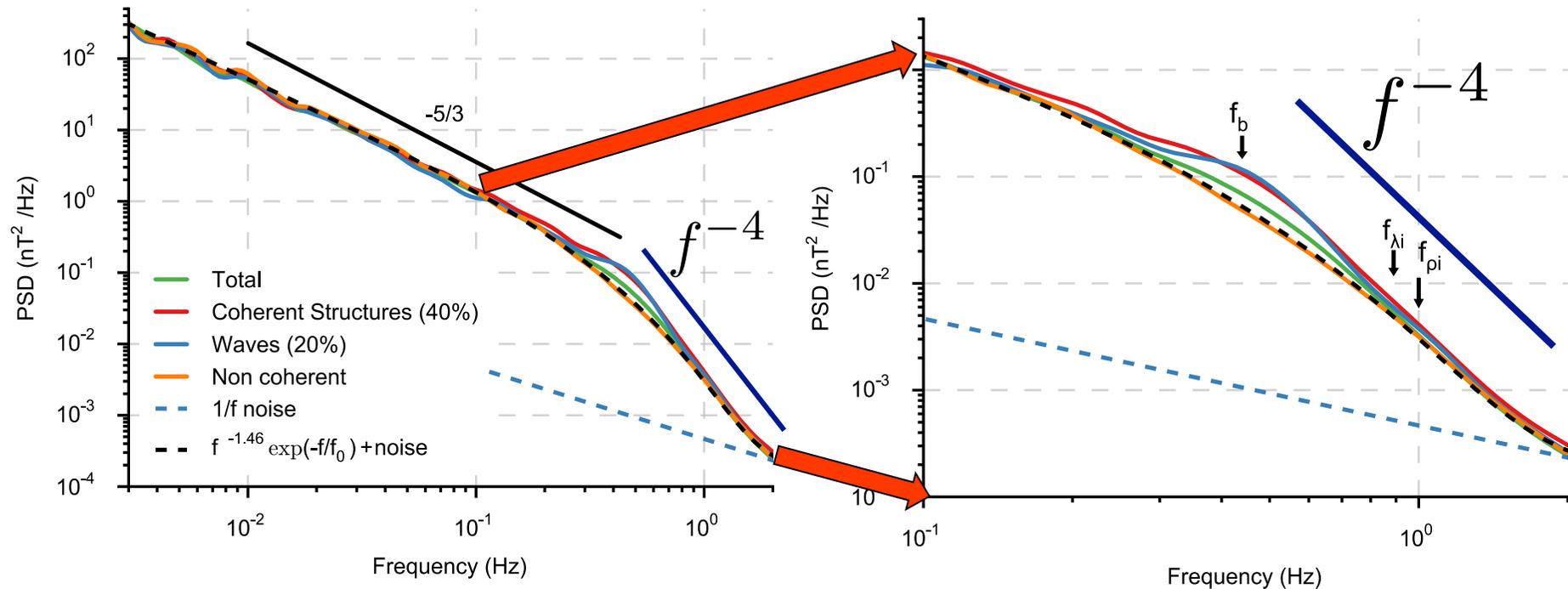


Localized spatial structures with  $k_{\text{perp}}$  at scale  $\sim$  ion Larmor radius



[Lion et al, 2016, ApJ]

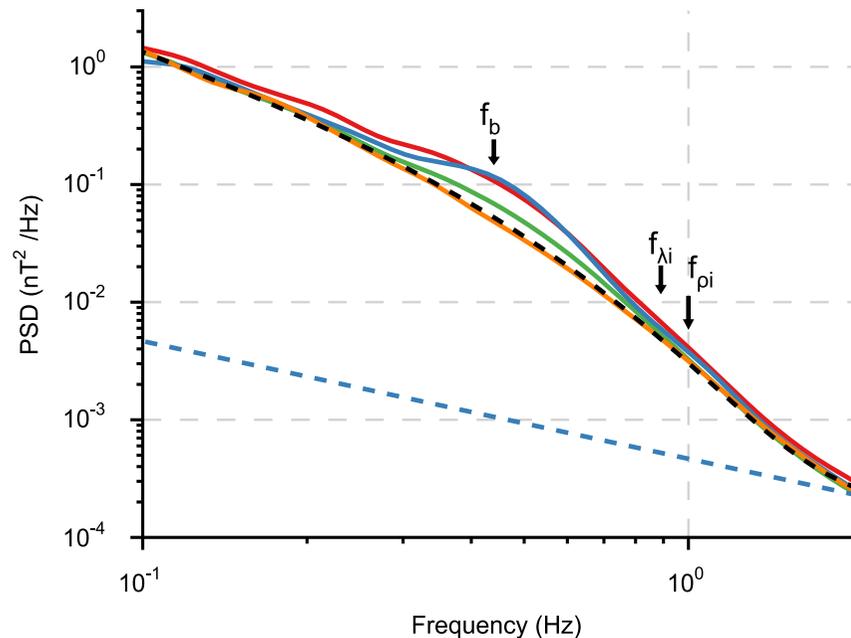
# Total and individual spectra of 3 families: waves, structures, background



- AIC waves (20%) spectrum has a bump around the break  $f_b$
- Structures (40%) spectrum has a knee around  $f_b$  and  $\sim f^{-4}$  power-law at  $f > f_b$
- Spectrum of non-coherent fluctuations (40%) has NO break, but a smooth transition which can be described by :

$$E_B \sim f^{-3/2} \exp(-f/f_0), \quad f_0 = 0.3\text{Hz}$$

# Nature of turbulence around ion scales: fast solar wind



- Alfvén Ion Cyclotron waves (with  $k_{\parallel}$ )
- Coherent structures (with  $k_{\perp}$ )
- Non coherent signal, which can be described by

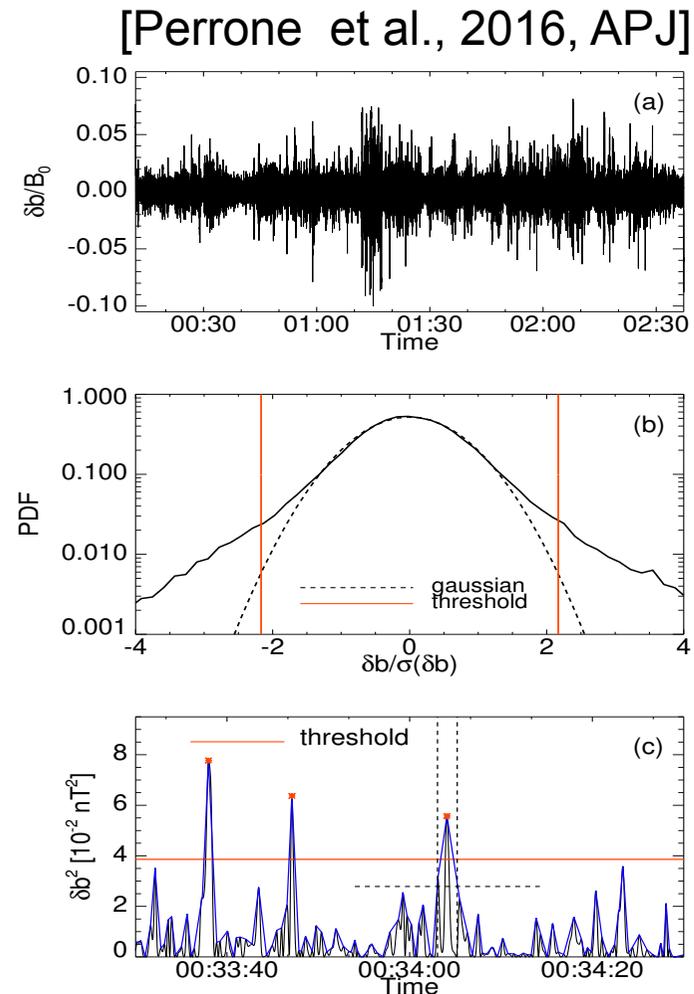
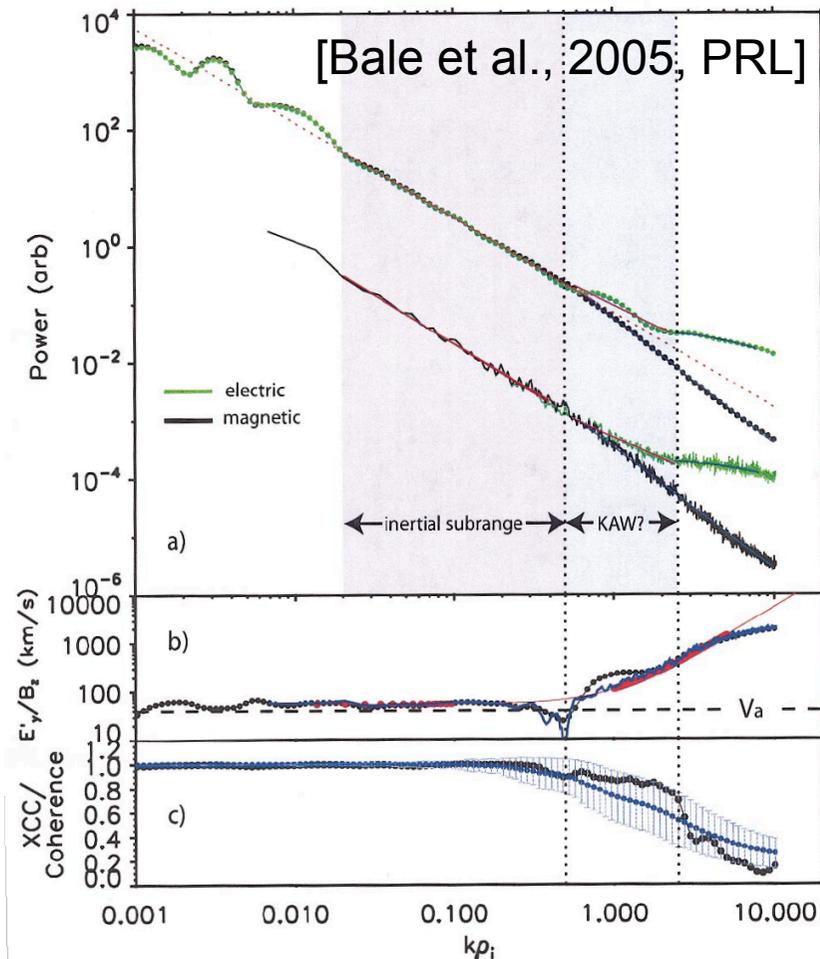
$$E_B \sim f^{-3/2} \exp(-f/f_0), \quad f_0 = 0.3Hz$$

⇒ The total observed spectrum depends on the contribution (percentage) of each event (which depends on the local plasma parameters and field-to-flow orientation)

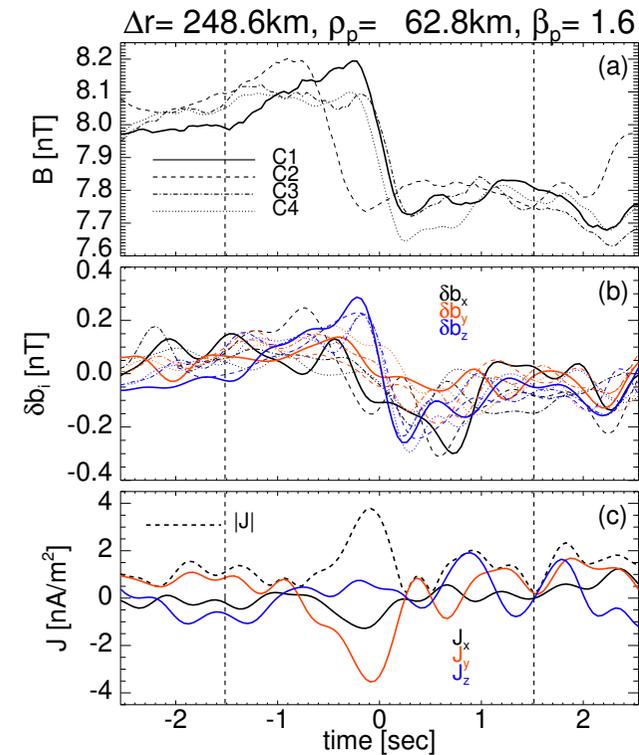
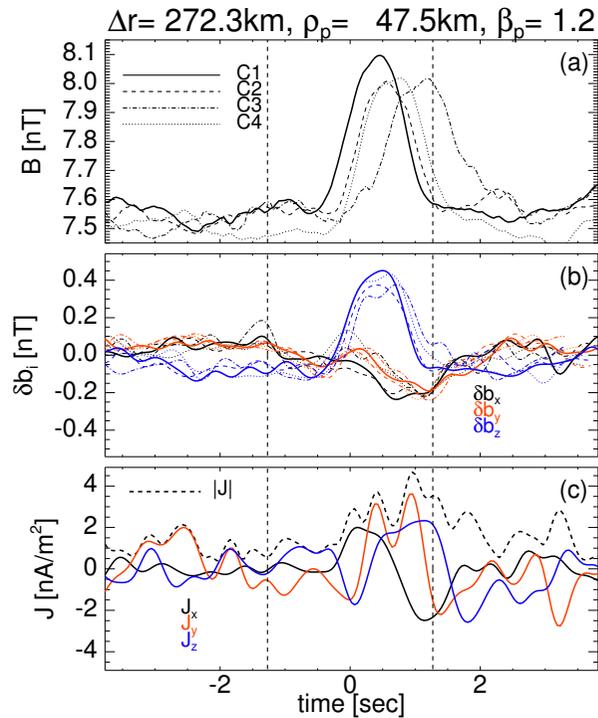
⇒ These results may explain spectral variability around ion scales.

# Nature of turbulence around ion scales in the slow solar wind ?

Let us consider Stuart Bale's 2005 time interval in the slow solar wind (no clear 'break', plasma  $\beta \sim 1-4$ ). At ion scales : KAWs ?



# Nature of turbulence around ion scales in the slow solar wind ?

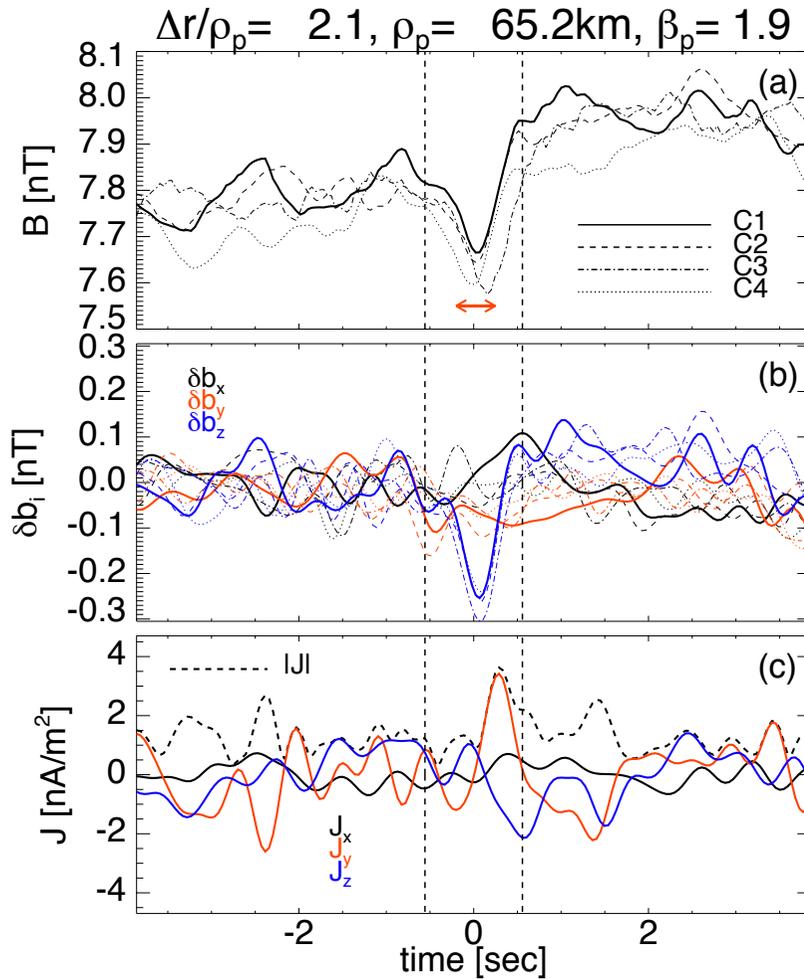


- Magnetic solitons**  
 $\beta_p \sim 1.2$ , we estimate  
 phase speed in the  
 plasma frame:  $V_\phi = V_f \sim V_{th}$

- Shocks** :  $\beta_p \geq 1, \text{Mach}_f = 3$

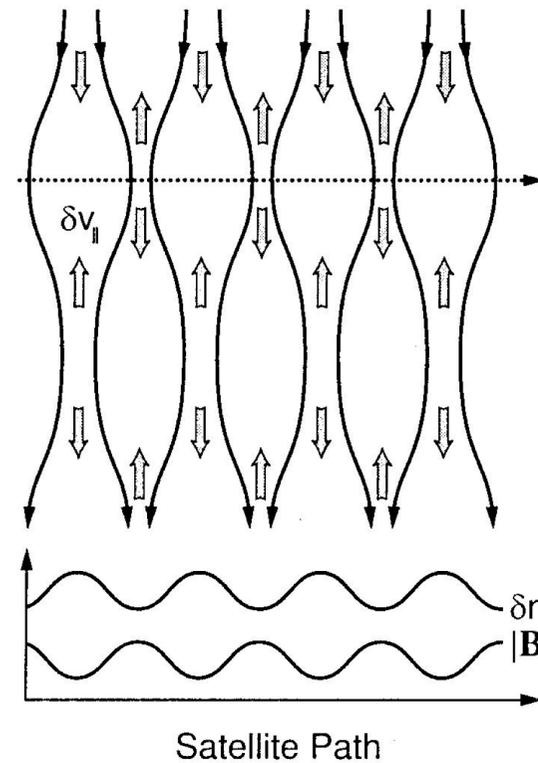
[Perrone et al., 2016, APJ]

# Nature of turbulence around ion scales in the slow solar wind ?



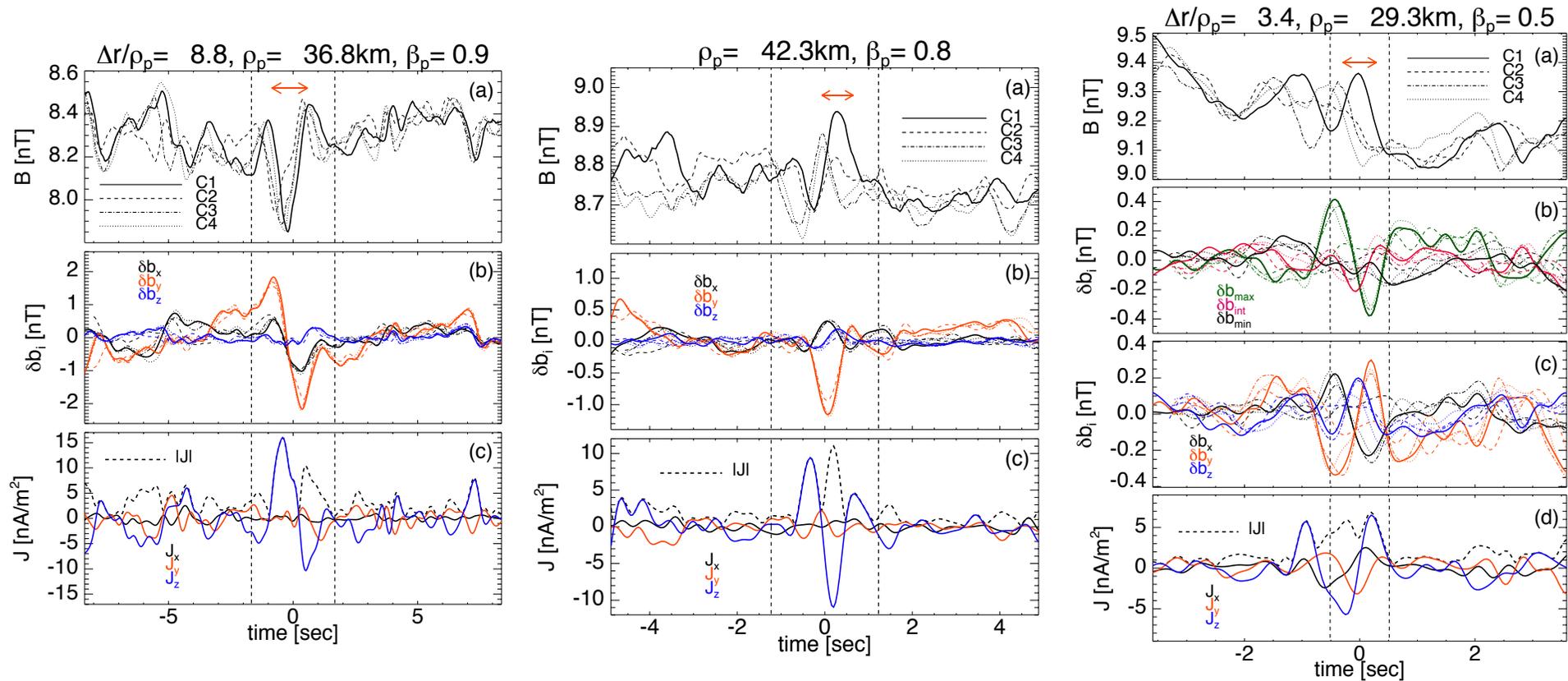
- **Magnetic holes** ( $\beta_p \sim 2$ ),  
 $V_\phi = 0, T_{\text{perp}}/T_{\parallel} > 1$

=> NL evolution of mirror instability ?



[Perrone et al., 2016, APJ]

# Nature of turbulence around ion scales in the slow solar wind ?



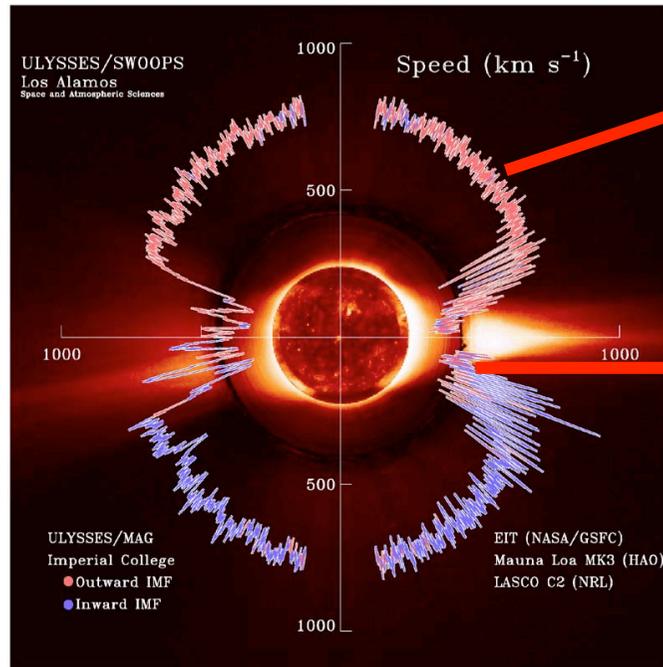
- **Current sheets** with  $\delta B_{\text{perp}} \gg \delta B_{\parallel}$  ( $\beta_p \leq 1$ ),  $V_{\phi} = 0$

[Perrone et al., 2016, APJ]

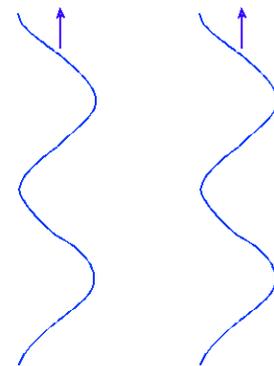
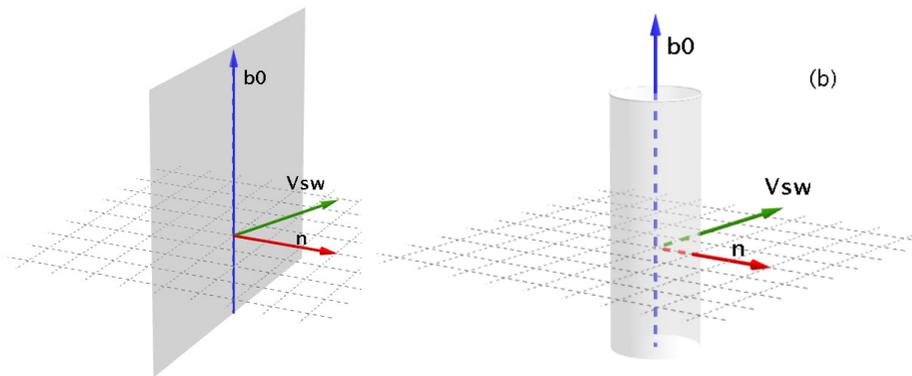
- **Alfven vortex** like structures with  $\delta B_{\text{perp}} \gg \delta B_{\parallel}$  ( $\beta_p \sim 1.2$ ),  $V_{\phi} \in [0.5, 2]V_A$

- **Compressible vortex** like structures with  $\delta B_{\text{perp}} \sim \delta B_{\parallel}$  ( $V_{\phi} = 0$ , or  $\in [1, 4]V_A$ )

# Nature of turbulence around ion scales in the fast and slow solar wind



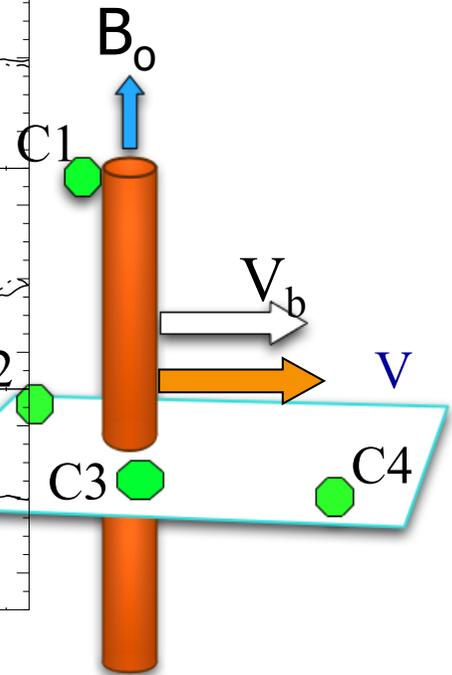
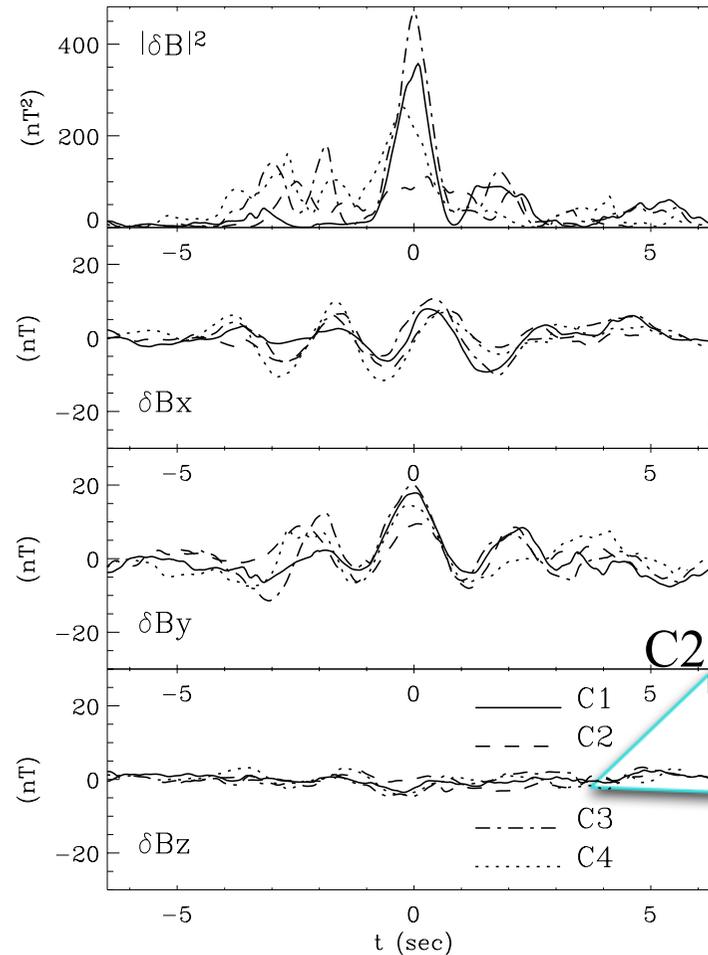
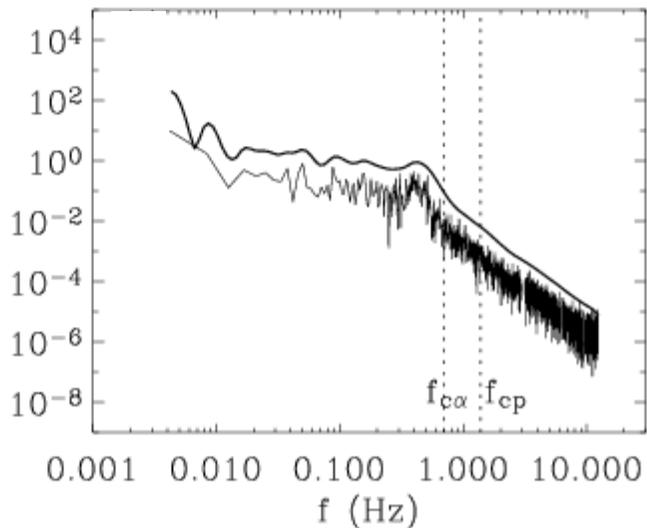
- **Fast wind:** Alfvénic structures with high amplitudes ( $\delta B/B > 0.5$ ) and  $k_{\text{perp}} + \text{Alfvén (IC) waves of small amplitudes with } k_{\parallel}$ .
- **Slow wind:** mixture of compressible (solitons and shocks) and Alfvénic (current sheets and vortices) coherent structures with small amplitudes  $\delta B/B \sim 0.1$  and quasi-perp wave vectors ( $k_{\text{perp}} \gg k_{\parallel}$ ).



Lion et al., 2016, ApJ  
Perrone et al., 2016, ApJ  
Roberts et al., 2016, JGR

# Alfven vortex: important ingredients of space plasma turbulence

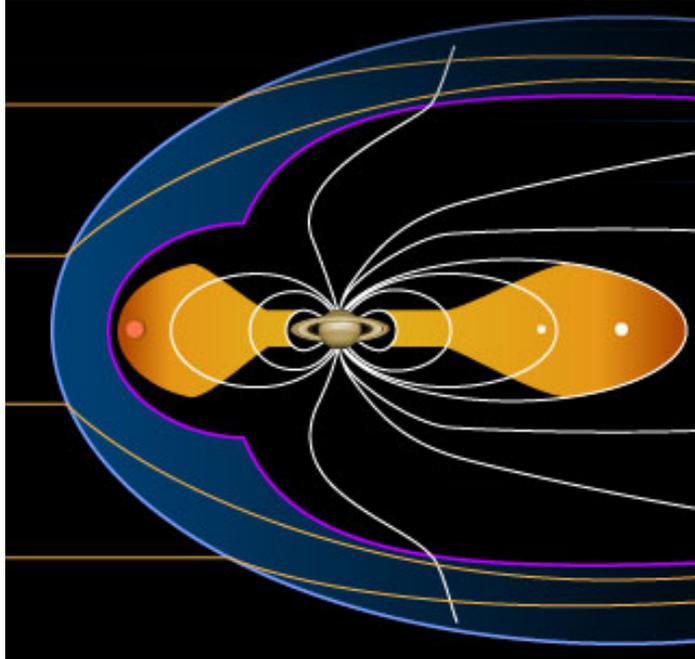
Ex: downstream of the Earth's bow shock



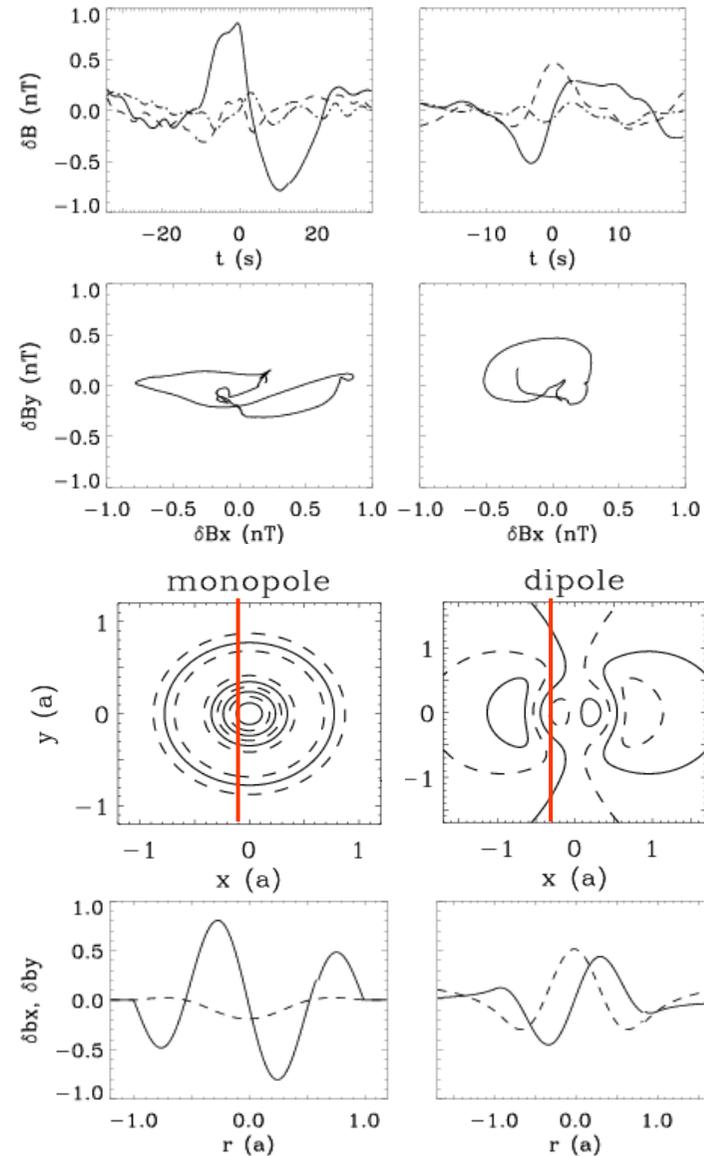
[Alexandrova et al. 2006, JGR]

$$\delta V_{\perp} / V_A = \xi \delta B_{\perp} / B_0$$

# Alfven vortices in the Saturn's Magnetosheath (*Cassini* observations)



- $B_{IMF} = 0.3 \text{ nT}$
- $B_{msh} = 1.2 \text{ nT}$
- $n_{msh} \sim 0.5 \text{ cm}^{-3}$  (Voyager-2)
- $V_{b,msh} \sim 130 \text{ km/s}$  (Voyager-2)
- $c/\omega_{pi} \sim 300 \text{ km}$
- $\text{Mach} \sim 15$



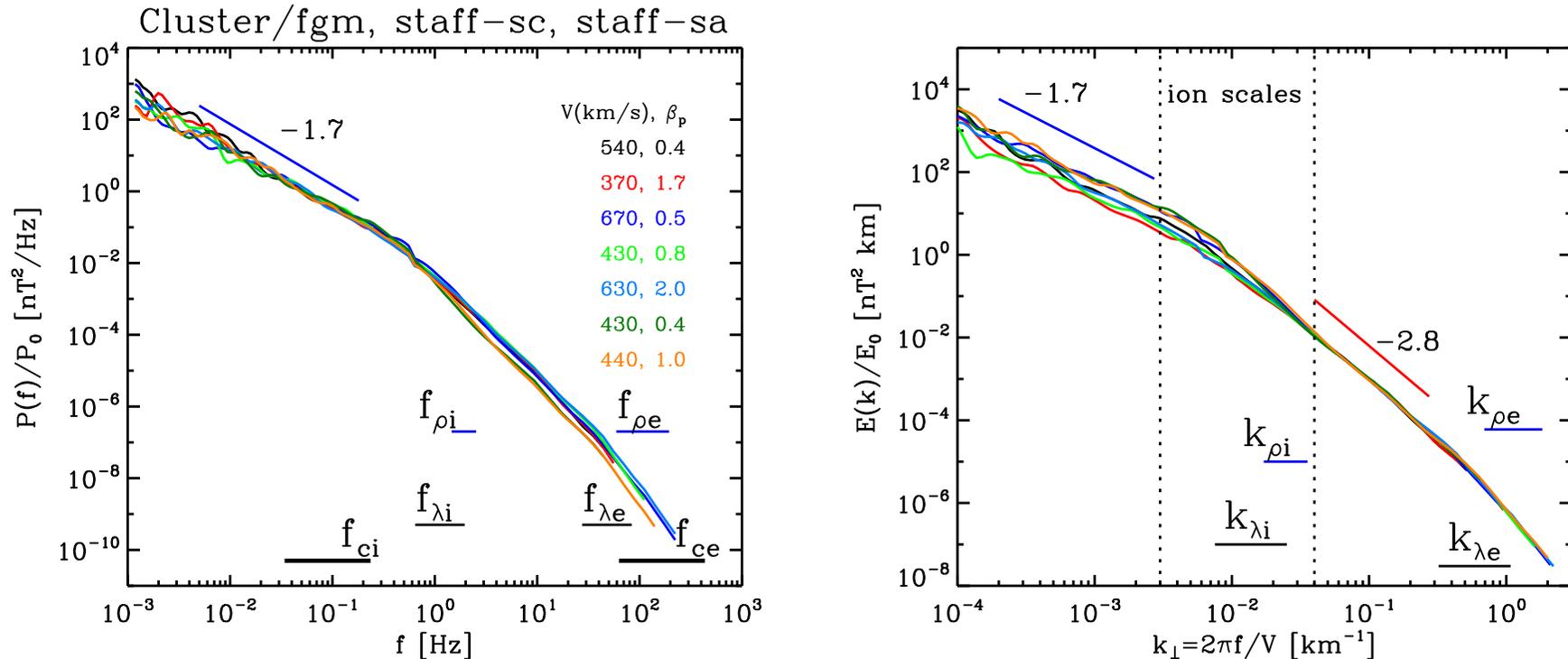
[Alexandrova & Saur, 2008, GRL]

# What is going on at electron scales?

- The best space mission (at the moment) to study sub-ion and electron scales in the solar wind is Cluster (4 identical satellites, 2000 - 2018).
- Cluster mission has the most sensitive SC magnetometer, better than on MMS (mag. fluctuations up to 400 Hz in the solar wind).
- Cluster is devoted to magnetospheric research, spend short time intervals in the solar wind/orbit.
- 1h of data in the solar wind: +/-limit for MHD scales, OK to study kinetic scales.

# Turbulent spectrum between MHD and electron scales (Cluster measurements)

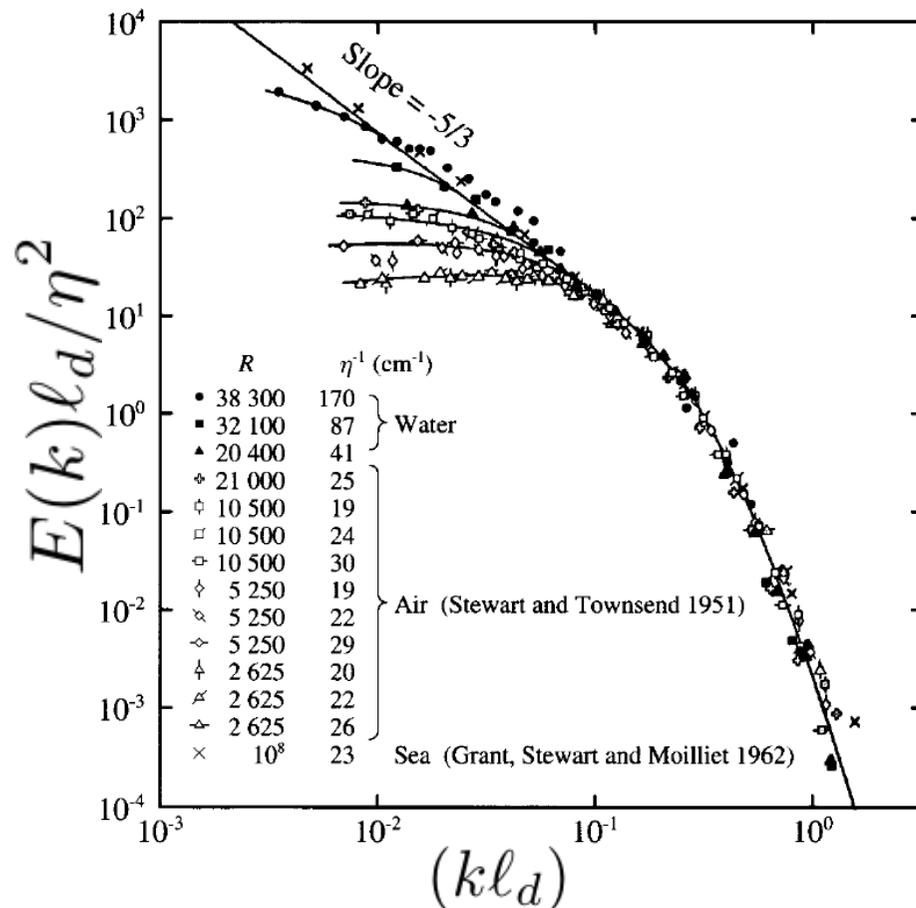
[Alexandrova et al. 2009, PRL; 2013, SSR]



- General spectra at MHD and between ion and electron scales ( $\sim k^{-2.8}$ ).
- Spectral variability around ion scales depends on coherent structures types and presence of ion instabilities [e.g. Matteini+'07, Bale+'09, Lion+'16, Perrone+'16, Roberts+'16].
- End of the cascade? Dissipation scales?

# Universal Kolmogorov's function:

Frisch, Turbulence: the legacy of Kolmogorov, 1996



$$E(k)\ell_d/\eta^2 = F(k\ell_d)$$

$$E(k)\ell_d/\eta^2 \sim (k\ell_d)^{-5/3}$$

$\ell_d$ : dissipation scale

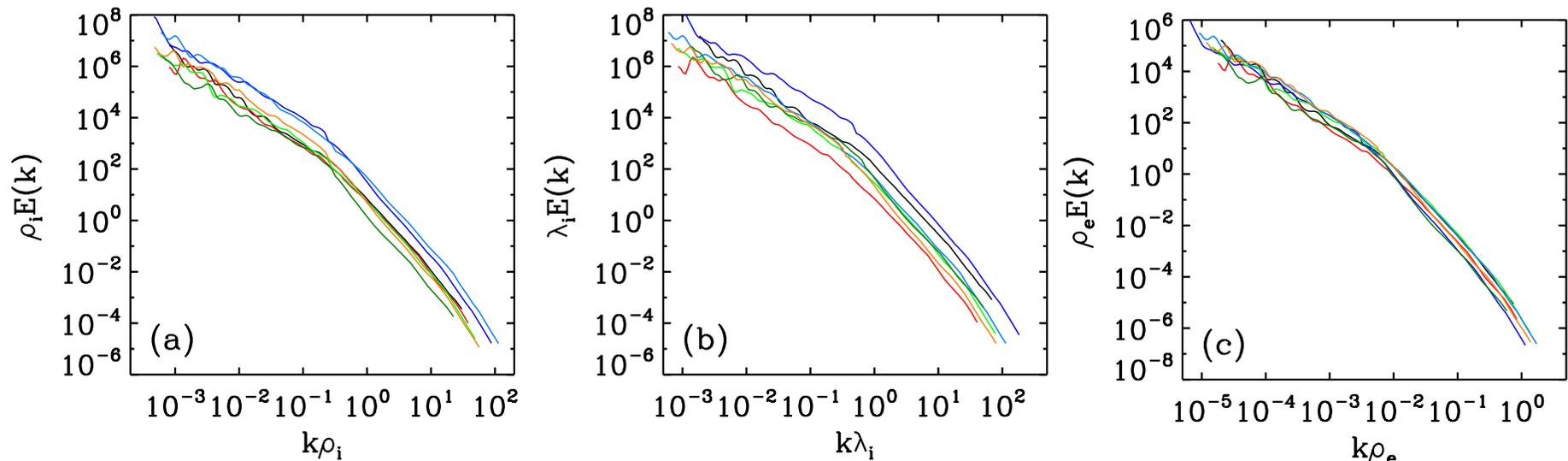
$\eta$ : viscosity

In HD turbulence, this normalization collapses spectra measured under different conditions.

# Dissipation scale?

Universal Kolmogorov's function:  $E(k)l_d/\eta^2 \sim (kl_d)^{-5/3}$

Let us try to apply this kind of normalization for sw spectra and for different candidates for the dissipation scale:  $l_d = \rho_{i,e}, \lambda_{i,e}$



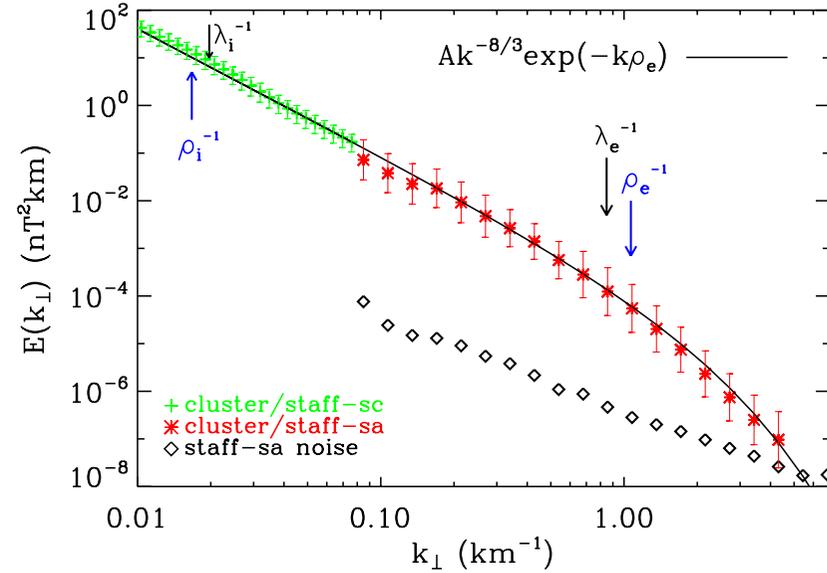
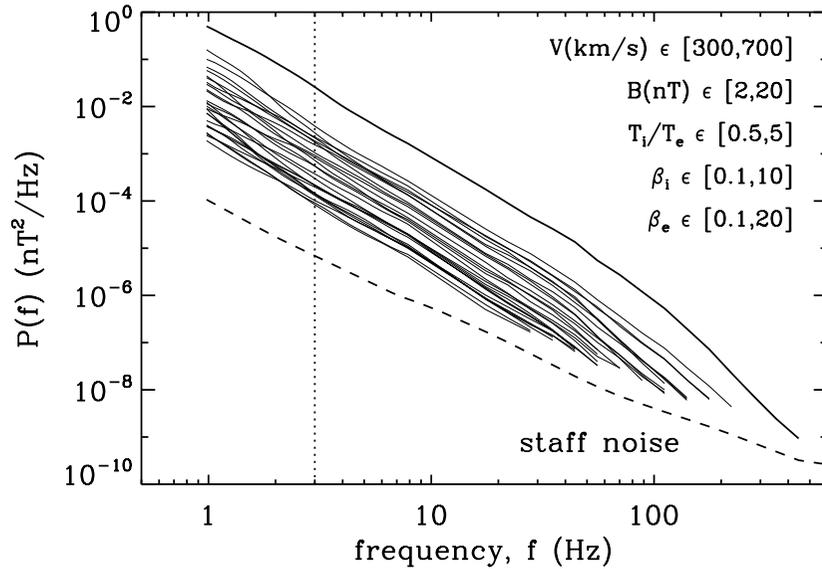
- Assumption:  $\eta = \text{Const}$
- $k\rho_i$  &  $k\lambda_i$  - normalizations are not efficient for collapse
- $k\rho_e$  normalization bring the spectra close to each other.

→  $l_d \sim \rho_e$

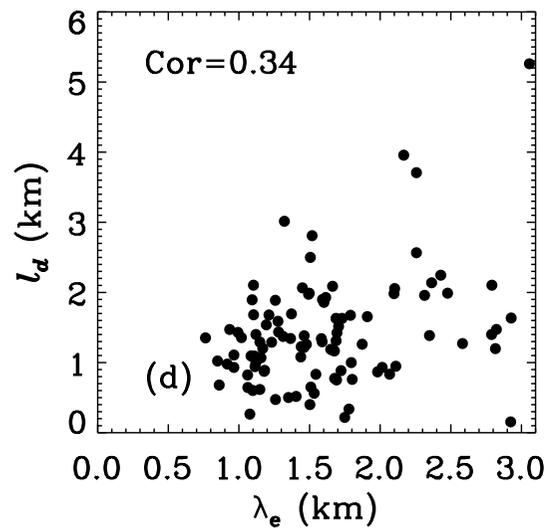
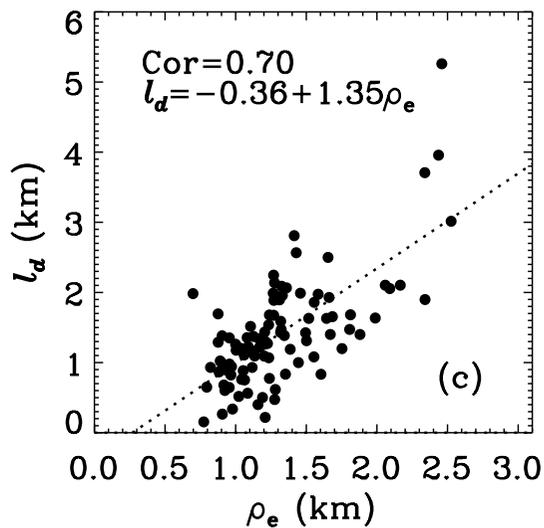
[Alexandrova et al., 2009, PRL]

# Spectrum at kinetic scales and dissip. scale

[Alexandrova et al., 2012, APJ]



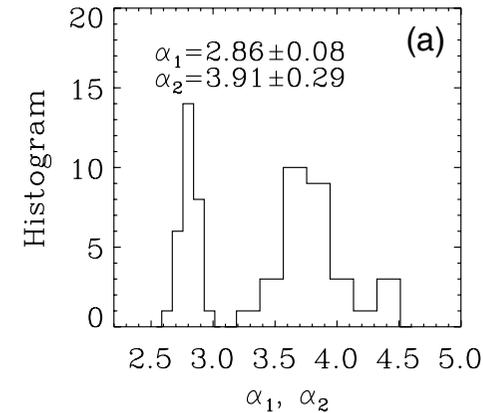
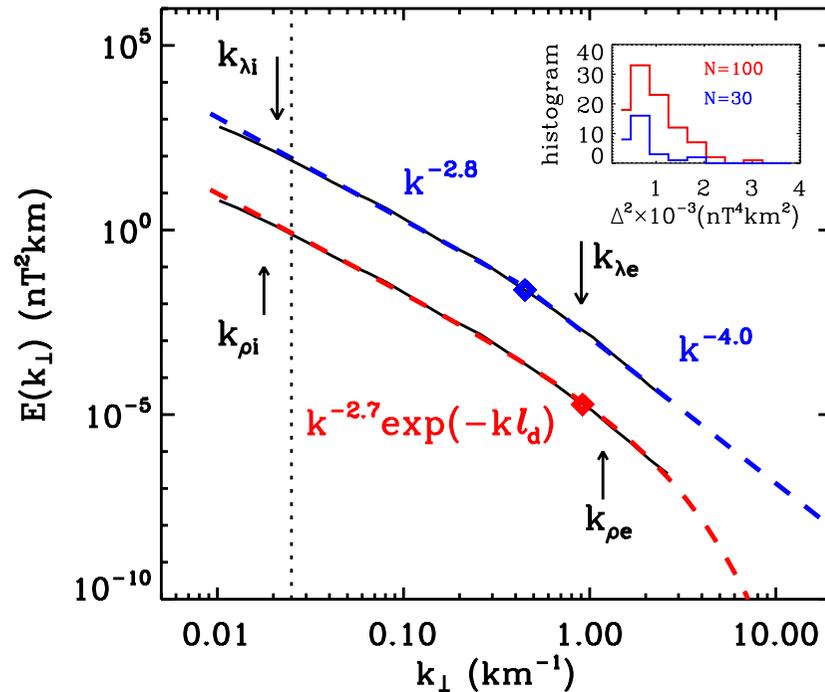
$$E(k) = Ak^{-\alpha} \exp(-k/k_d) \quad \text{[Chen et al., 1993, PRL]}$$



→  $l_d \sim \rho_e$

# Spectral shape: 2 alternatives approaches

[Alexandrova et al., 2012, APJ]

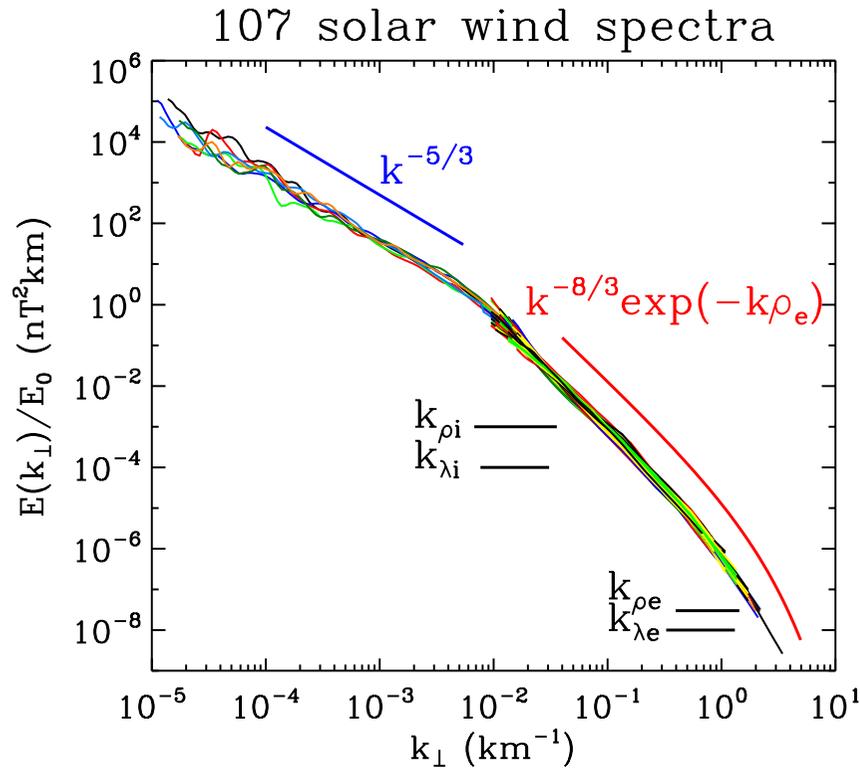


[Alexandrova et al., 2012, APJ]  
Sahraoui et al. [2013] show similar results.



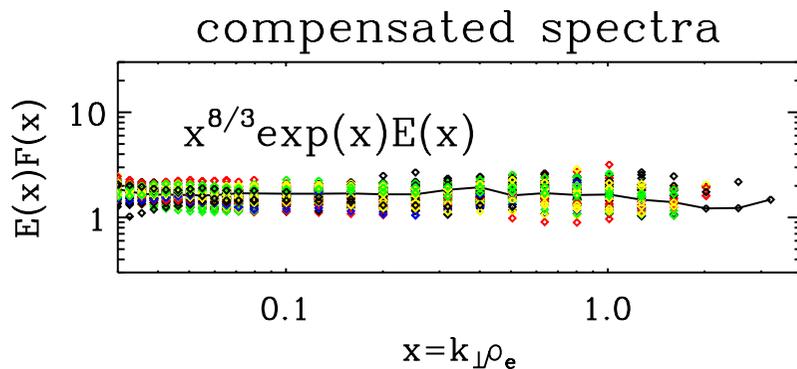
- small dispersion for  $\alpha_1$  around 2.86 and high dispersion for  $\alpha_2$
- Exponential-model has 3 free parameters ( $A, \alpha, l_d$ )
- 2-power-law model has 5 free parameters ( $A_1, \alpha_1, A_2, \alpha_2, k_{\text{break}}$ )

# General spectrum at kinetic scales



- For different solar wind conditions we find a general spectrum with “fluid-like” roll-off at electron scales [Alexandrova+'12].
- Electron Larmor radius seems to play a role of the dissipation scale in collisionless solar wind [Alexandrova +'09,12; Sahraroui+10,13]

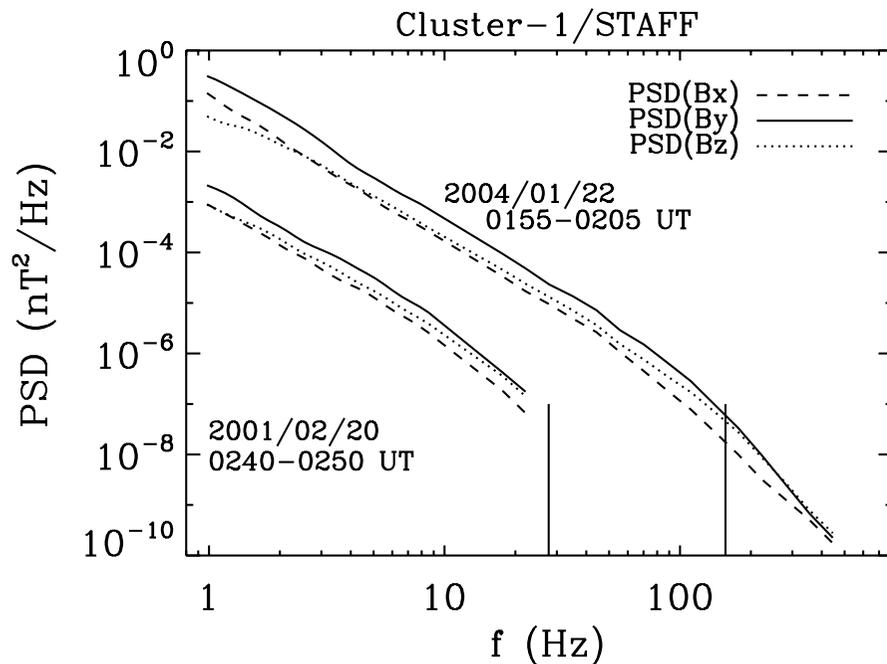
$$E(k) = Ak^{-8/3} \exp(-k\rho_e)$$



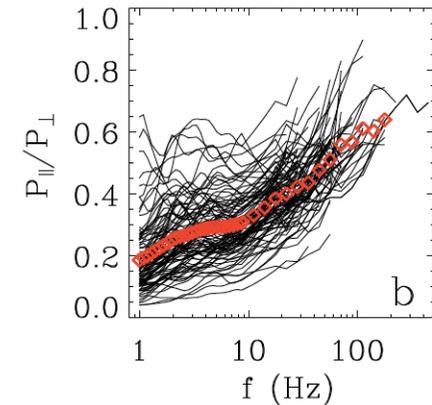
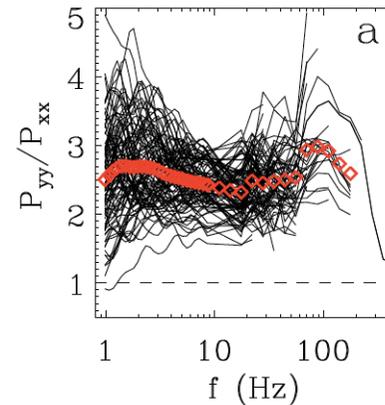
# Nature of turbulence at sub-ion scales?

(See as well the lectures of S. Boldyrev and M. Kunz)

[Lacombe, Alexandrova, Matteini, under rev., APJ]



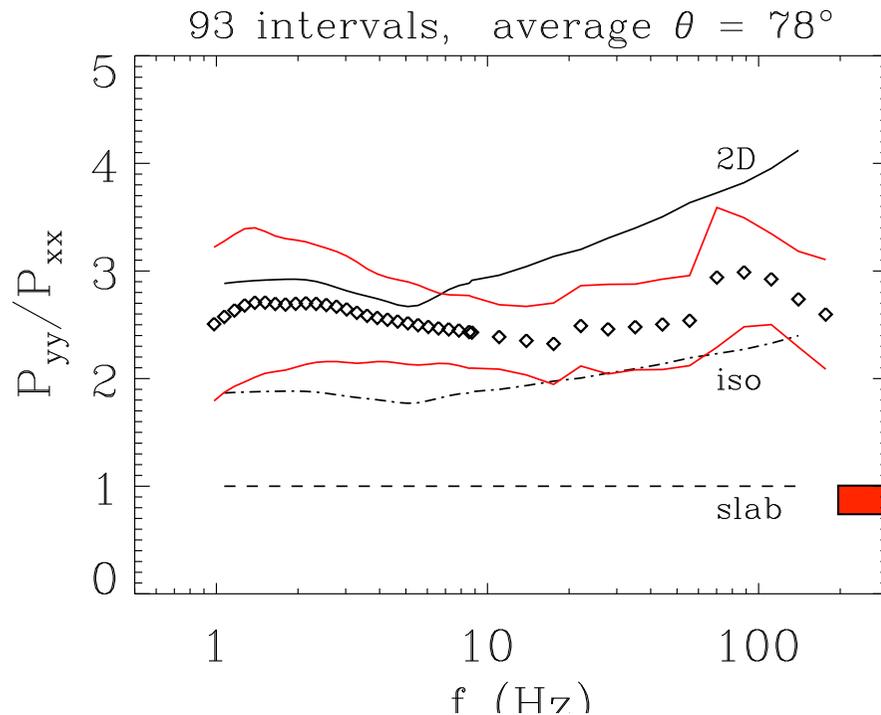
1. Anisotropy of  $\delta B$  in B-V frame
2. non-gyrotropy of B-fluctuations with  $\text{PSD}(B_y) > \text{PSD}(B_x)$ .
3. Compressibility increases with frequency.



Dashed line:  $\text{PSD}(B_x) = P_{xx}$ ,  $B_x$  is perp to  $B_0$ , in the plane  $(R, B_0)$ , where  $R$  is the radial direction (or solar wind velocity  $V$ -direction).  
 Solid line:  $\text{PSD}(B_y) = P_{yy}$ ,  $B_y$  is perp to the plane  $(R, B_0)$ .  
 Dotted line:  $\text{PSD}(B_z) = P_{zz} = P_{\parallel}$ ;  $B_z$  is parallel to  $B_0$ ;  $P_{\text{perp}} = P_{xx} + P_{yy}$

# Non-gyrotropy of $\delta\mathbf{B} \Rightarrow \mathbf{k}$ -anisotropy

[Saur & Bieber 1999]



2D  $\mathbf{k}$  ( $q$ =spectral index):

$$P_{yy}/P_{xx} = q$$

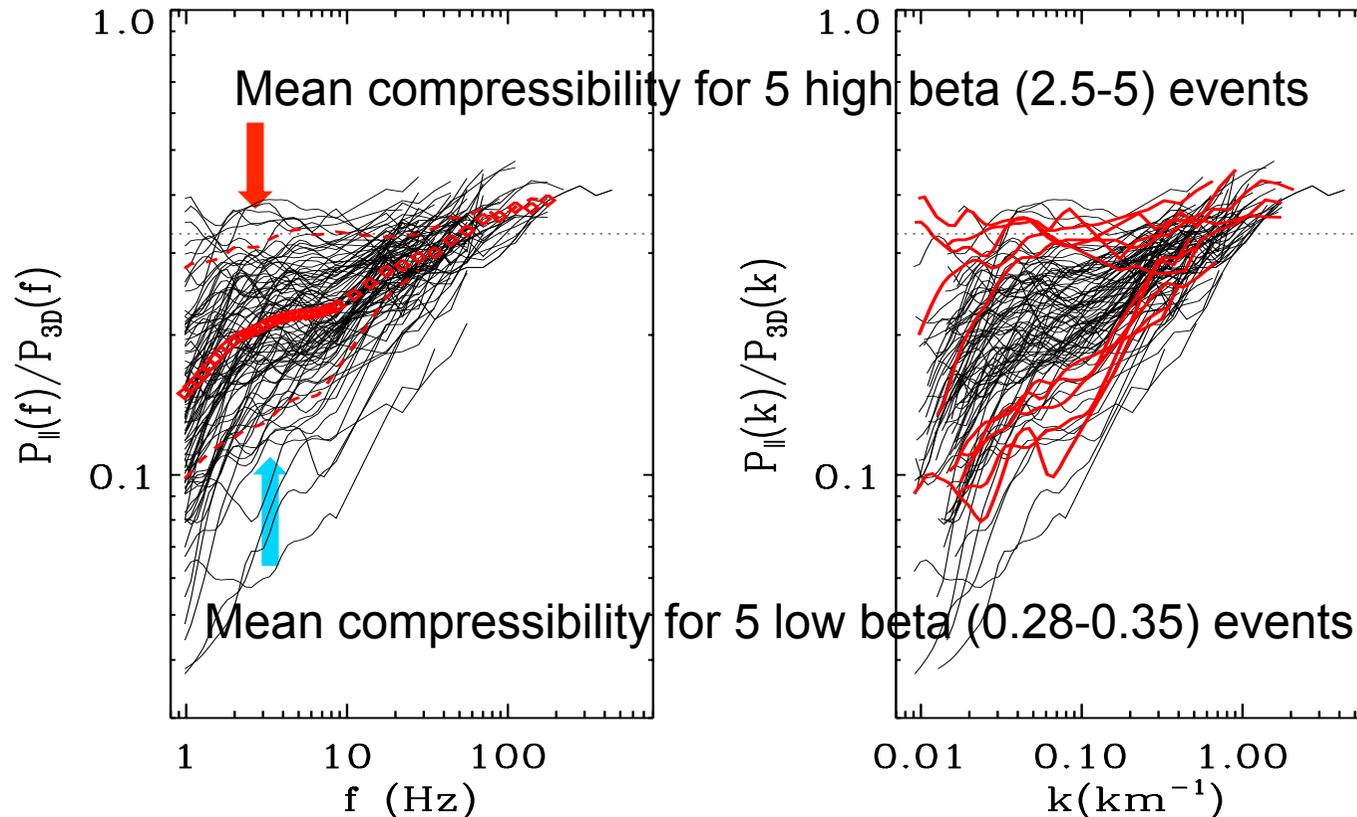
Isotropic  $\mathbf{k}$  ( $\Theta$  is the flow-to-field angle):

$$P_{yy}/P_{xx} = \frac{(1+q)/2}{\cos^2\Theta(1+q)/2 + \sin^2\Theta}$$

Slab :  $\mathbf{k} \parallel \mathbf{B}_0$   $P_{yy}/P_{xx} = 1$ ,  $\delta B_x \simeq \delta B_y$

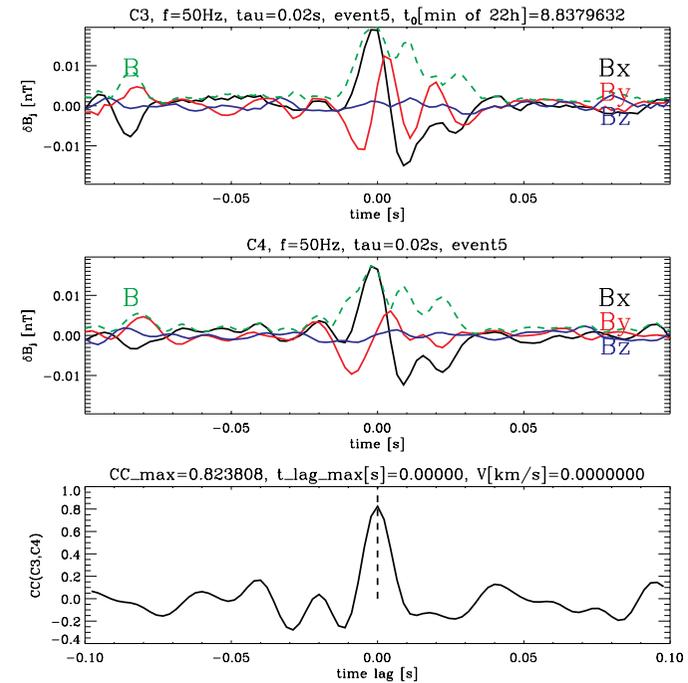
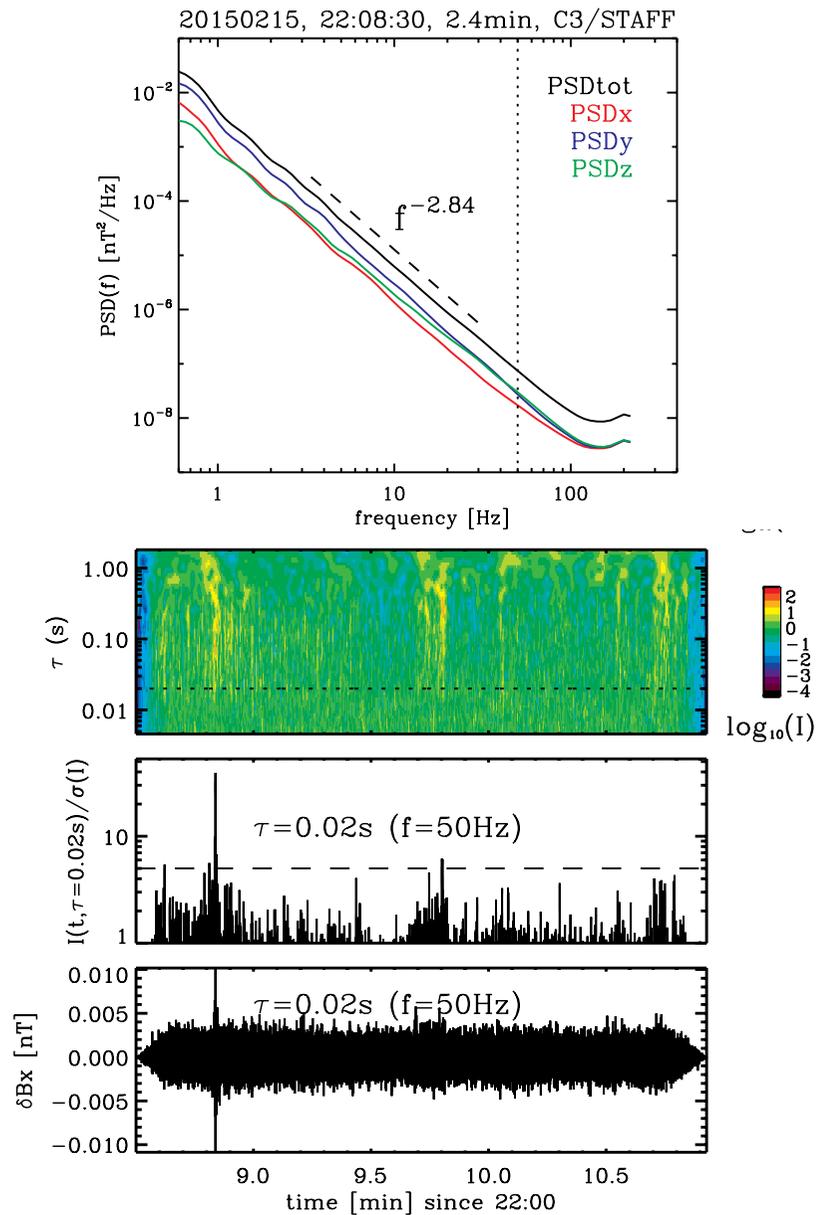
- at  $f < 10$  Hz : 2D turbulence ( $k_{\text{perp}} \gg k_{\parallel}$ ).
- at  $f > 10$  Hz : a more isotropic  $\mathbf{k}$ -distribution.

# Compressibility of $\delta B$ at kinetic scales



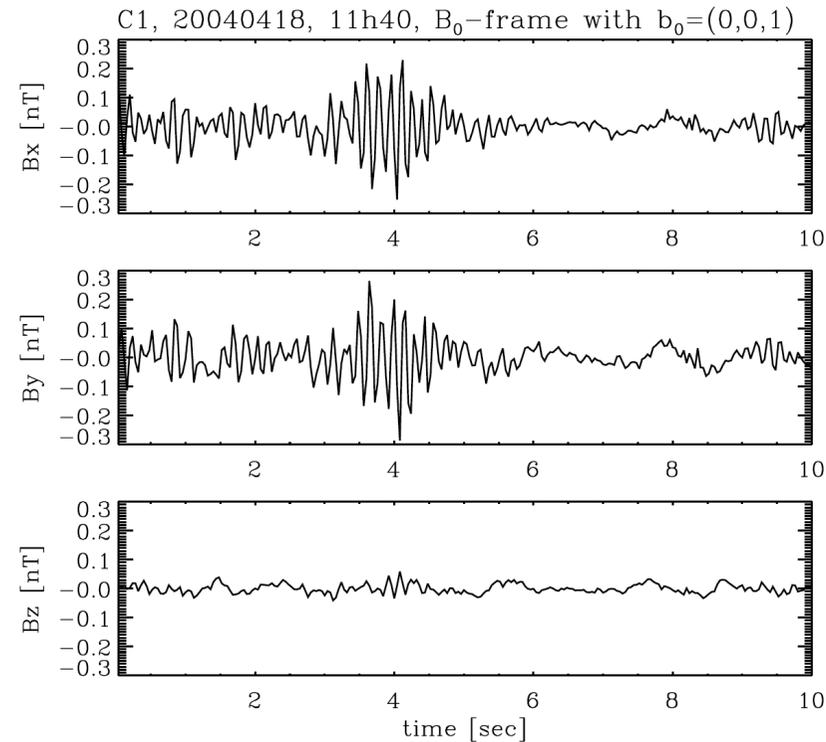
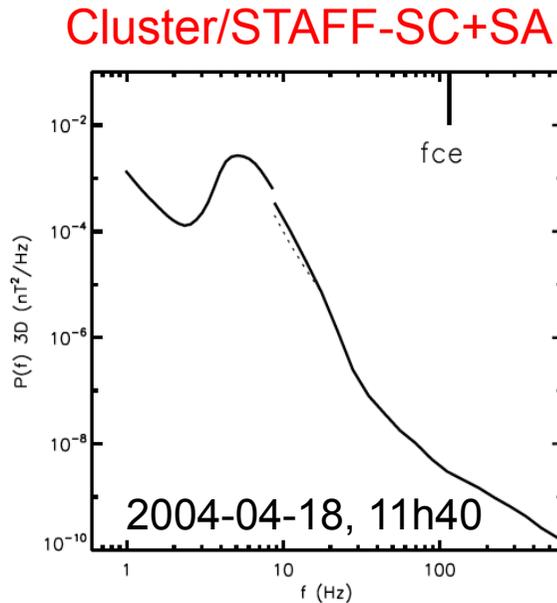
- $S_{\parallel}/S$  at  $f < 20$  Hz is compatible with KAW fluctuations [Boldyrev et al. 2013, TenBarge et al. 2012, Schekochihin et al. 2009].
- At  $f > 20$  Hz, i.e. at electron scales,  $S_{\parallel}/S$  is not compatible with KAWs, rather with slow-ion acoustic modes [Gary, 1992; Krauss-Varban et al., 1994; Camporeale and Burgess, 2011, Schreiner and Saur, 2017].
- Fluctuations  $\sim$  superposition of q-linear waves?

# Nature of solar wind sub-ion scales turbulence ?



- Example of intermittent structure at scales close to electron scales (10 km): Kinetic Alfvén vortex ?
- Signature of strong turbulence..
- **Turbulence nature: superposition of modes + coherent structures...**

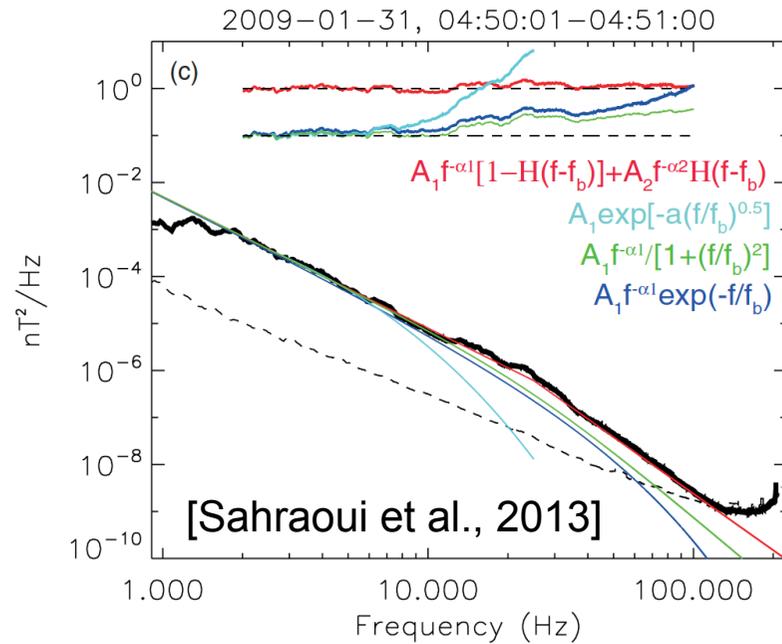
# Parallel whistlers in the solar wind



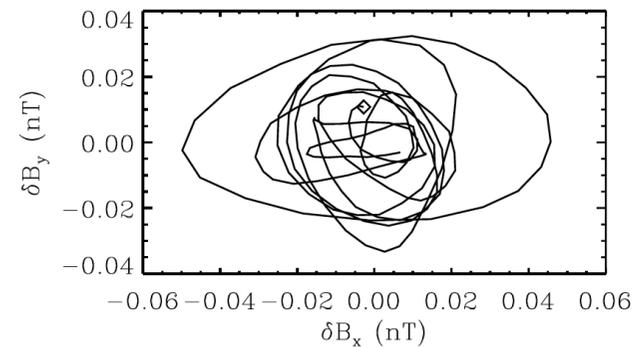
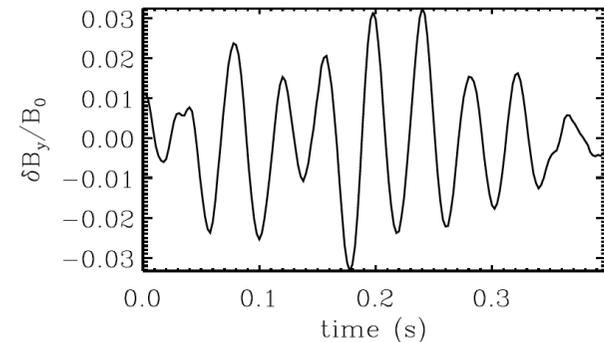
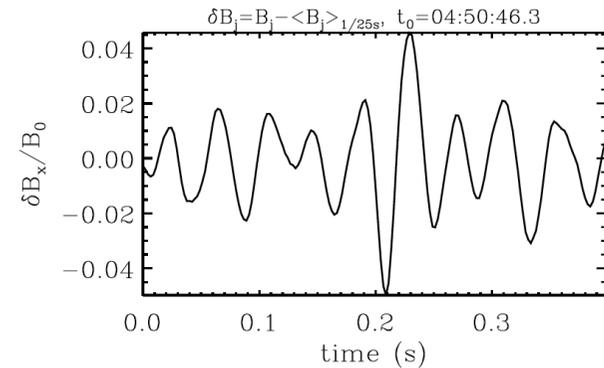
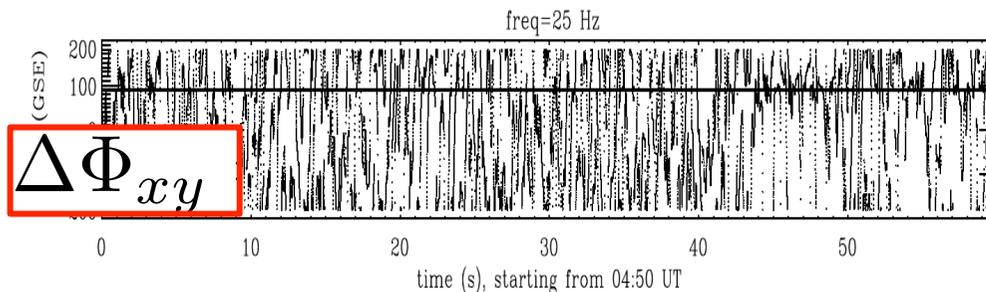
- The spectral bump at  $f < f_{ce}$  is observed by both, STAFF-SC and SA;
- it corresponds to coherent wave-packets of whistlers (RH polarization):

$$\Delta\Phi_{xy} = \Phi_y - \Phi_x = 90^\circ \rightarrow RH$$

# Sporadic parallel whistlers

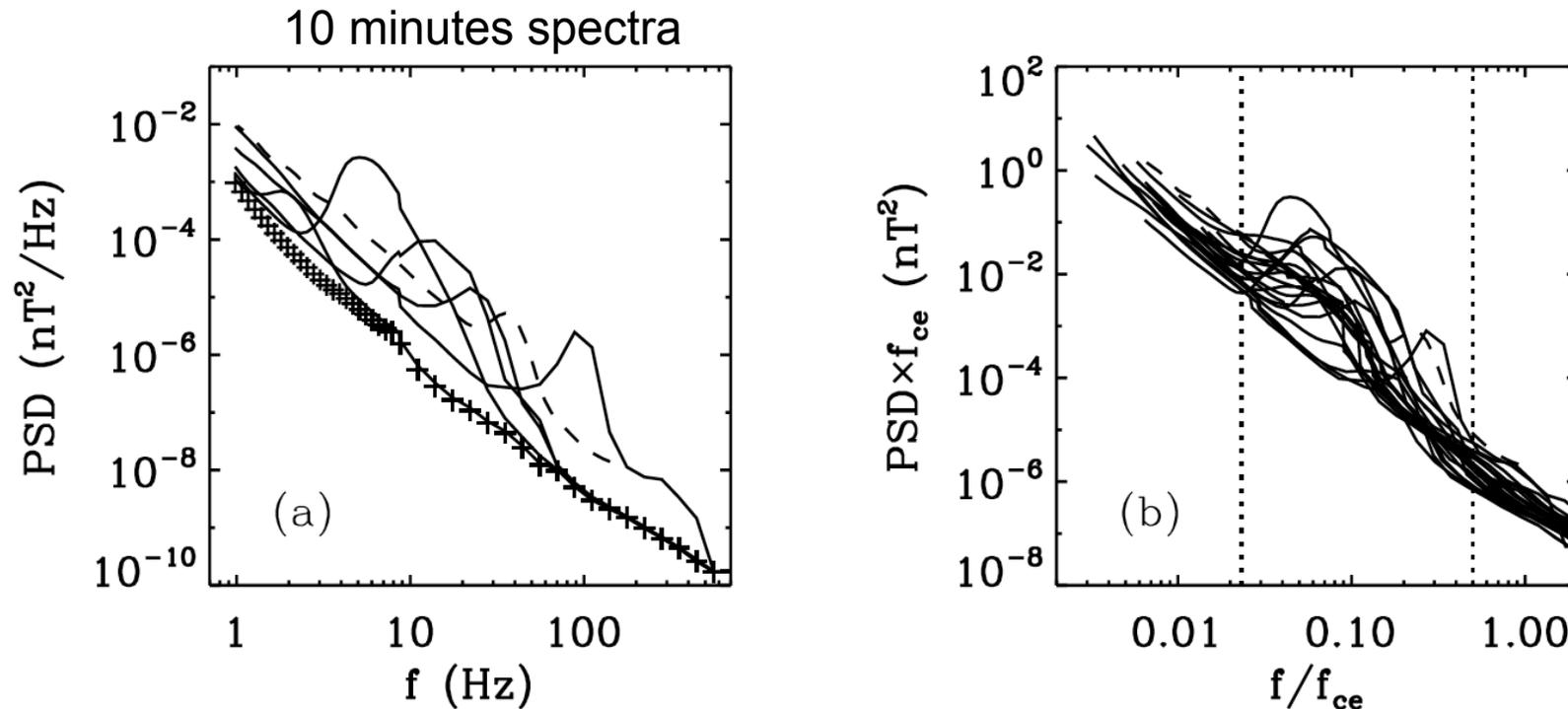


Phase difference in the plane perp to  $B_0$  around  $90^\circ$  is a signature of whistlers:



Permanent turbulence + sporadic whistler waves => spectral break/knee at the frequency of whistlers

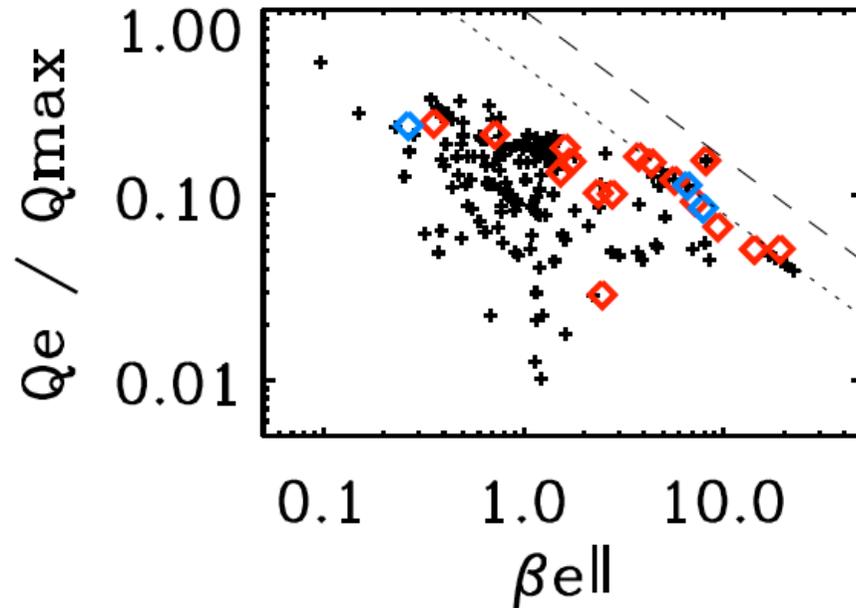
# Magnetic spectra with long-lived whistlers



- $f/f_{ce}$  normalization is more appropriate
- Spectral bumps are observed between the lower hybrid frequency  $\sqrt{f_{ce} * f_{ci}}$  and  $0.5f_{ce}$
- These frequencies are typical of whistler mode waves.

[Lacombe et al. 2014, APJ]

## The electron heat flux instability



$$Q_e = \int \frac{m}{2} \mathbf{U} U^2 f(v) d^3v,$$
$$\mathbf{U} = \mathbf{v} - \langle \mathbf{v} \rangle$$

$$Q_{max} = \frac{3}{2} m_e n_e v^3$$

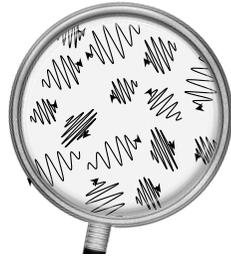
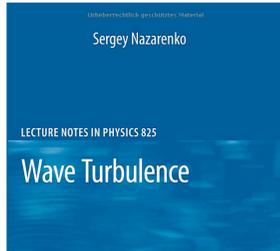
Whistlers (diamonds) are observed at the threshold for the whistler heat flux instability (dashed line, Gary et al.,99)

The whistler heat flux instability contributes to the regulation of the electron heat flux, at least for  $\beta_e > 3$  at 1 AU.

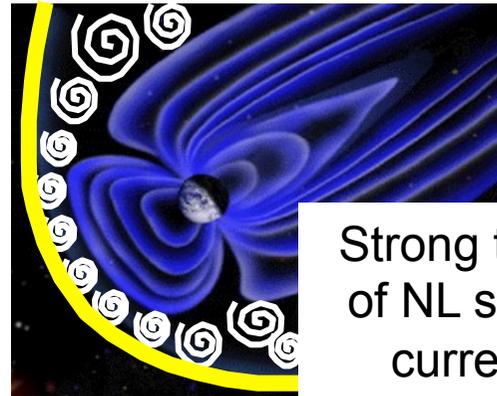
[Lacombe et al. 2014, APJ]

# Turbulence nature: weak (or wave) vs strong

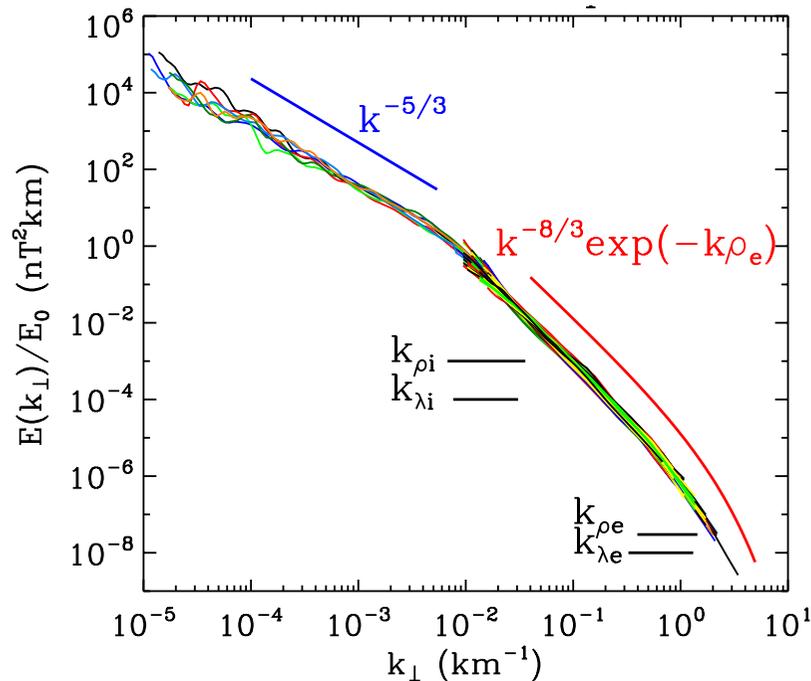
Courtesy S. Galtier



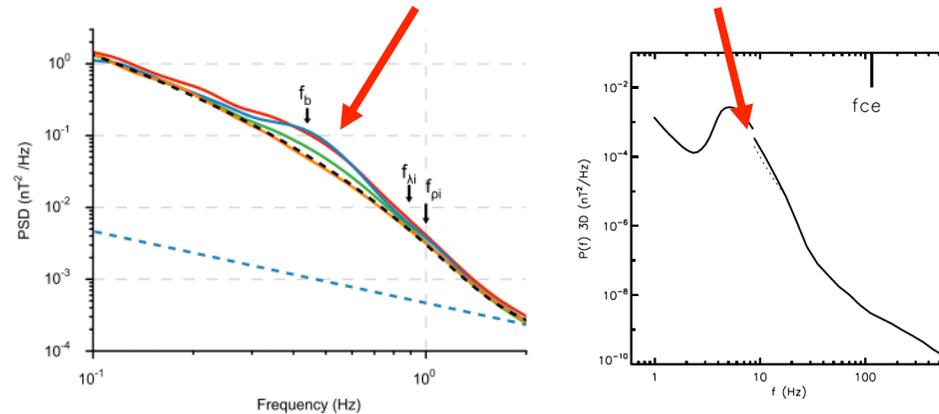
Weak turbulence:  
mixture of waves with  
+/-random phases



Strong turbulence: mixture  
of NL structures (vortices,  
current sheets, ect...)



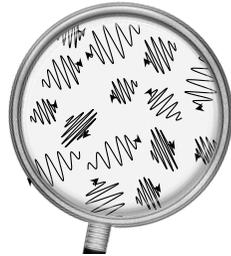
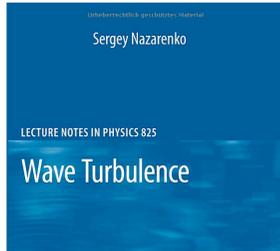
Zooms around ion and electron scales:



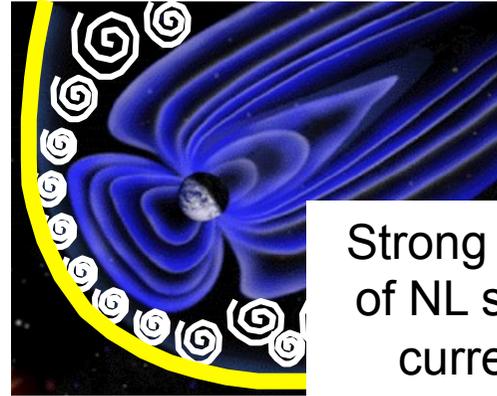
$k^{-5/3}$  and  $k^{-8/3}$  ranges: strong  
turbulence with 'waves properties' +  
sometimes instabilities at ion and  
electron scales (Q-linear waves).

# Turbulence nature: weak (or wave) vs strong

Courtesy S. Galtier

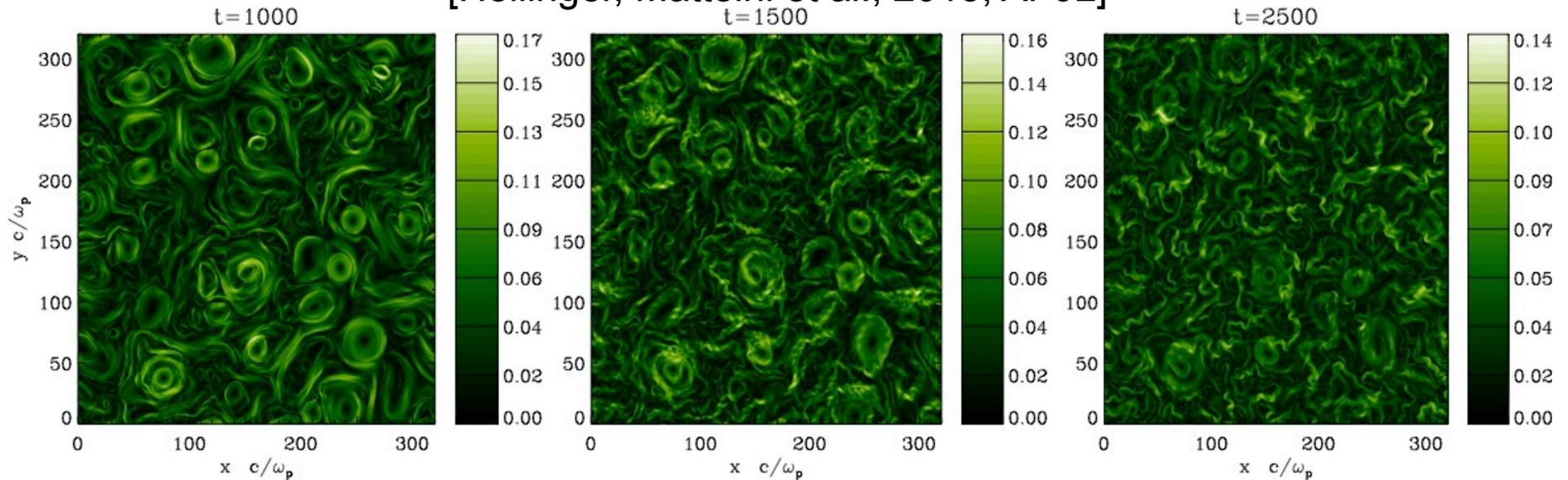


Weak turbulence:  
mixture of waves with  
+/-random phases



Strong turbulence: mixture  
of NL structures (vortices,  
current sheets, ect...)

[Hellinger, Matteini et al., 2015, APJL]

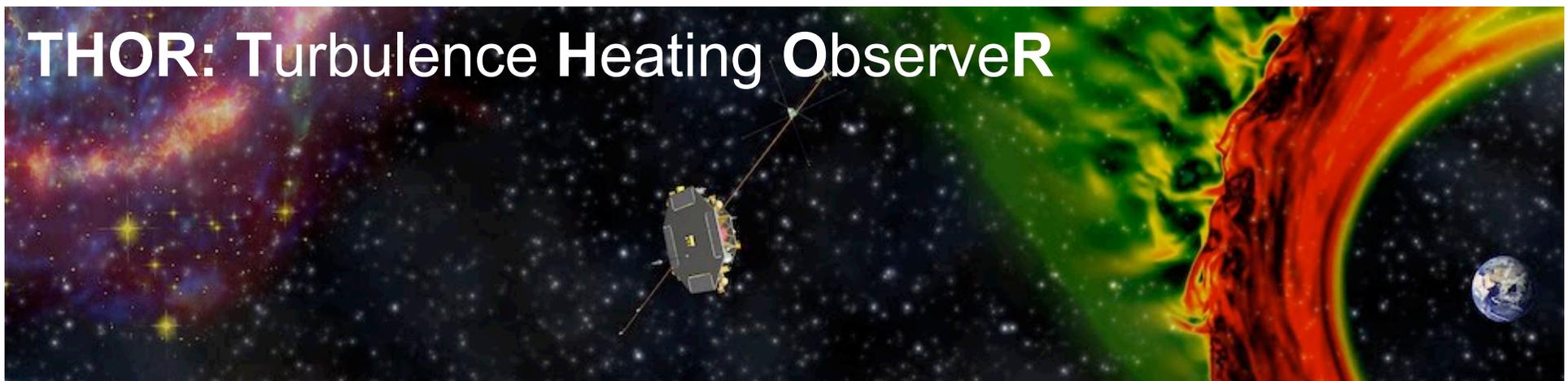


Courtesy of Lorenzo Matteini: **2D hybrid expanding box simulations** showing development of strong turbulence (vortices) with superposed waves at ion scales.

# Conclusion and discussion

- Plasma turbulence is an important ingredient in many astrophysical systems.
  - Solar wind is one of the best laboratories of space plasma turbulence.
  - We resolve turbulent fluctuations from MHD ( $10^7$  km) to sub-electron scales (300 m).
  - General turbulent spectrum for  $k_{\text{perp}}$  fluctuations + variability at plasma scales;
  - Variety of coherent structures; quasi-linear instabilities at ion and e-scales
  - NB: Polarization/waveform analysis in time is very important. We can not make conclusions on the nature of turbulence by looking only at spectra !
- 
- Dissipation of turbulent energy ?

Recent/future space missions: MMS, Solar Orbiter (ESA, 2019), Solar Probe Plus (NASA, 2018), THOR (ESA?) will answer this question ?



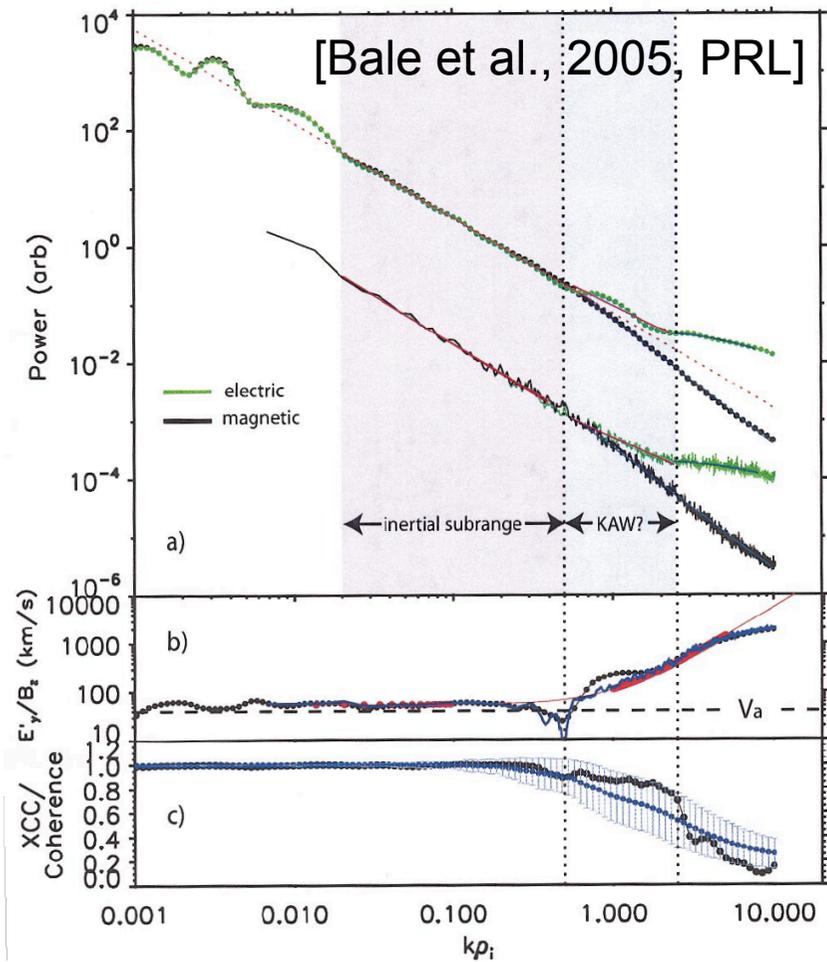


# Bonus

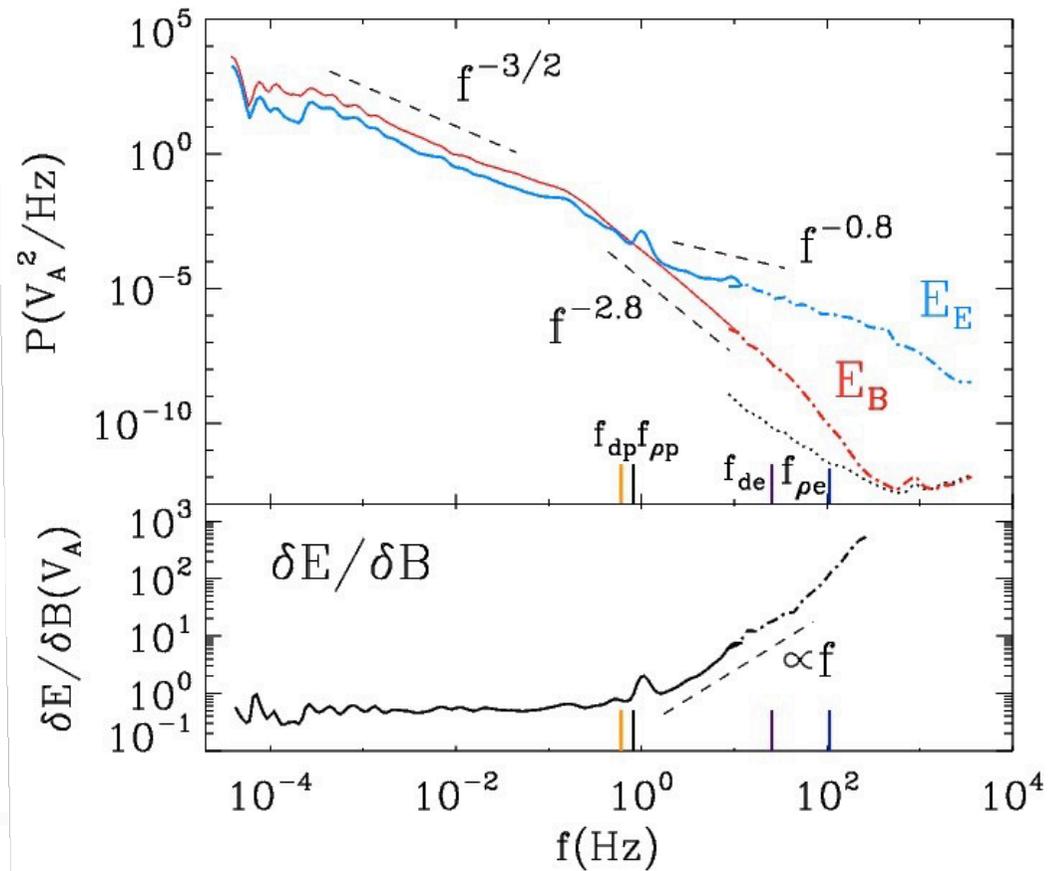
- E field spectrum
- Phase coupling between magnetic field components
- Separation  $k_{\perp}$  turbulence from whistlers
- Observed Compressibility VS KAWs compressibility
- Wavelets and coherent structures

# E field spectrum

Slow solar wind



Magnetosheath (Cluster)

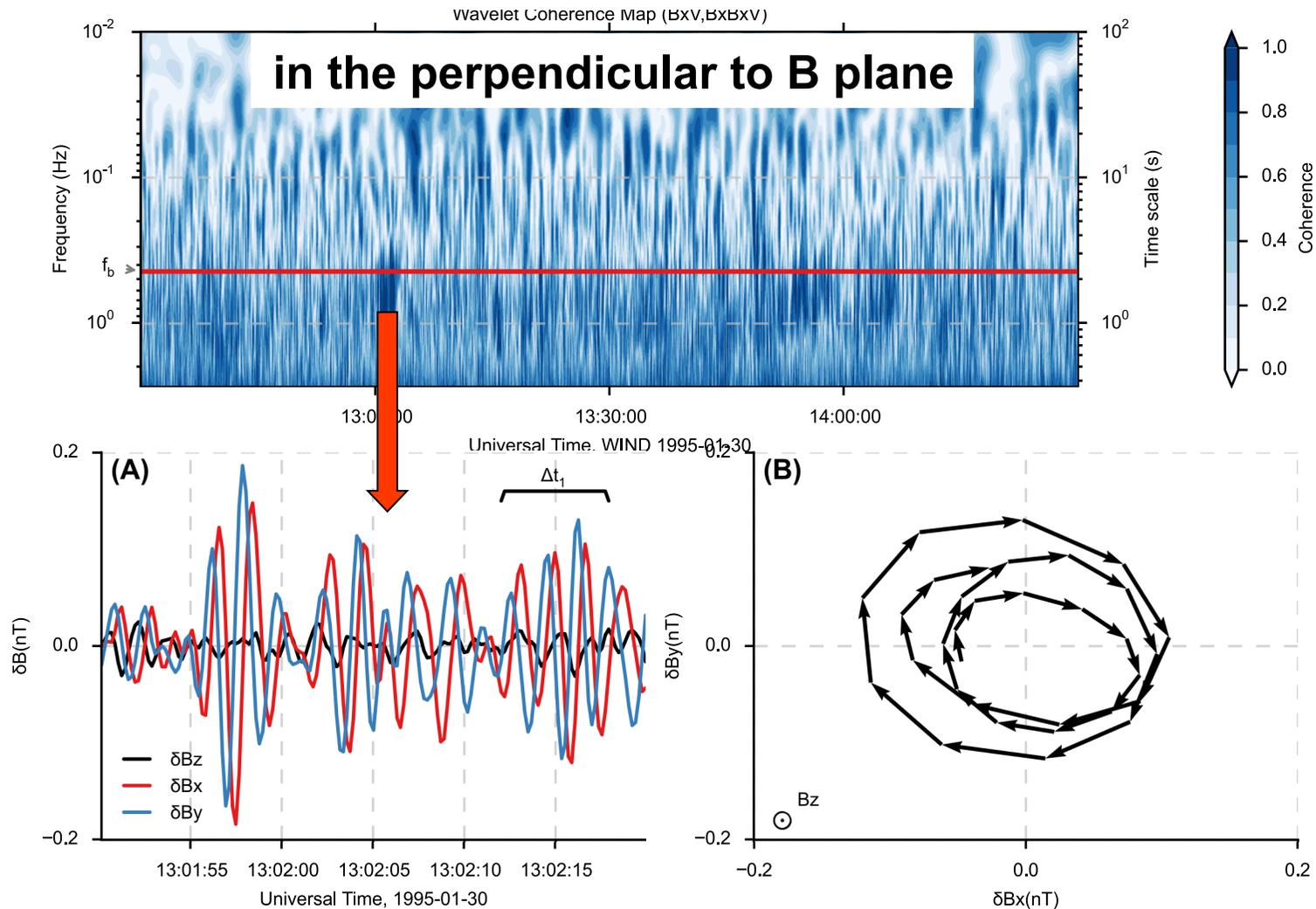


[Matteini et al., MNRAS, 2017]

[Mangeney et al., 2006; Lacombe et al., 2006]

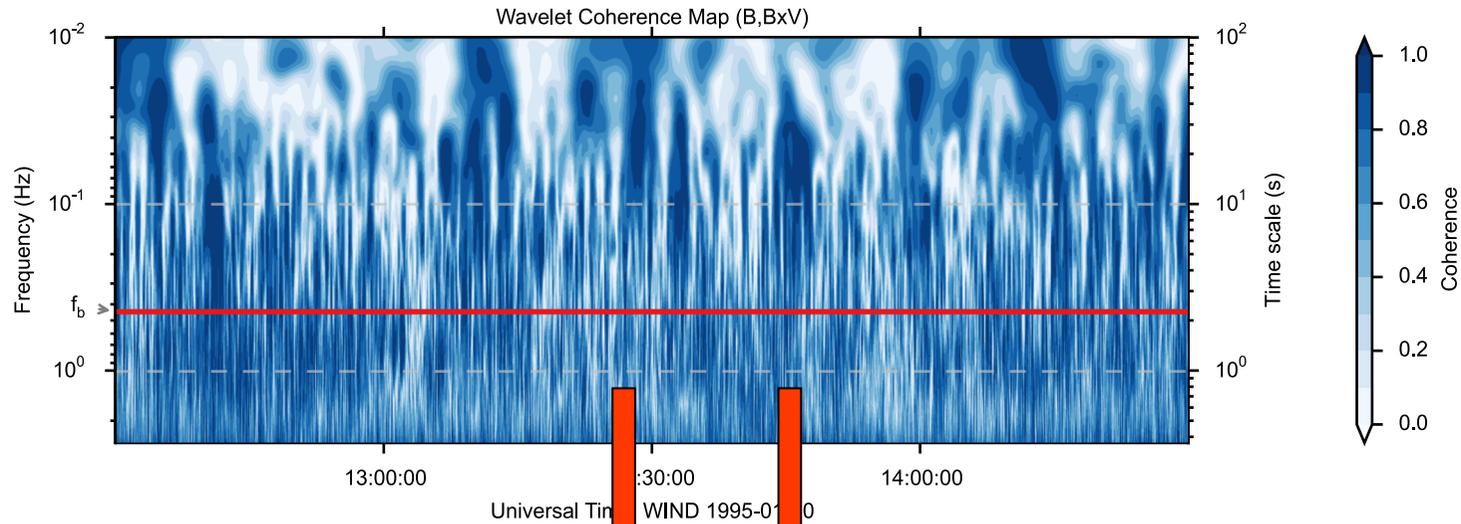
# Phase coupling between 2 magnetic components

[Grinsted et al., 2004] : 
$$R_{ij}^2(f, t) = \frac{|S(fW_i(f, t)W_j^*(f, t))|^2}{S(f|W_i(f, t)|^2) \cdot S(f|W_j(f, t)|^2)}$$

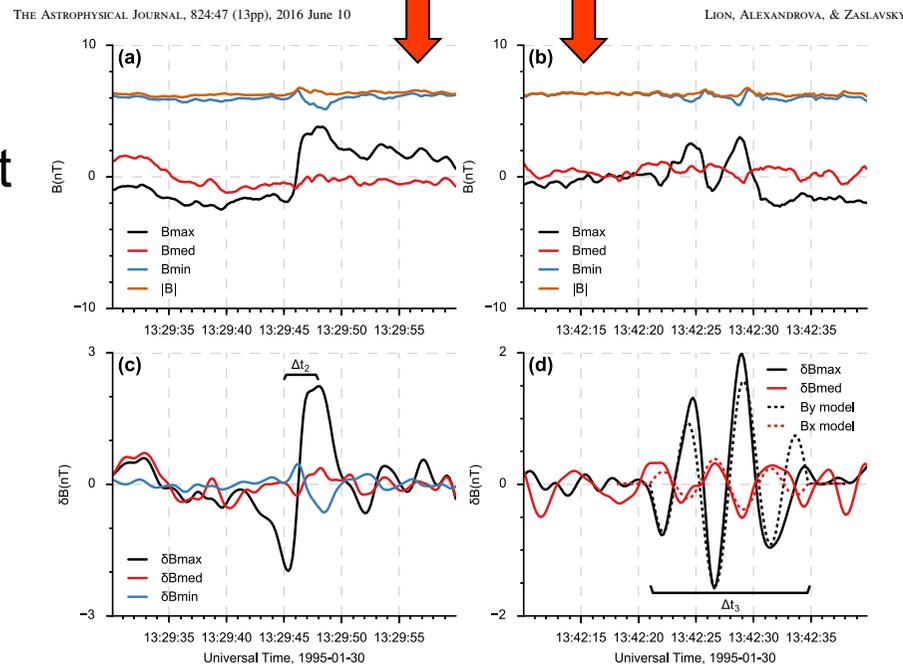


Alfven Ion Cyclotron waves with  $k_{||}$  and  $w \sim w_{ci}$ , instable for  $T_{perp} > T_{||}$   
 [Lion et al, 2016, APJ].

# Phase coupling between Bx and Bz ( $\parallel$ to $B_0$ )



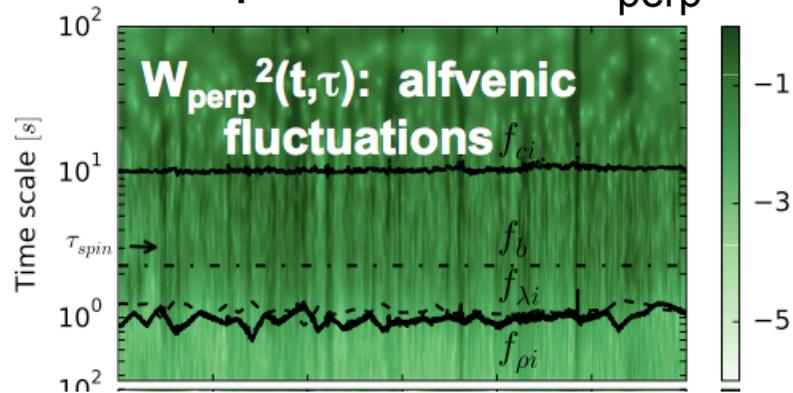
Current sheet



~Alfven vortex  
(1<sup>st</sup> direct observation  
at ion scales in the  
fast SW),  
Lion et al. 2016, APJ

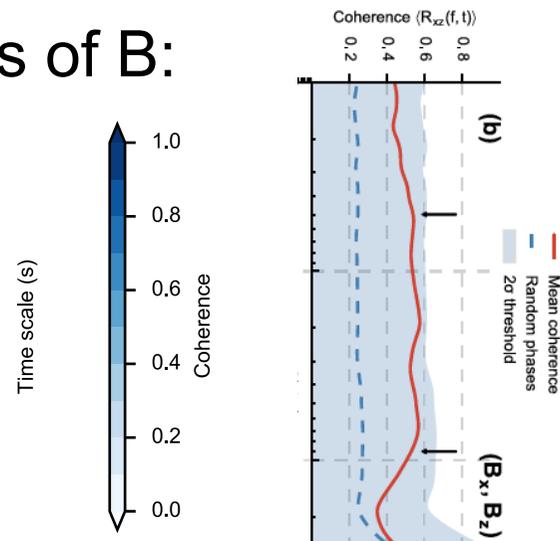
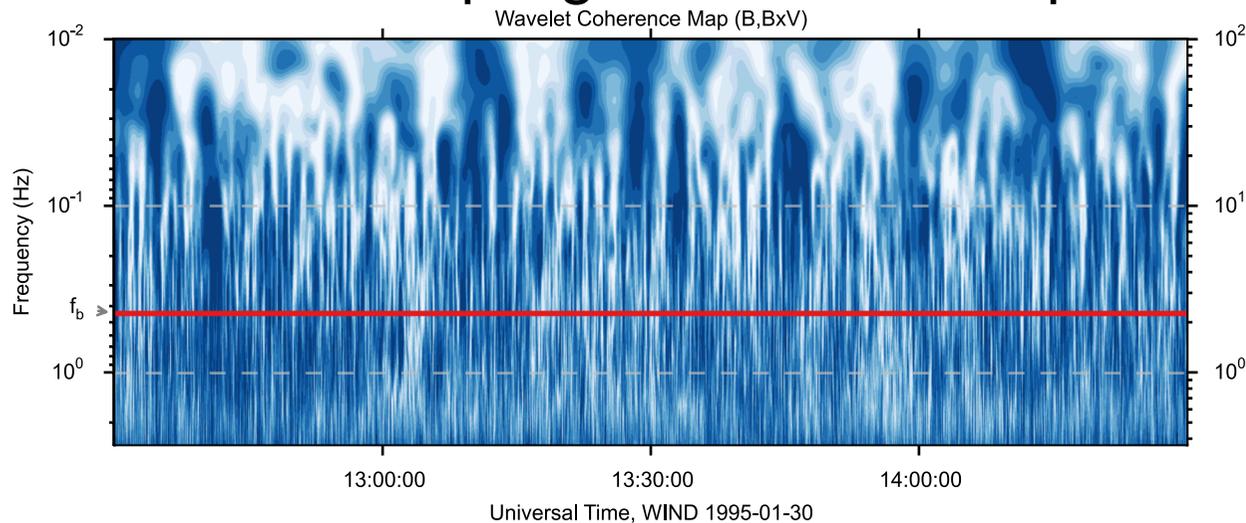
# Scales covered by coherent structures ? Influence on the spectral shape ?

Amplitudes of  $\delta B_{\text{perp}}$ :



- Structures cover all observed scales?
- Dominate in the turbulent spectrum?

Phase coupling between 2 components of B:

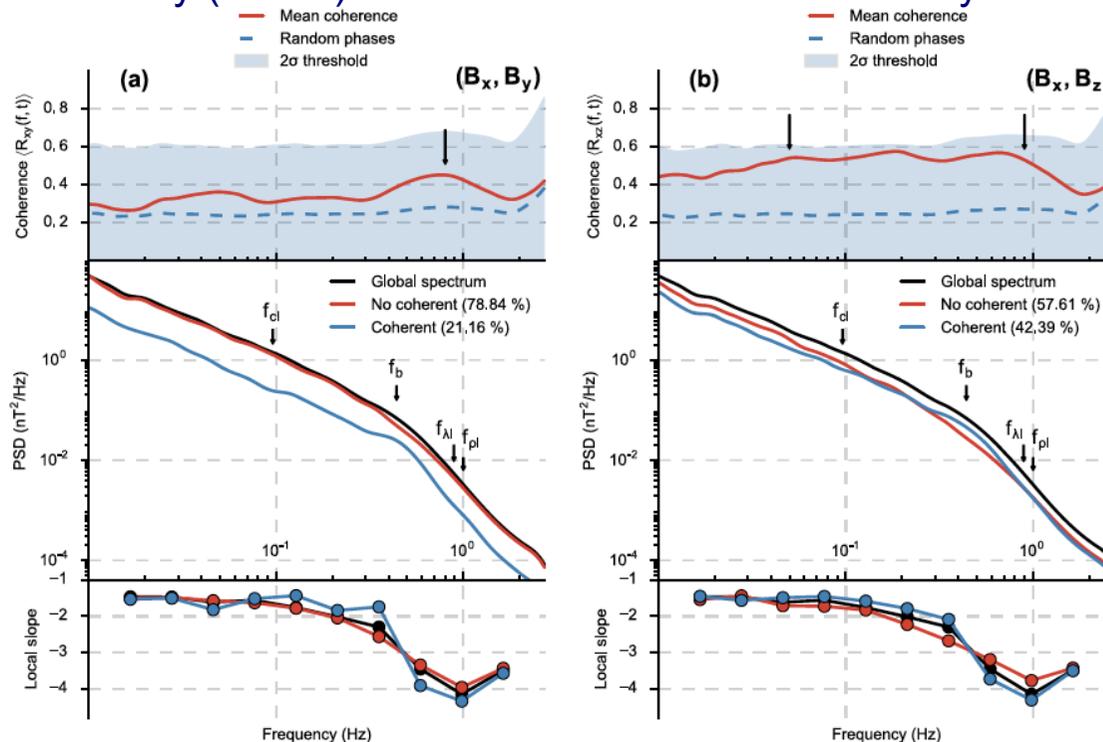


# Scales covered by coherent structures ? Influence on the spectral shape ?

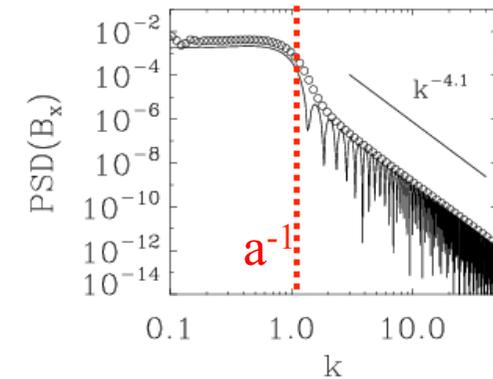
Mean coherence as a function of frequency => ranges of scales, where coherent events dominate :

- Coherency (Bx-By): waves signatures cover a narrow frequency range around  $f_b$ .
- Coherency (Bx-Bz): coherent structures cover nearly 2 decades and 'stops' at  $f_b$ .

[Lion et al. 2016, APJ]



Spectrum of a vortex  
[Alexandrova, 2008]:

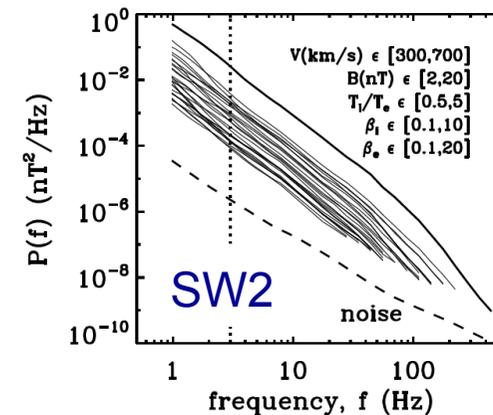
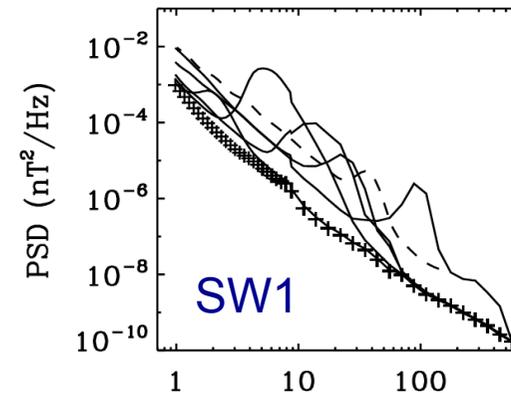
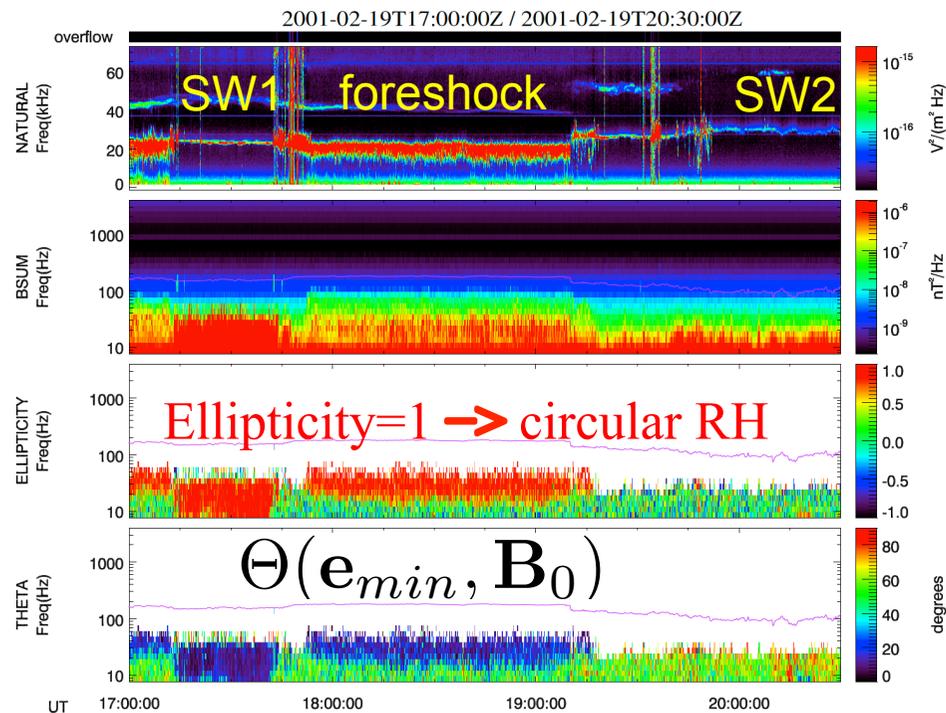


- **Local slopes of individual spectra** show that coherent structures have flatter spectrum in the inertial range and the spectrum has -4 index at  $f > f_b$  !
- Non-coherent spectrum **has no break** and **no -4 slope** at small scales.



# Data selection at sub-ion scales

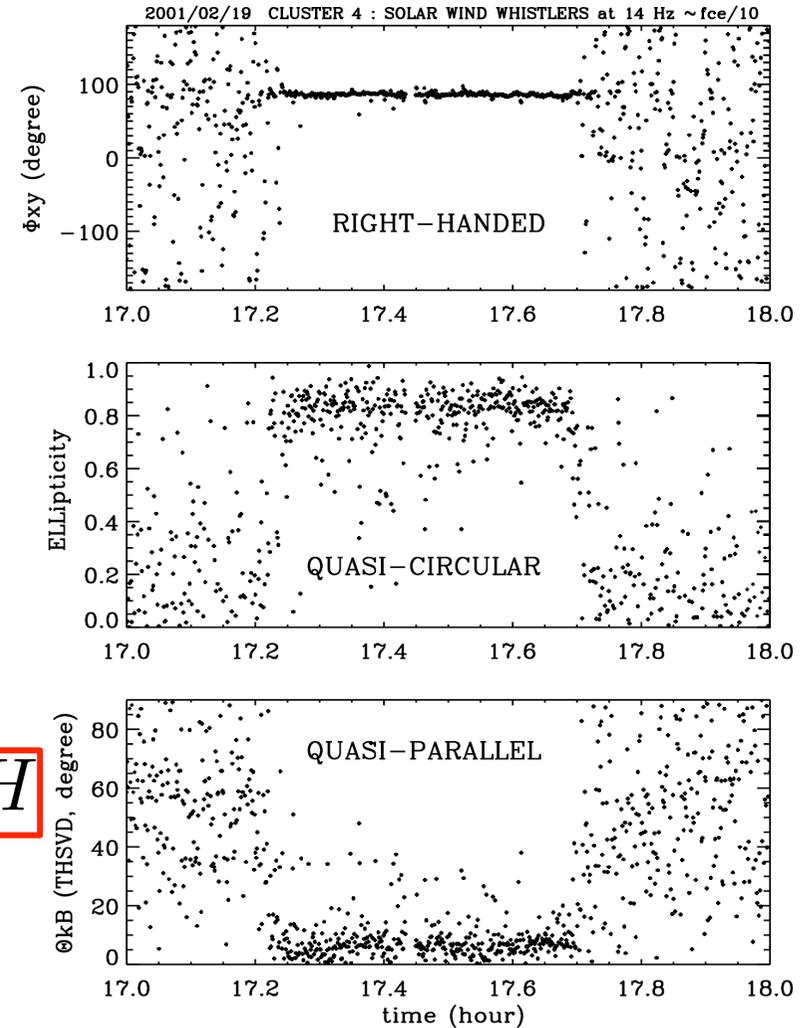
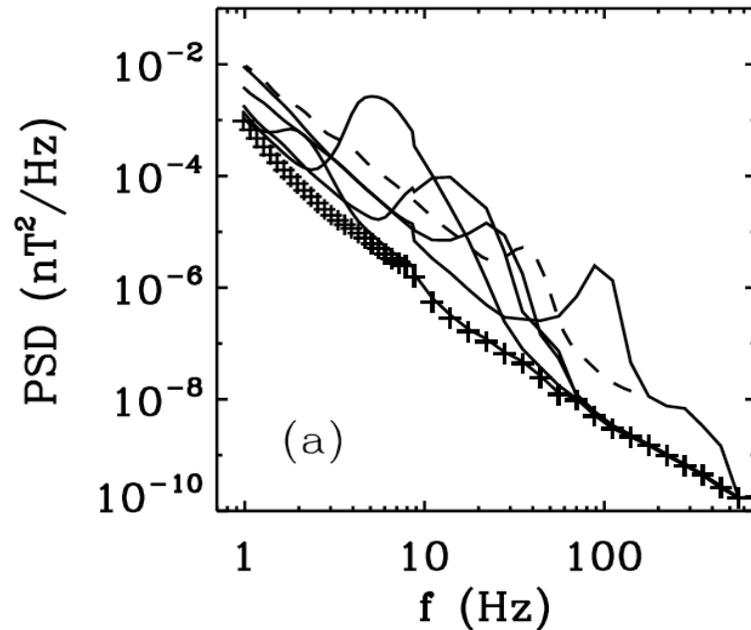
1. Cluster/Whisper measures electric field fluctuations around the plasma frequency (check for bow-shock connection)
2. Cluster/STAFF-SA: magnetic fluctuations within the (8Hz,4kHz) range, polarization and propagation direction



- Polarized fluctuations => spectra with bumps (10% of data)
- Non-polarized fluctuations => permanent (or background) turbulence (90% of data, Alexandrova et al. 2012, 2013)
- Permanent turbulence + sporadic polarized fluctuations => “intermediate” spectral shape with a break or a small bump (as spectra in Sahraoui et al. 2009, 2013)

# Polarized fluctuations (spectra with bumps, knees, breaks...)

[Lacombe et al. 2014]



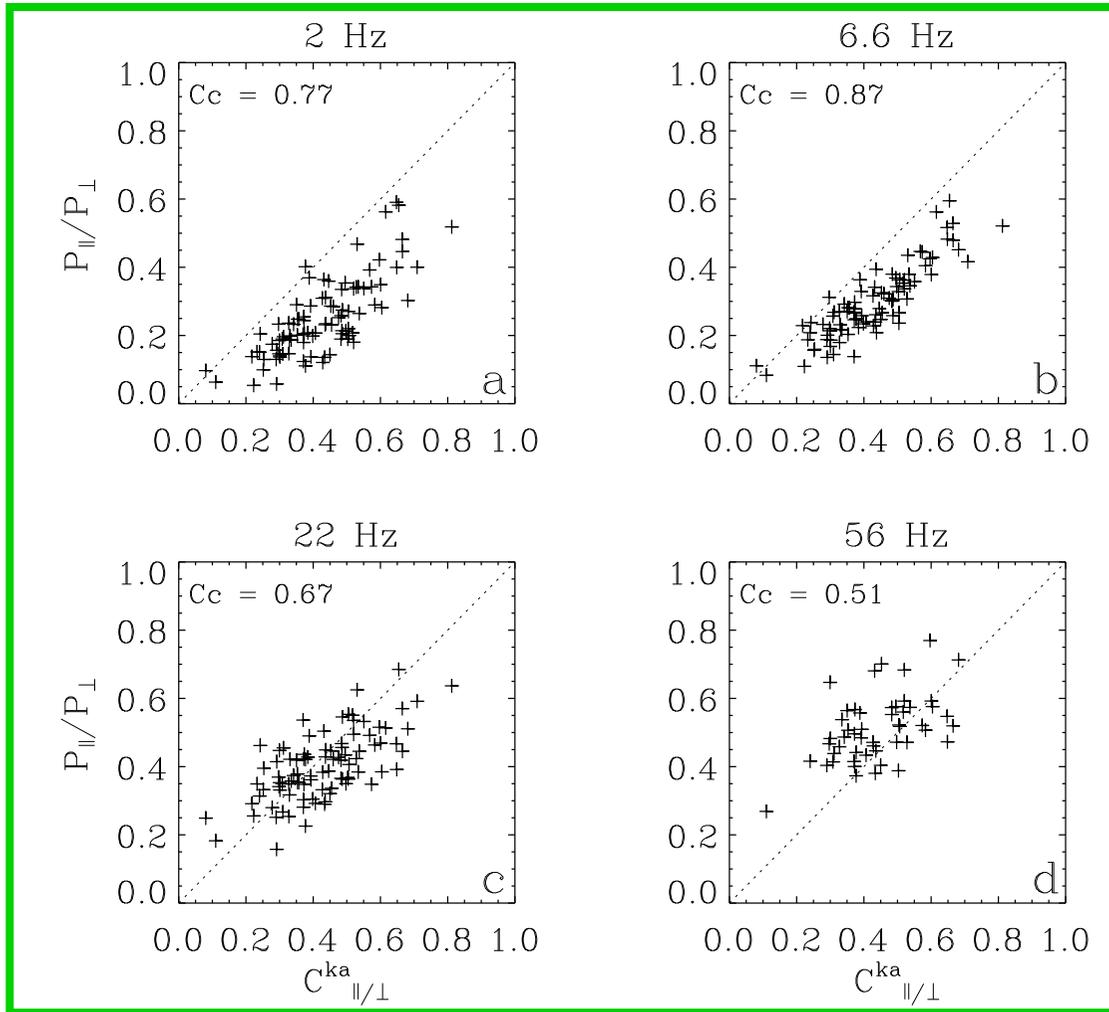
$$\Delta\Phi_{xy} = \Phi_y - \Phi_x = 90^\circ \rightarrow RH$$



**Parallel whistlers with RH-polarization**

# Observed compressibility vs expected one for KAWs

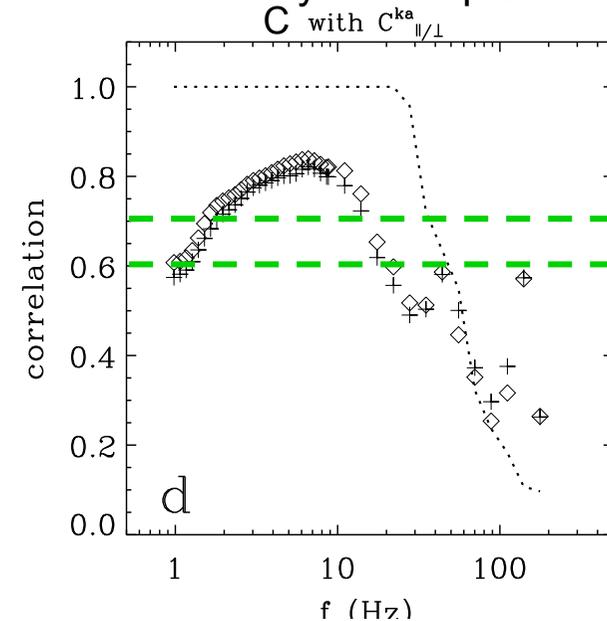
Whistler mode compressibility depends on  $\mathbf{k}$ -direction, but not on plasma  $\beta$ .  
 KAWs compressibility (in electron-reduced MHD)  $\sim$  on  $\beta_p$  and  $\beta_e$ , not on  $k$ :



$$C^{ka}_{\parallel/\perp} = \frac{\beta_p + \beta_e}{2 + \beta_p + \beta_e}$$

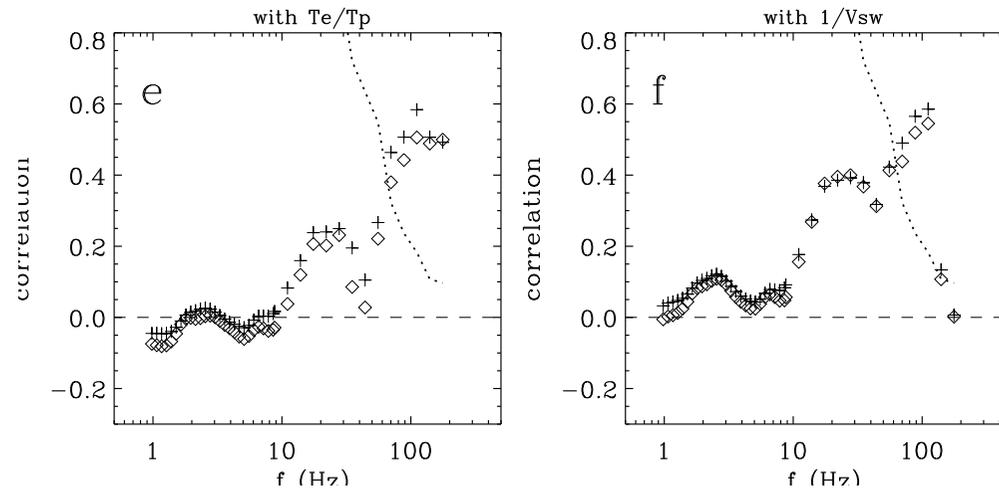
[Boldyrev et al. 2013,  
 TenBarge et al. 2012,  
 Schekochihin et al. 2009]

For all analysed frequencies:



The observed compressibility  $C = P_{\parallel}/P_{\text{perp}}$  is the one expected for KAW,  $f < 20$  Hz.

# Nature of fluctuations at $f > 15\text{-}20$ Hz: slow-ion acoustic modes ?



**We observe correlations of  $P_{\parallel}/P_{\text{perp}}$  with  $1/V_{sw}$  or with  $T_e/T_p$  at  $f > 20$  Hz.**

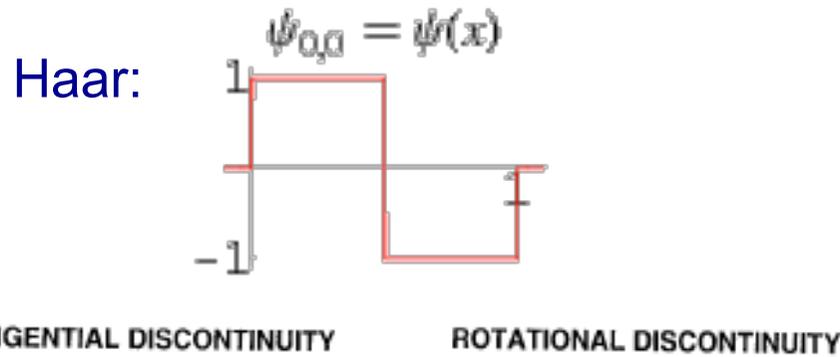
Larger values of  $T_e/T_p$  increase the damping of KAW for  $\beta_p > 0.3$  [Schreiner and Saur, 2017] favour the presence of slow-ion acoustic modes [Gary, 1992; Krauss-Varban et al., 1994; Camporeale and Burgess, 2011].

So, we suggest that at  $f > 15\text{-}20$  Hz, turbulence is dominated by slow-ion acoustic type of fluctuations, which

- (i) can be less damped than KAW for large enough  $T_e/T_p$
- (ii) have a compressibility larger than the KAW compressibility
- (iii) can propagate without strong damping at oblique or small angles with  $B_0$ .

# Wavelets is a powerful tool to detect coherent structures !

- Haar wavelets (Step function ~ structure functions) => planar coherent structures
- Morlet wavelets => cylindrical structures, solitons, ect...



Morlet:

