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Magnetic helicity, eruptivity and the need for good 3D NLFFF extrapolations

É. Pariat[1], G. Valori[2] , L. Linan[1],
J. Leake[3], K. Dalmasse[4], M. Linton[5], F. Zuccarello[6]

[1] LESIA, Observatoire de Paris, PSL*, CNRS, UPMC, Univ. D. Diderot, France
[2] MSSL, University College London, UK
[3] NASA GSFC, Greenbelt MD, USA
[4] IRAP, Univ. Toulouse, CNRS, UPS, CNES, France
[5] NRL, Washington DC, USA
[5] KU Leuven, Belgium
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Outline

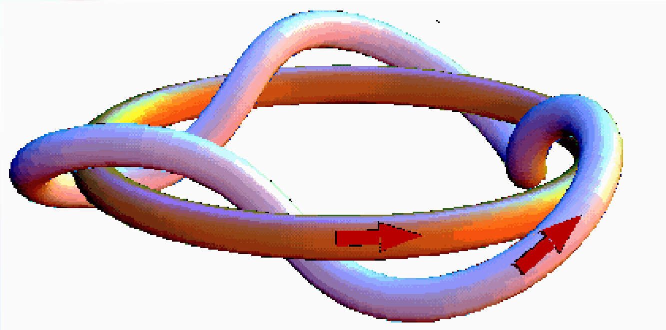
- Magnetic helicity: definition and properties
- Magnetic helicity-based eruptivity proxy
- Measurement of magnetic helicity from solar observational data

Definition of Magnetic Helicity

- **Helicity of the magnetic field in MHD plasmas** (Elsasser 56)

$$H = \int_{\mathcal{V}} \vec{A} \cdot \vec{B} \, \mathrm{d}V \quad , \quad \vec{B} = \vec{\nabla} \times \vec{A}$$ ← **Magnetic vector potential**

  – Unique signed scalar value for volume considered



(Berger 00)

- **Magnetic helicity: signed level of knotedness and twist of magnetic field lines**
  – Magnetic flux weighted **Gauss Linking Number** of pairs of magnetic field lines (Moffatt 1968)

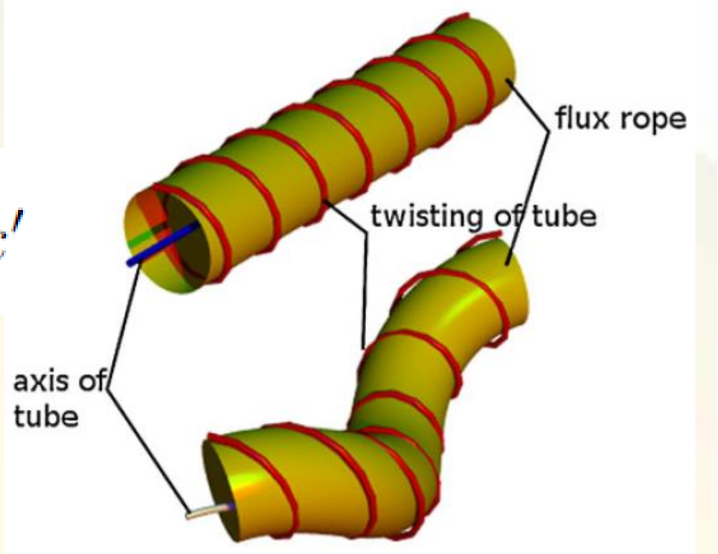
$$L_{12} = -\frac{1}{4\pi} \oint_1 \oint_2 \frac{d\mathbf{x}}{d\sigma} \cdot \frac{\mathbf{r}}{r^3} \times \frac{d\mathbf{y}}{d\tau} \, d\tau \, d\sigma$$

$$H = -\frac{1}{4\pi} \int \int \mathbf{B}(\mathbf{x}) \cdot \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{x}') \, d^3x \, d^3x'$$

  – For a uniformly twisted flux tube

$$H = N \, \Phi_{ax}^2$$

N:nbr of turns, Φ_{ax}: axial flux



(Prior & Berger 12)

Magnetic helicity properties

- **Magnetic helicity is an ideal MHD invariant. For E⊥B: no dissipation ➔ magnetic helicity is conserved (Woltjer 1958).**

Time variations — **Surface Flux** — **Dissipation**

$$\frac{dH_m}{dt} = \int_{\partial \mathcal{V}} \left(\mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t} \right) \cdot d\mathbf{S} - 2 \int_{\partial \mathcal{V}} (\mathbf{E} \times \mathbf{A}) \cdot d\mathbf{S} - 2 \int_{\mathcal{V}} \mathbf{E} \cdot \mathbf{B} \; d\mathcal{V}$$

- **Taylor 1974: hypothesis helicity conservation true even in non-ideal MHD**
  - **Pariat et al. 16 : verified for a solar like active event**

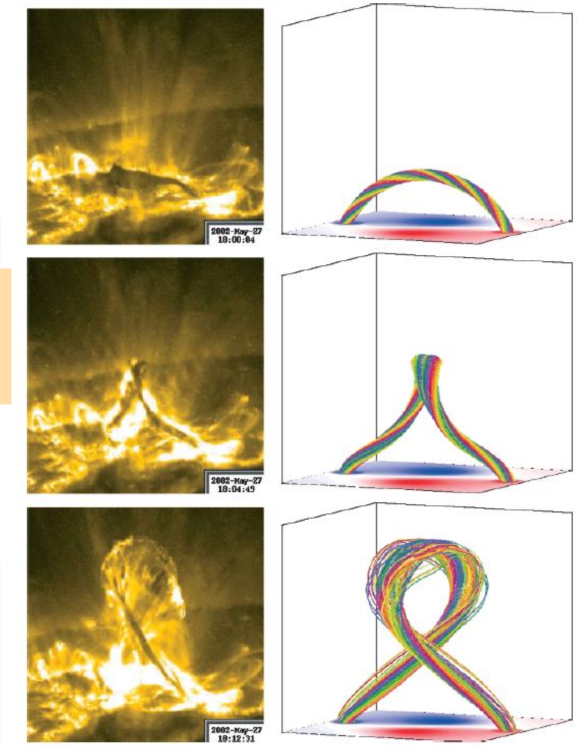
- Magnetic helicity bounds the system E distribution:

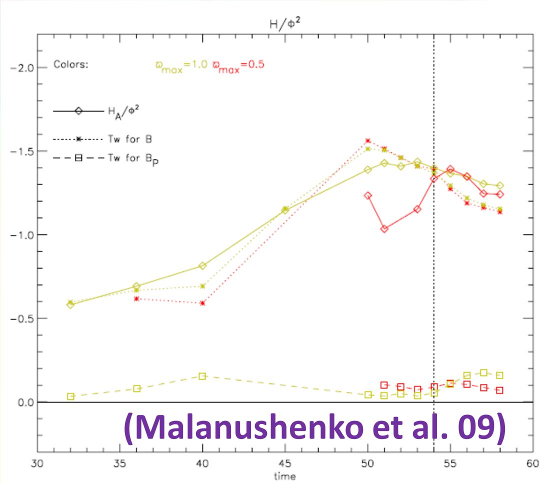
$$\mu_0 \hat{E}(k) > k \hat{H}(k)$$ **(Frisch et al. 75)**

- **Inverse helicity cascade:** Helicity goes from small to large spatial scales. **(Frisch et al. 1975, Alexakis et al. 06)**
  - e.g. kink instability **(Malanushenko et al. 09)**

- Impact on dynamic of magnetic reconnection:
  e.g. Linton et al. 01, Del Soro et al. 10

(Török et al. 05)

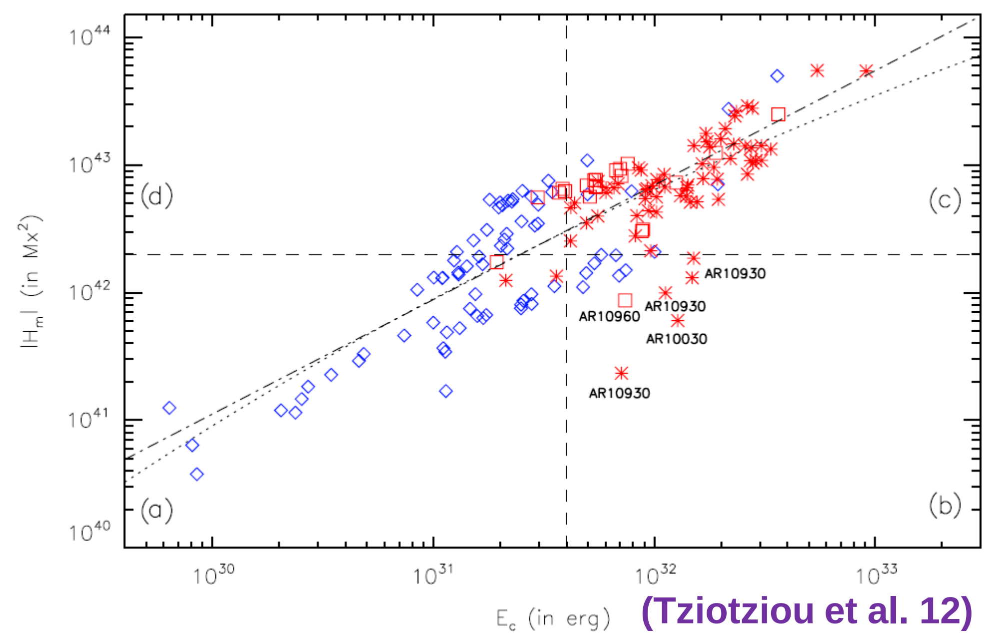




(Malanushenko et al. 09)

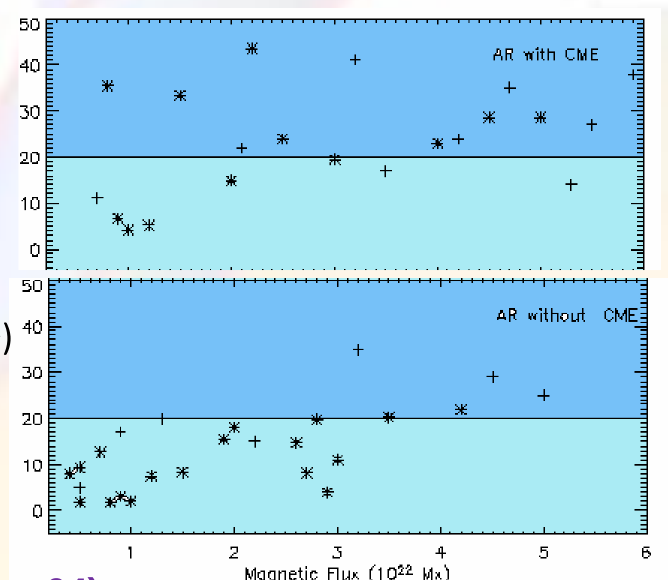
Helicity and solar eruption

- Helicity conservation could be the "raison d'être" of coronal mass ejections. **(Rust 94, Low 96).**
- Several observational studies have shown diverse indications that magnetic helicity can be tightly linked with enhanced eruptivity: **(Nindos et al. 04, Labonte et al. 07, Park et al. 08, 10, Tziotziou et al. 12)**

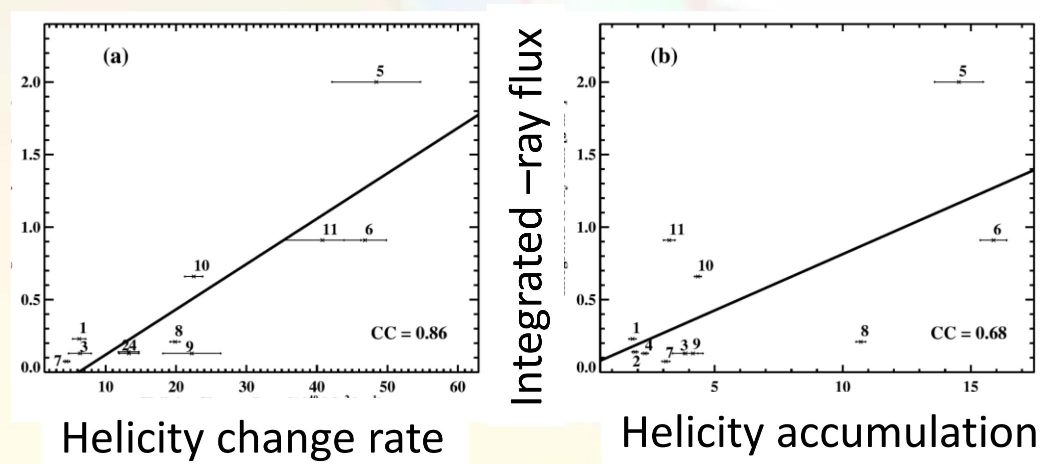


(Tziotziou et al. 12)

Total Coronal Helicity (10^{42} Mx^2)



(Nindos 04)



(Park et al. 08)

Gauge invariance of magnetic helicity

- Gauge transformation of magnetic helicity: $H = \int_{\mathcal{V}} \vec{A} \cdot \vec{B} \, \mathrm{d}V$

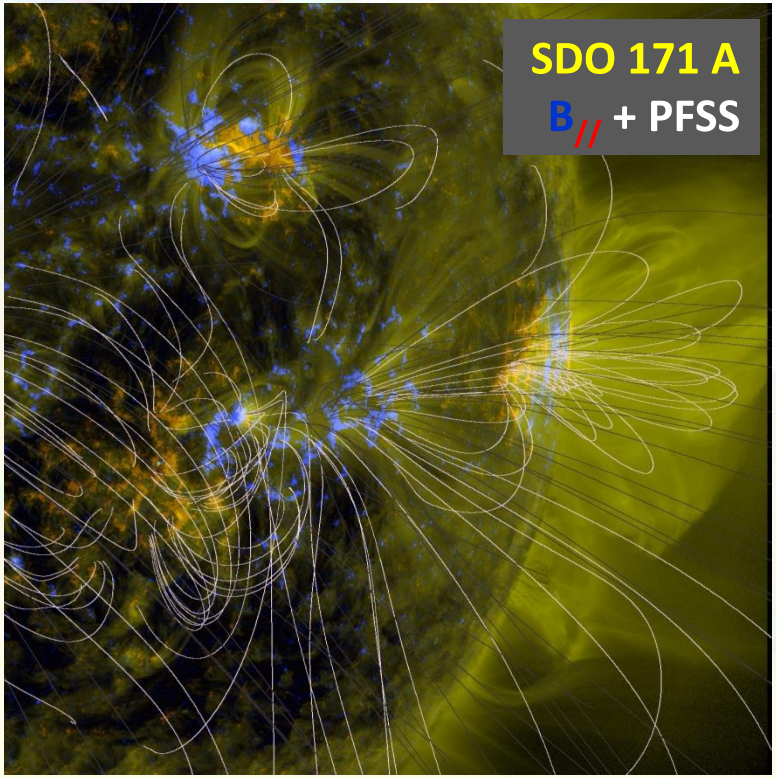
$$\mathbf{A}' \longrightarrow \mathbf{A} + \nabla\phi,$$

$$H'_m = \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V + \int_V \nabla\phi \cdot \mathbf{B} \, \mathrm{d}V = H_m + \int_S \phi \mathbf{B} \cdot \mathrm{d}S$$

- Magnetic helicity is gauge invariant only for magnetically bounded systems:

$$\mathbf{B \cdot dS}\big|_S = 0$$

- Strict definition of magnetic helicity useless for numerous applications:
    - e.g. natural plasmas, like the solar corona have boundaries threaded by magnetic fields



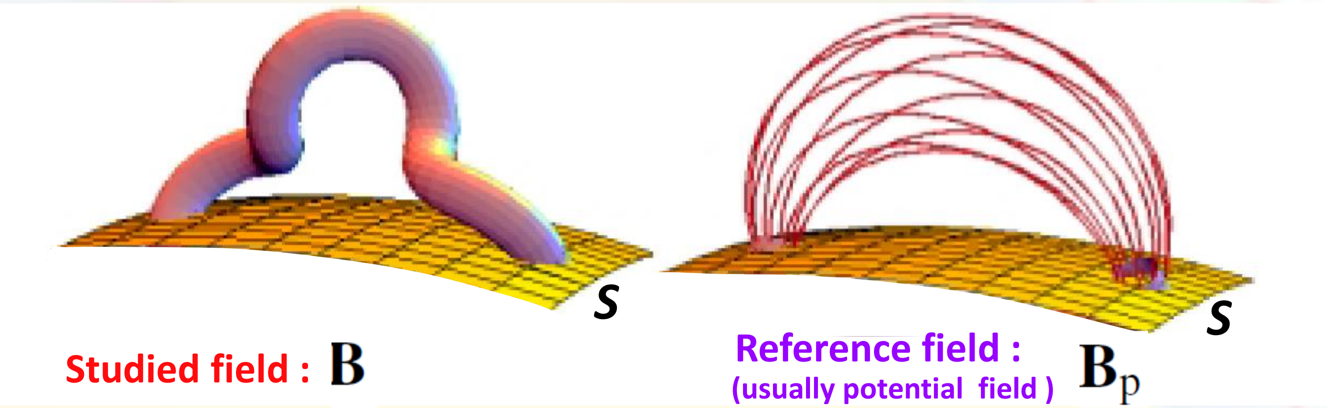
Relative Magnetic Helicity

➔ Useful quantity: **Relative Magnetic Helicity:** helicity of a studied field relative to a reference field **(Berger 84, Finn & Antonsen 85).**

$$H_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\rm p}) \cdot (\mathbf{B} - \mathbf{B}_{\rm p}) \ {\rm d}\mathcal{V}$$ **(Finn & Antonsen 85)**

with boundary condition : $(\mathbf{B}_{\rm p} \cdot {\rm d}\mathbf{S})|_{\partial\mathcal{V}} = (\mathbf{B} \cdot {\rm d}\mathbf{S})|_{\partial\mathcal{V}}$ $\nabla \times \mathbf{A} = \mathbf{B}$

- **Gauge invariant provided that studied and reference fields share the same magnetic-flux distribution <u>on the whole boundary</u>.**



Relative Magnetic Helicity Estimations

- The computation of relative magnetic helicity is not straightforward:
  - **Computation of reference field must be done imposing boundary conditions on the whole domain boundary.**
  - Many previous methods assumed semi-infinite volumes while all existing datasets are bounded volumes: could lead to incorrect results **(Valori et al. 11, 12)** error in intensity, even in sign!

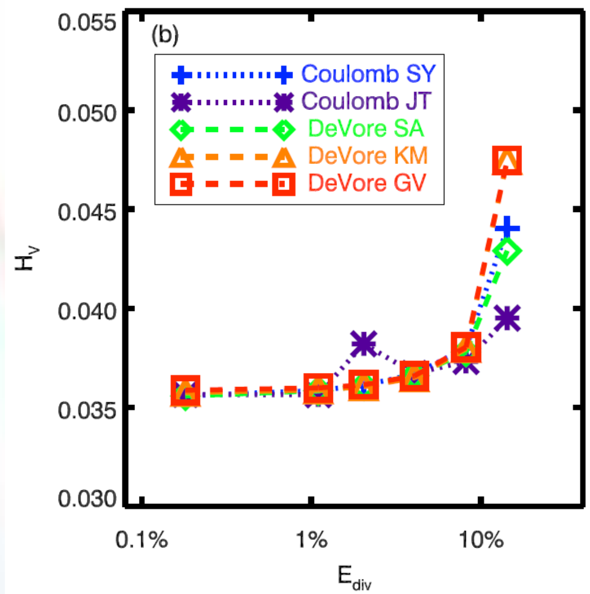
- **Several methods recently developed on 3D cuboid system** **(Valori et al. 2016)**
  - Using Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

    Thalmann et al. 2011, **Rudenko & Myshyakov 2011, Yang et al. 2013**
    - Simpler theoretical formulation
    - Harder to implement numerically
  - Using DeVore gauge **(DeVore et al. 2000)** : $A_z = 0$

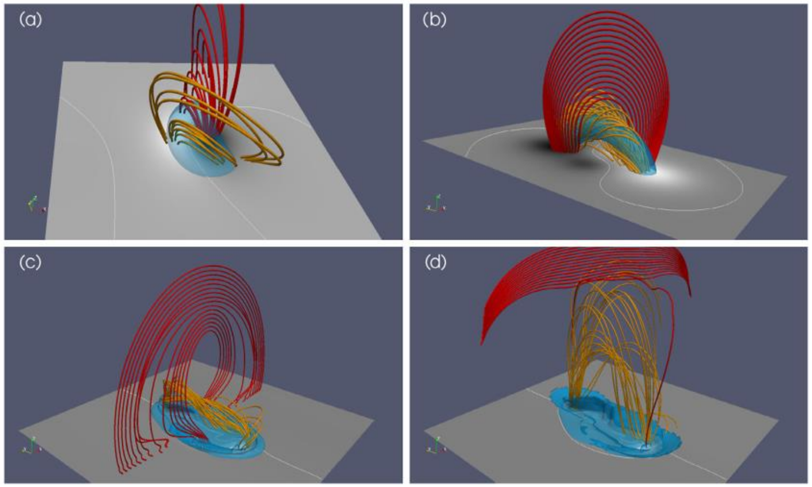
    Valori, Démoulin & Pariat 2012, Moraitis et al. 2014
    - More complex theoretical formulation
    - Simpler to implement numerically: more precise
- **New method to compute relative magnetic helicity in spherical wedge domains**. **(Moraitis et al. submitted)**

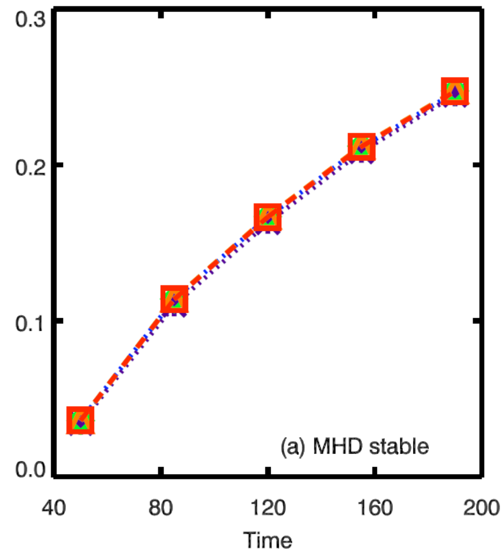
Relative magnetic helicity estimations

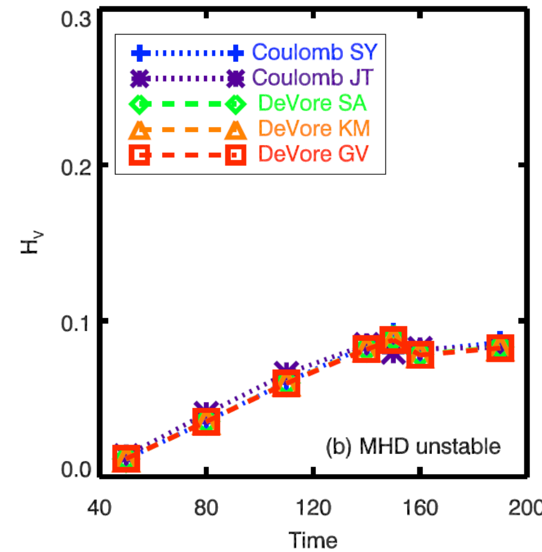
- **Benchmarking of these methods performed by ISSI team on "Helicity estimations in models and observations“: Valori et al. 2016**
- Numerous tests: sensibility to resolution, twist, solenoidality using various types of data.
  - Force free fields **(Low & Lou 1990)**
  - Stable flux rope (**Titov & Démoulin 1999**, data fromT. Török)
  - Flux emergence simulations **(Leake et al. 2013, 2014)**
- **Methods perform very consistently when B sufficiently solenoidal**



(Valori et al. 16)







Outline

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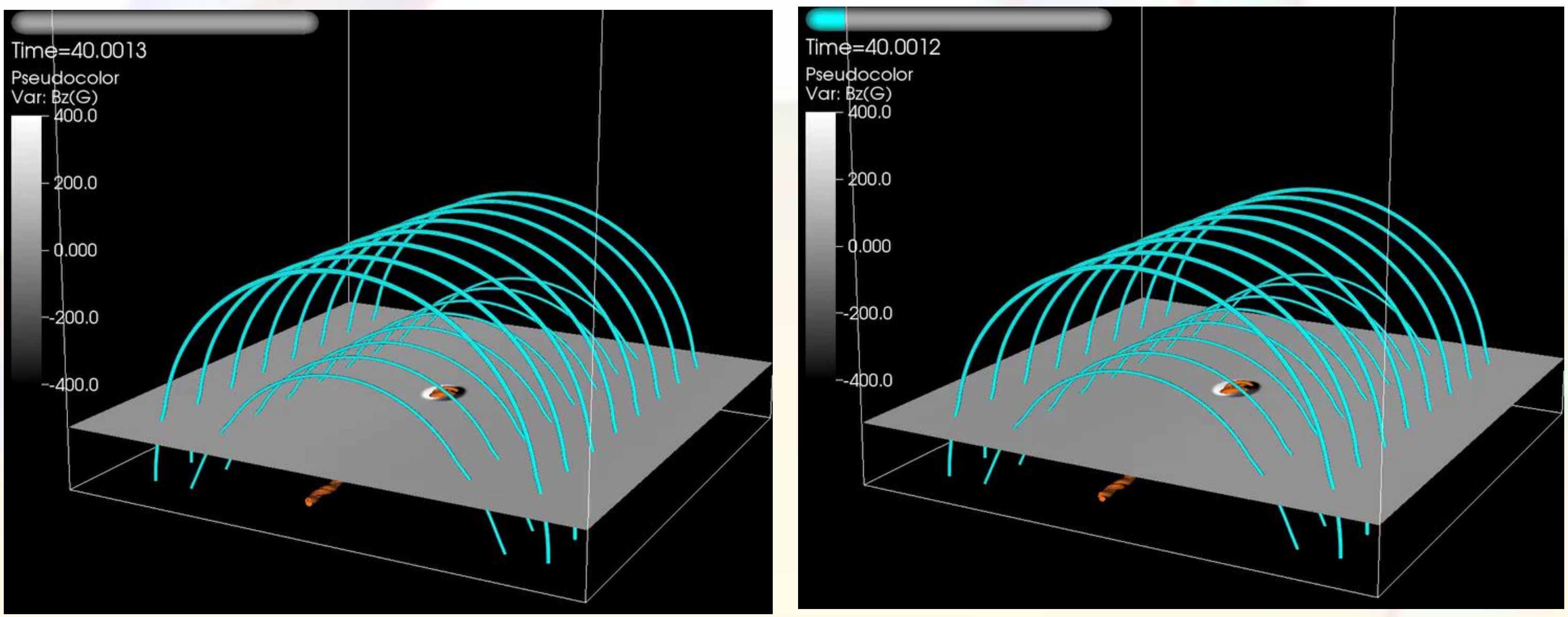
Motivations & Methodology

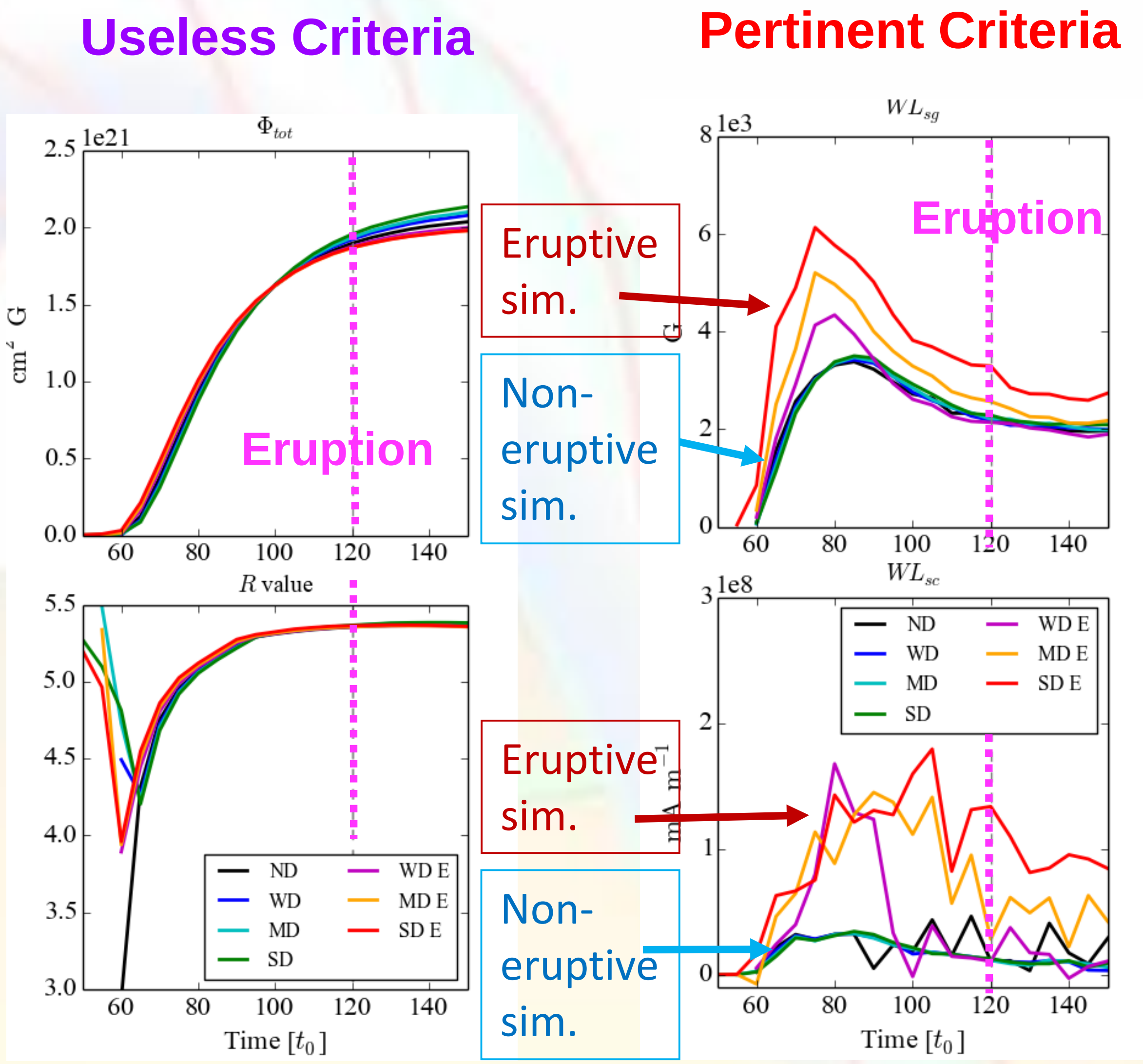
- **Goal: use flux emergence simulations to look for efficient eruptivity criterion**
  - **7 flux emergence simulations** obtained with 3D visco-resistive MHD eq. solved with Lagrangian-remap code **(Arber et al. 2001)**
  - **either lead to eruptive or non-eruptive dynamics (Leake et al. 2013, 2014)**
- Methodology: - extract part of the magnetic field,
  - compute different physical quantities,
  - search those that discriminates between the eruptive and non-eruptive case



(Leake et al. 13, 14)

Search for eruptivity criterion

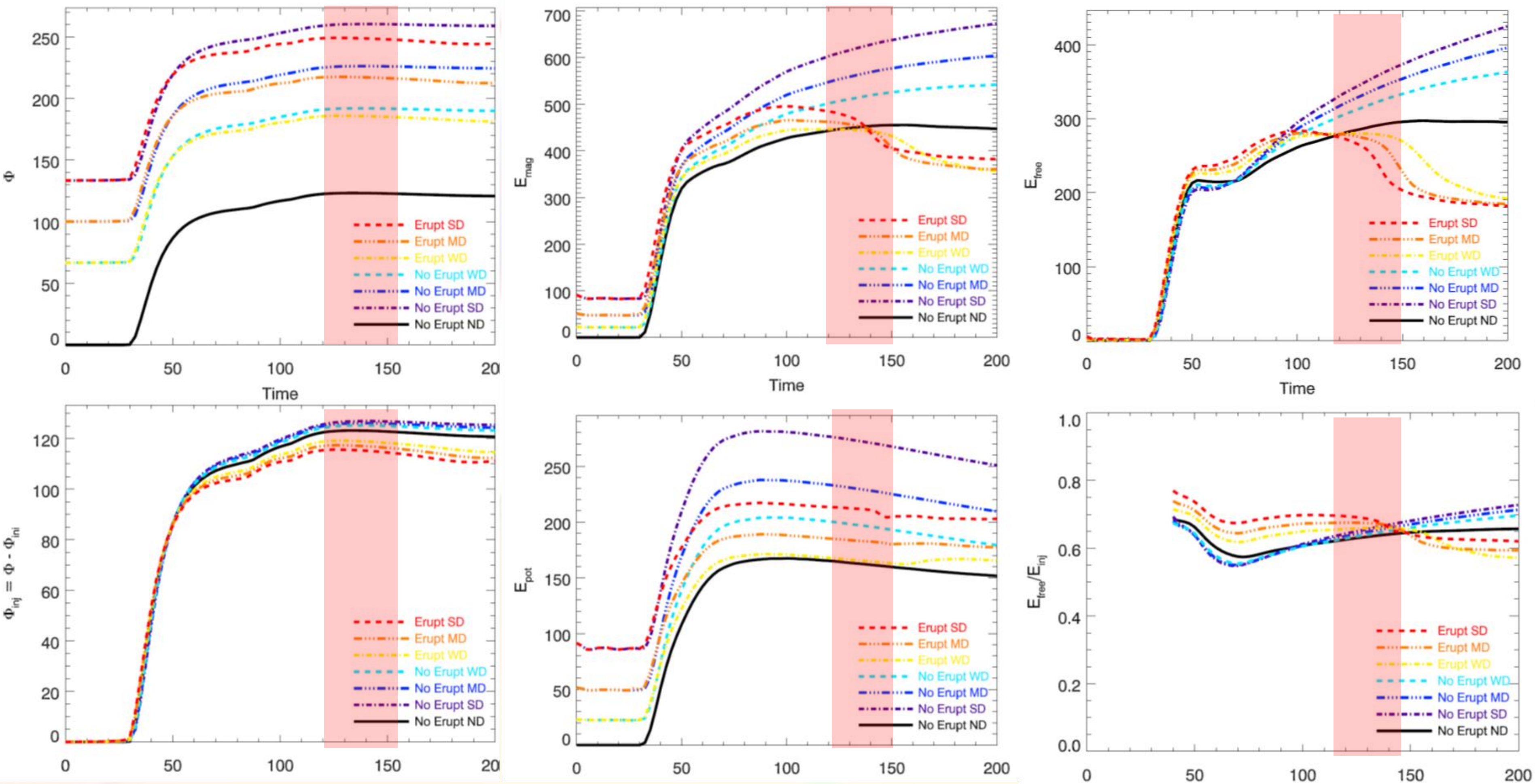
- **Goal: search for eruptivity indicators from 3D coronal magnetic datacube**
- Good eruptivity criterion should:
  - Discriminate eruptive and non-eruptive sim. during pre-eruptive phase
  - Reach its highest value
    - for eruptive simulation only,
    - during the pre-eruptive phase only.
  - Present similar trend for eruptive and non-eruptive sim. in post-eruptive phase



(Guennou et al. 17)

Magnetic fluxes and energies

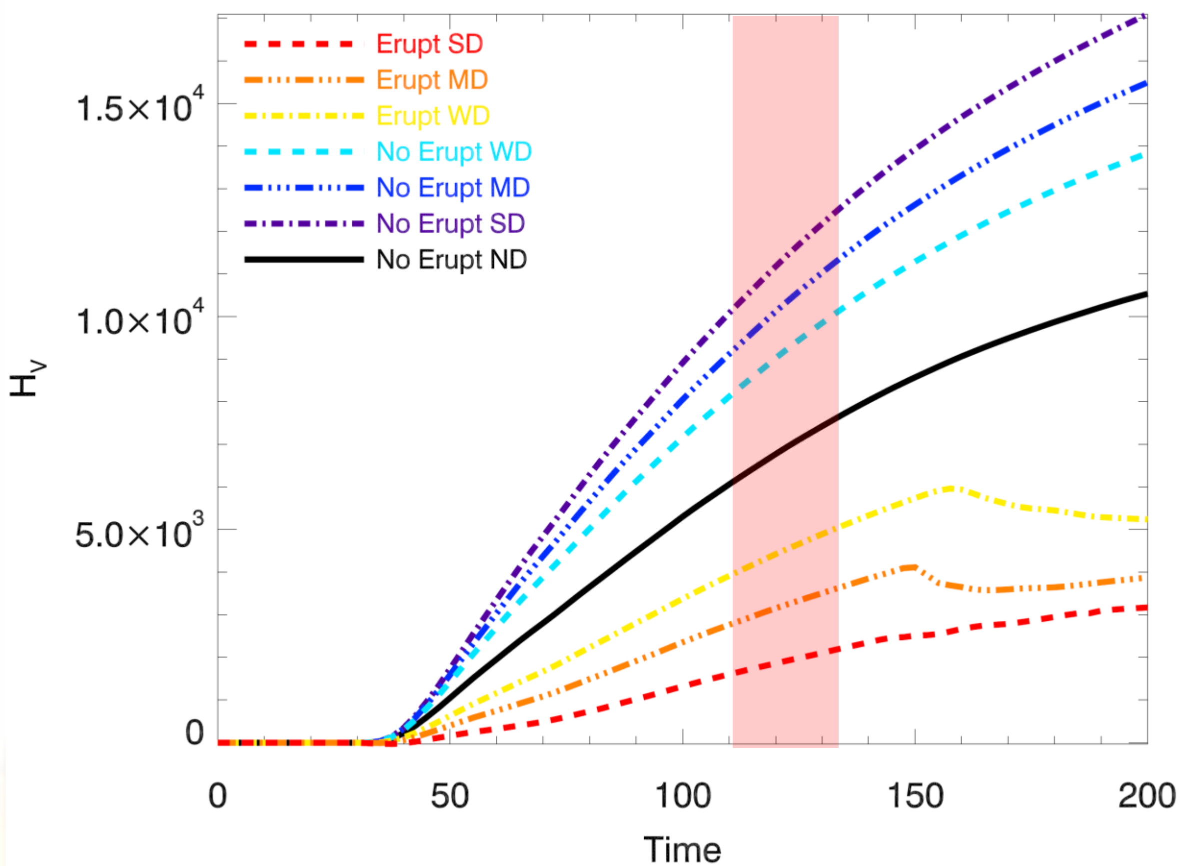
(Pariat et al. 17)



- **Neither injected magnetic flux nor magnetic energies are properly discriminating between the different simulations and do not provide reliable eruptivity diagnostics**

Relative magnetic helicity evolution

(Pariat et al. 17)



- **Unlike with magnetic flux & free energy, helicity discriminates strongly the cases**
  - Total helicity depends
    - on dipole strength
    - on dipole orientation

- The surrounding (potential) field influences the helicity content!

  ← magnetic helicity is a non-local quantity!

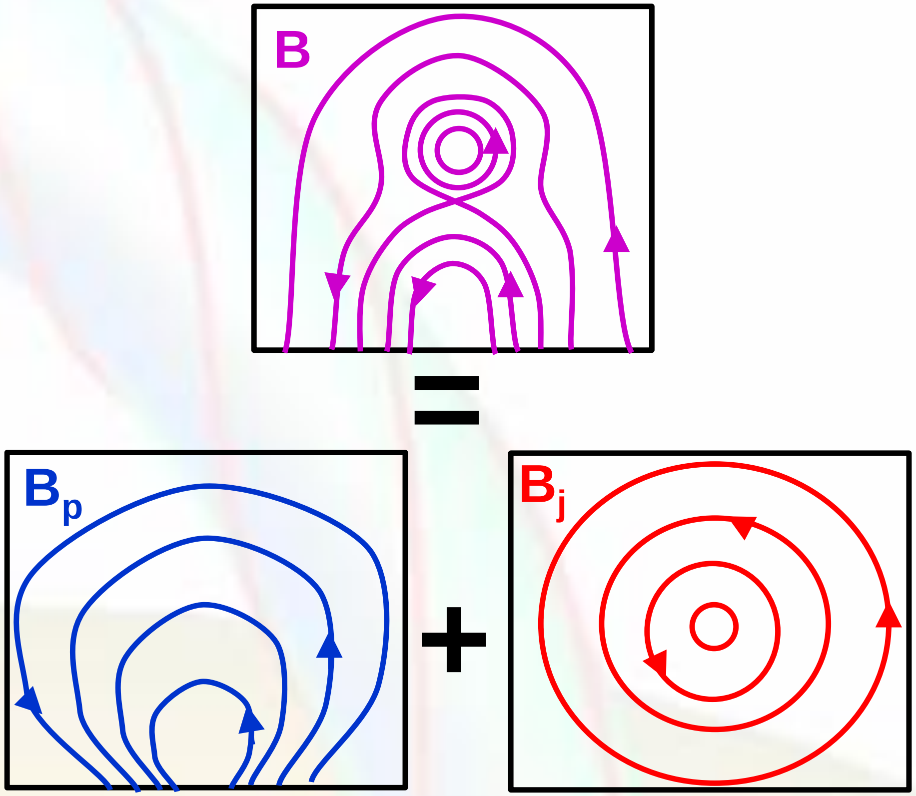
- Here, eruptive simulations have lower helicity than non-eruptive one

  ➔ unlike what is commonly believed/expected, large total helicity is not a sufficient condition of eruptivity.

Relative magnetic helicity decomposition

- Based on the decomposition of a **magnetic field** into **potential** and **non-potential** fields….

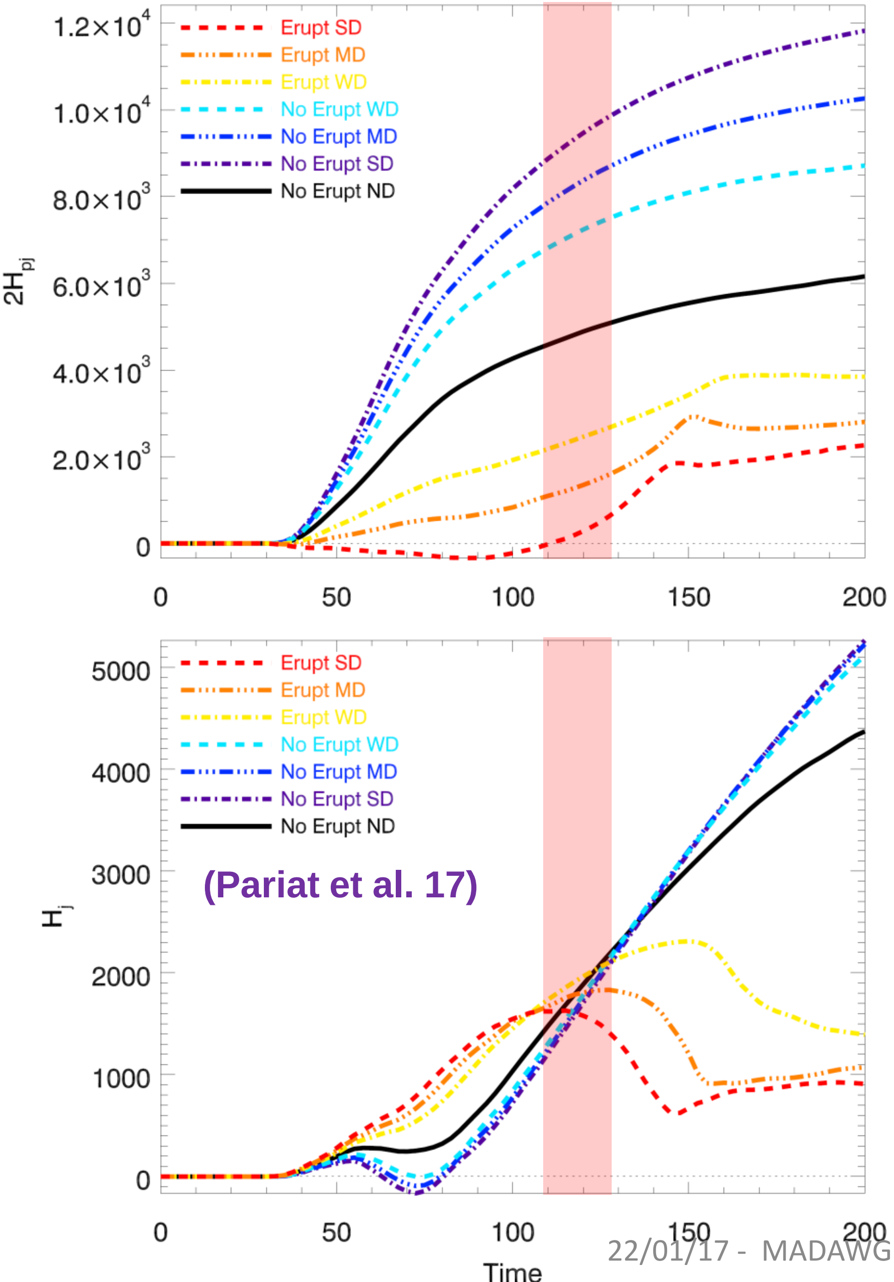
- **Relative magnetic helicity can be decomposed in 2 gauge-invariants quantities (Berger et al. 2003) :**
  - H_j = magnetic helicity of the current-carrying field B_j (non-potential field)
  - H_{pj} = volume-threading helicity, between potential and current-carrying fields



$$H_V = H_j + 2H_{pj} \quad \text{with}$$
$$H_j = \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_p) \cdot (\mathbf{B} - \mathbf{B}_p) \, d\mathcal{V}$$
$$H_{pj} = \int_{\mathcal{V}} \mathbf{A}_p \cdot (\mathbf{B} - \mathbf{B}_p) \, d\mathcal{V}$$

- Remark for the heli-aware: H_j & H_{pj} are different from the “self” and “mutual” helicities

Helicity decomposition evolution

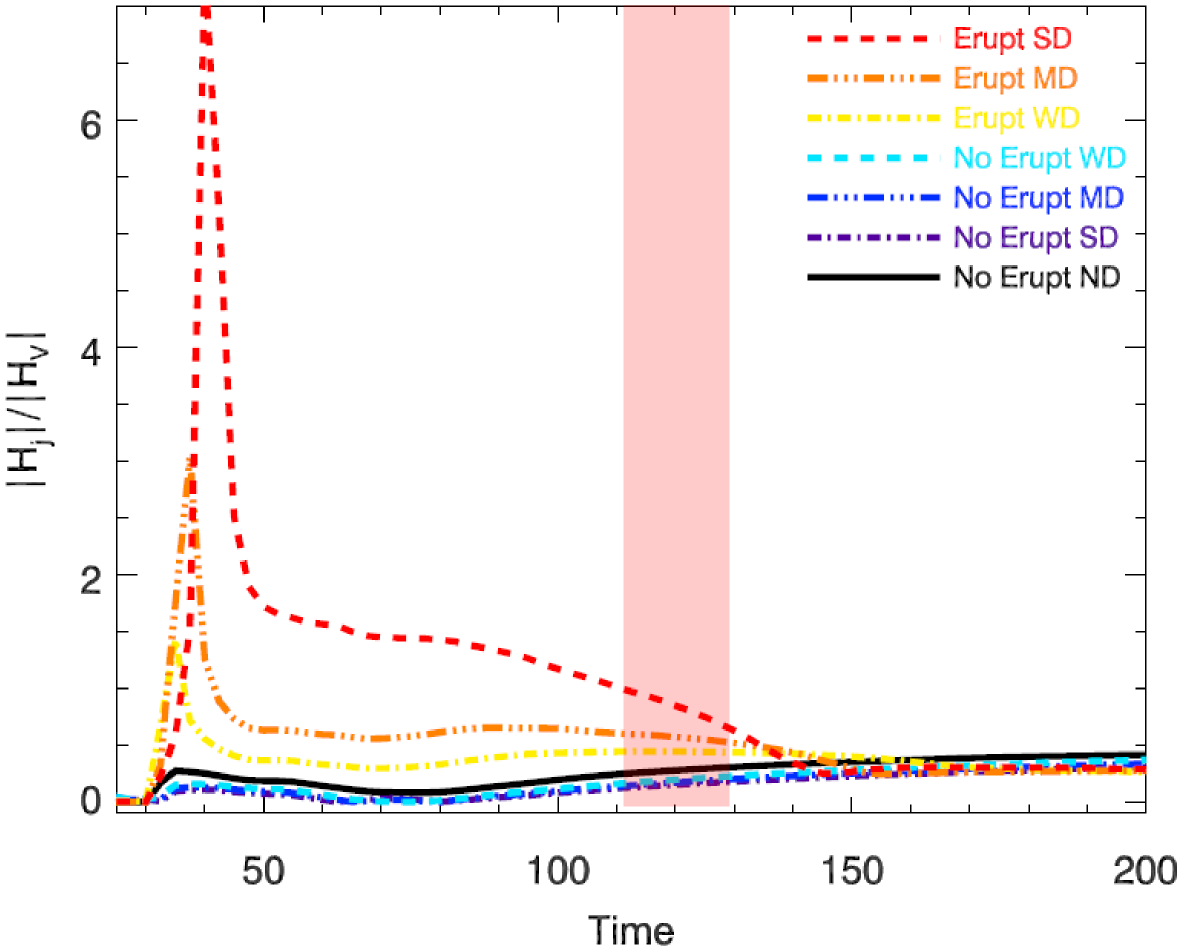


$$H_V = H_j + 2H_{pj} \quad \text{with}$$
$$H_j = \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_p) \cdot (\mathbf{B} - \mathbf{B}_p)\, d\mathcal{V}$$
$$H_{pj} = \int_{\mathcal{V}} \mathbf{A}_p \cdot (\mathbf{B} - \mathbf{B}_p)\, d\mathcal{V}$$

- Total helicity is overall dominated by $2H_{pj}$
- $2H_{pj}$ has same properties than total helicity ➔ not a good eruptivity proxy
- **H_j behaves similarly to E_{free}**
  - **higher for the eruptive simulations in the pre-eruptive phase**
  - **however higest values reached by non-eruptive simulations**
- **H_j is not a good eruptivity proxy.**

$|H_j|/|H_V|$: excellent eruptivity indicators

(Pariat et al. 17)

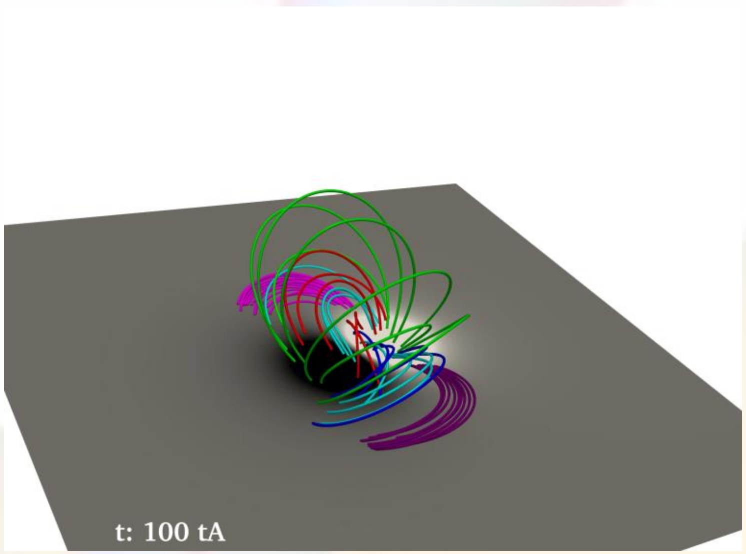


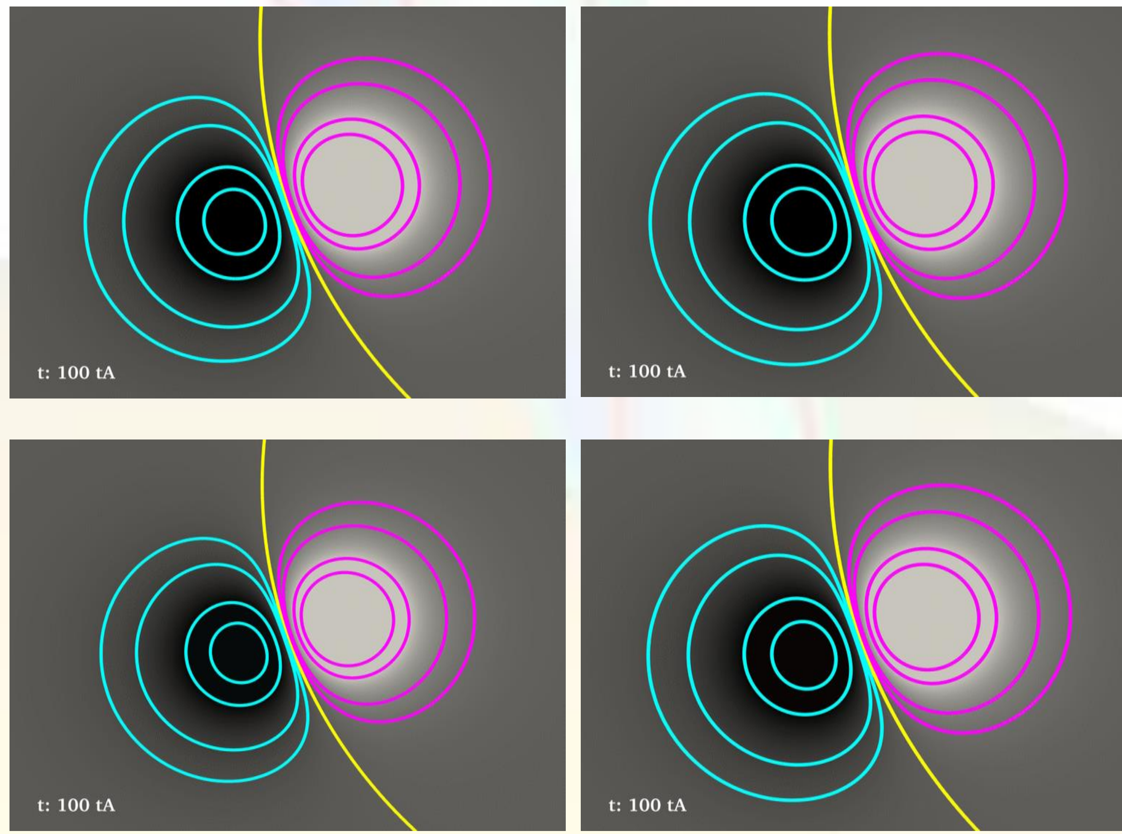
$$H_V = H_j + 2H_{pj} \quad \text{with}$$
$$H_j = \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_p) \cdot (\mathbf{B} - \mathbf{B}_p) \, d\mathcal{V}$$
$$H_{pj} = \int_{\mathcal{V}} \mathbf{A}_p \cdot (\mathbf{B} - \mathbf{B}_p) \, d\mathcal{V}$$

- **$|H_j|/|H_V|$ appears as an excellent eruptivity predictor of these sims.**
  - **Highest value for the eruptive simulations in the pre-eruptive phase**
  - **Eruptive and non-eruptive simulations have similar values in post-eruption phase**
- $|H_j|/|H_V|$ is also sensitive to dipole strength which fits with promptness to erupt

Further evidences : torus-instability triggered eruptive simulations

- **Zuccarello et al. 2015**: parametric eruptive simulations
- **4 different line-tied boundary driving patterns** with different: shear around the PIL magnetic flux dispersion + 1 non-eruptive control case (diffusion)

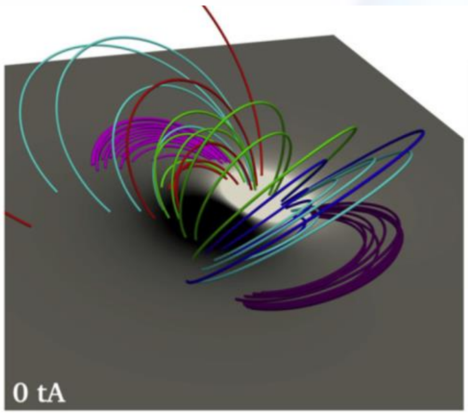


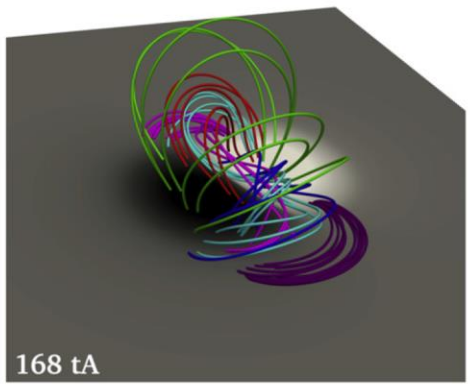


(Aulanier et al. 10, Zuccarello et al. 16)

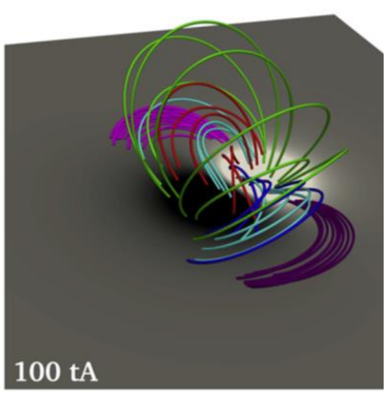
Further evidences :
torus-instability triggered eruptive simulations

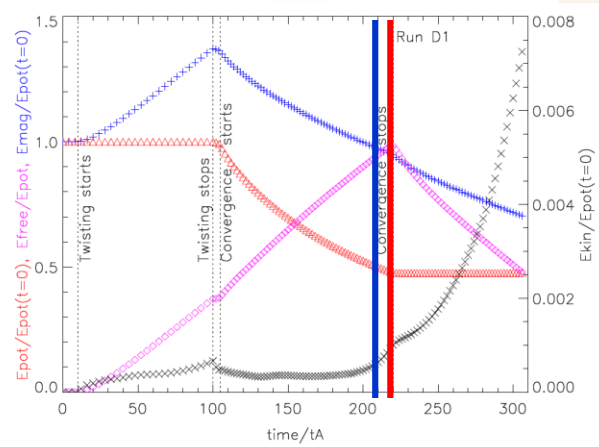
- **For each simulation, precise determination of the onset time, t_{erupt}, thanks to numerous relaxation runs initiated at regular instants.**



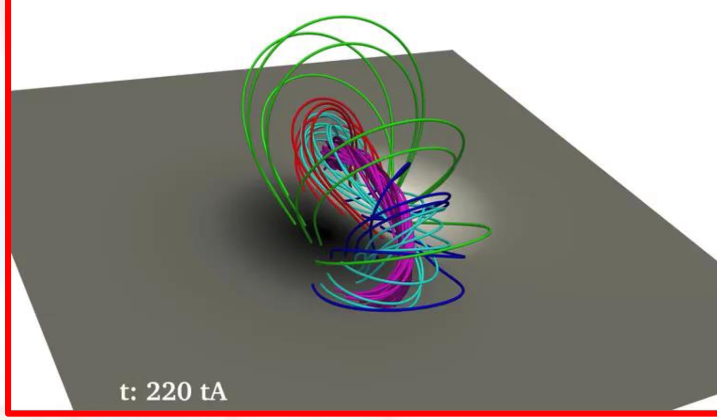


(Aulanier et al. 10, Zuccarello et al. 16)

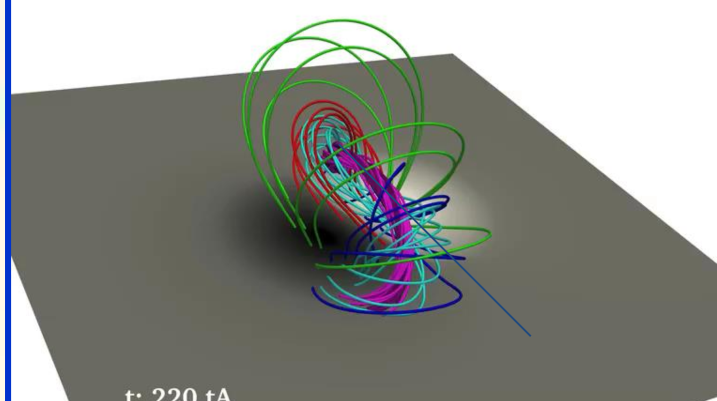




Relaxation run at t_{erupt}



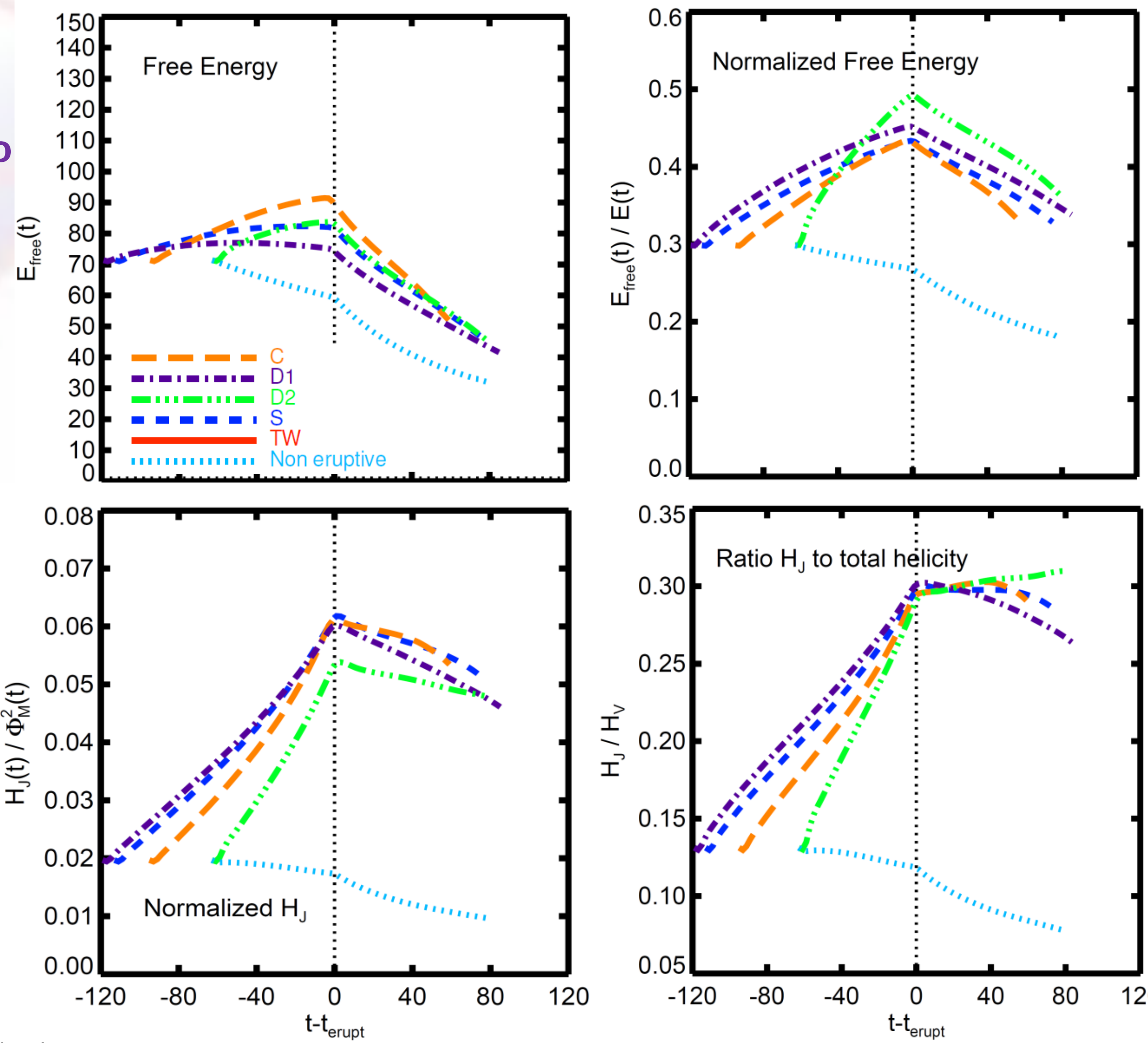
Relaxation run at $t_{erupt} - 4t_A$



Further evidences :
torus-instability triggered eruptive simulations

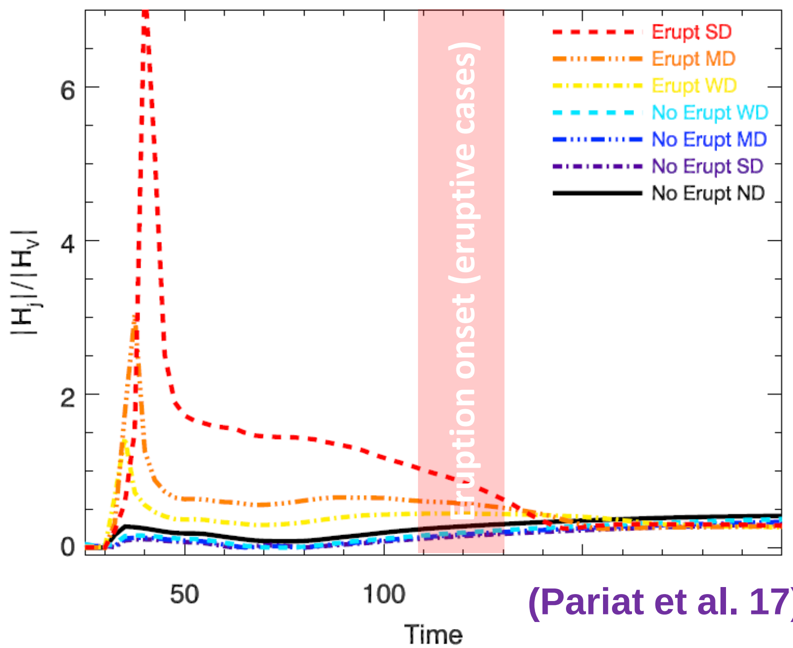
(Zuccarello et al. tbs)

- Computation of several quantities at the sim. respective t_{erupt} : **Zuccarello et al. to be submitted.**
- **Despites different boundary drivers and t_{erupt}, eruptions are triggered when $|H_j|/|H_V|$ reaches the same value:**
  - <4% dispersion
  - within measurement precision of helicity
- All other quantities have dispersions of values above 8 % at t_{erupt} , including torus instability criteria

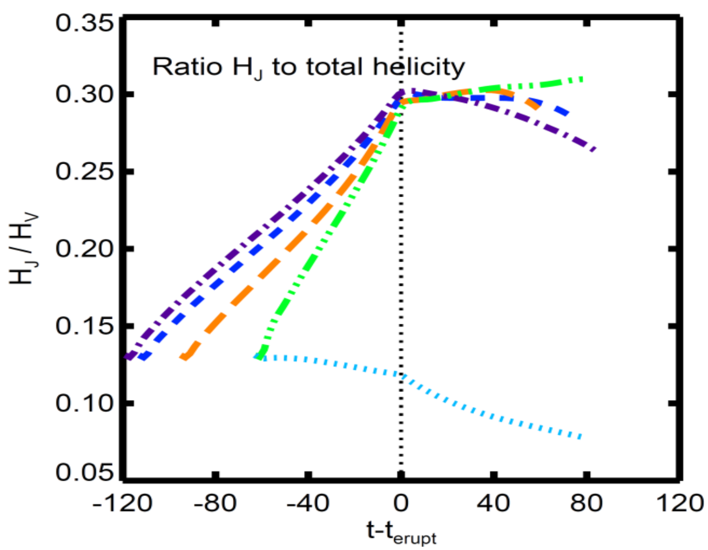


Partial - conclusions

- (too) Rare attempts to use parametric numerical simulation to study eruptivity proxy of solar active events.
- **The ratio |Hj|/|Hv| is an excellent indicator of the eruptivity state in several numerical models**
  - 15 different numerical simulations
  - inducing 11 eruptions & 6 stable systems
  - in 4 very different magnetic configuration
  - performed by 3 different MHD numerical codes
- Now needs to be validated against proper observational datasets



(Pariat et al. 17)



(Zuccarello et al. tbs)

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Helicity estimation from observations

- Two main methods: cf review **Valori et al., Space Science Rev. 2016**

Finite volume (FV)

$$\mathscr{H}_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V}$$

– Requires **B** in $\mathcal{V}$ *e.g.*, from MHD simulations or NLFFF
– Compute $\mathscr{H}_{\mathcal{V}}$ at one time
– May employ different gauges (

- Input: 3D magnetic field!
- No direct estimation from observations
- Requires 3D reconstruction of the coronal magnetic field from 2D magnetogram

Helicity-flux integration (FI)

$$\frac{\mathrm{d}\mathscr{H}_{\mathcal{V}}}{\mathrm{d}t} = 2 \int_{\partial\mathcal{V}} [(\mathbf{A}_{\mathrm{p}} \cdot \mathbf{B})\, v_n - (\mathbf{A}_{\mathrm{p}} \cdot \mathbf{v}_{\mathrm{t}})\, B_n]\, \mathrm{d}S$$

– Requires time evolution of vector field on $\partial\mathcal{V}$
– Requires knowledge or model of flows on $\partial\mathcal{V}$
– Valid for a specific set of gauge and assumptions,

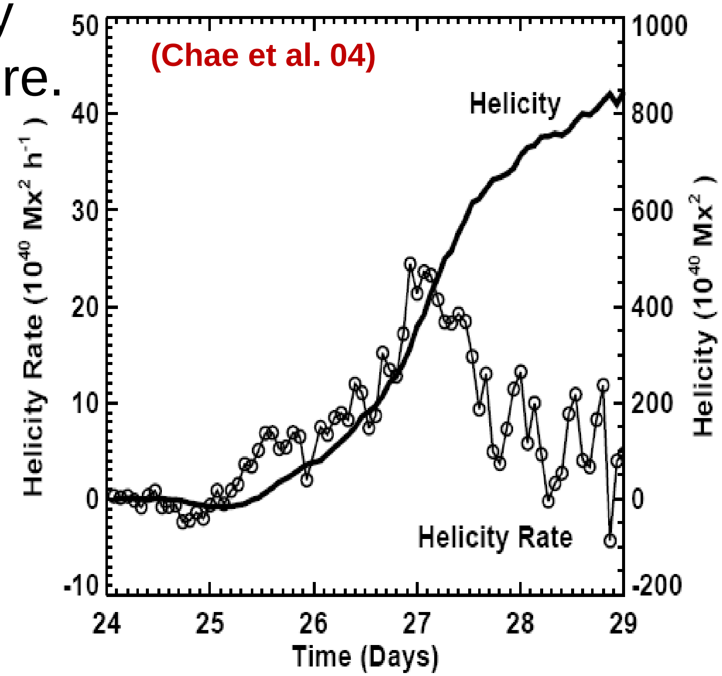
- Input: time series of 2D magnetograms
- Direct estimations from observed data

Helicity flux integration methods

- Relative magnetic helicity can be estimated by time-integrating its flux through the photosphere.

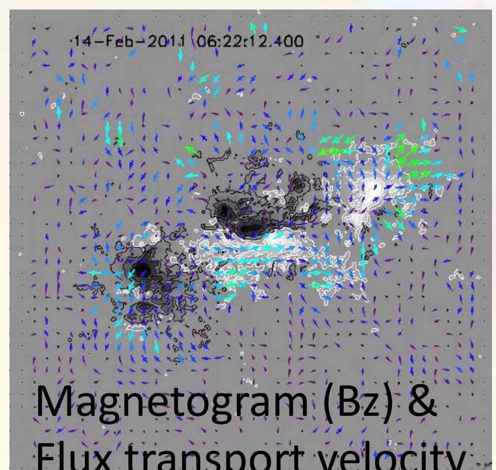
$$\frac{\mathrm{d}\mathcal{H}_{\mathcal{V}}}{\mathrm{d}t} = 2\int_{\partial\mathcal{V}} [(\mathbf{A}_{\mathrm{p}}\cdot\mathbf{B})\,v_n - (\mathbf{A}_{\mathrm{p}}\cdot\mathbf{v}_{\mathrm{t}})\,B_n]\,\mathrm{d}S$$

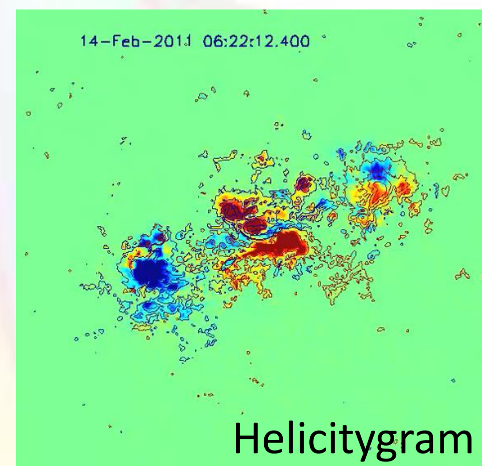
  - Most commonly used method to measure magnetic helicity : review **Démoulin & Pariat 09**



- How to measure the helicity flux?
  - B is given from spectropolarimetry (magnetograms)
  - A_P is inferred from Bn maps by:
    - Fourier Transform methods **(e.g. Chae 01)** or Green functions **(e.g. Liu & Schuck 14)**
  - V may be deduced from optical flow method:
    - Local Correlation Tracking Methods
    - DAVE & DAVE4VM **(Schuck 06; 08)**

- **➔ All what is needed are time series of magnetograms at the highest possible cadence & resolution**





Flux of non-potential magnetic helicity

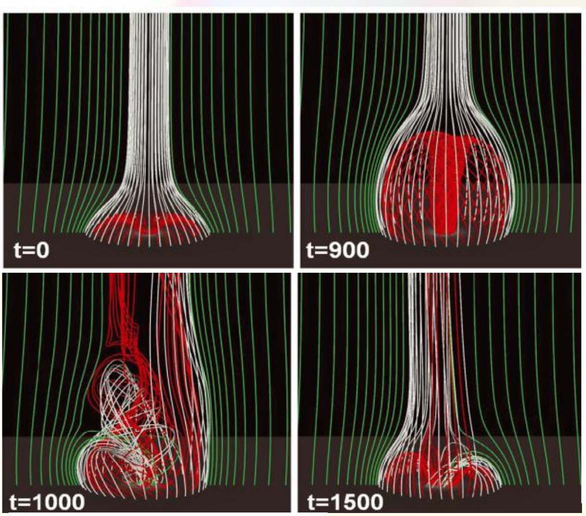
- While relative magnetic helicity is a quasi-conserved, the terms of its decomposition are not **(Linan et al. 18)** !

$$H_V = H_j + 2H_{pj}$$

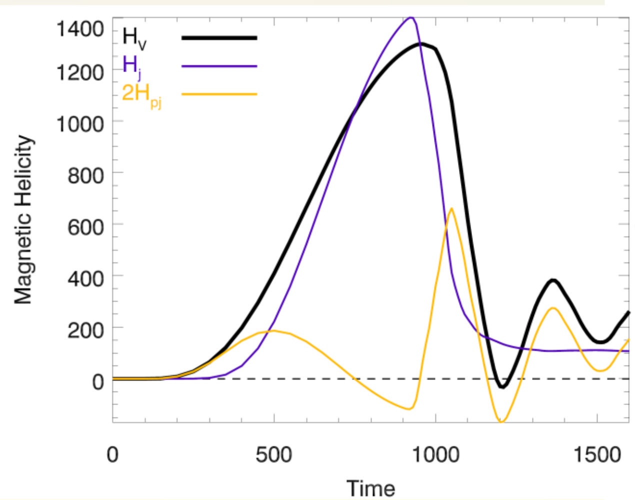
$$\frac{\mathrm{d}H_j}{\mathrm{d}t} = 2\int_V \mathbf{R}\cdot\mathbf{B}_j\,\mathrm{d}V + 2\int_V ((\mathbf{v}\times\mathbf{B})\cdot\mathbf{B}_p)\,\mathrm{d}V + 2\int_V \frac{\partial\phi}{\partial t}\nabla\cdot\mathbf{A}_j\,\mathrm{d}V$$

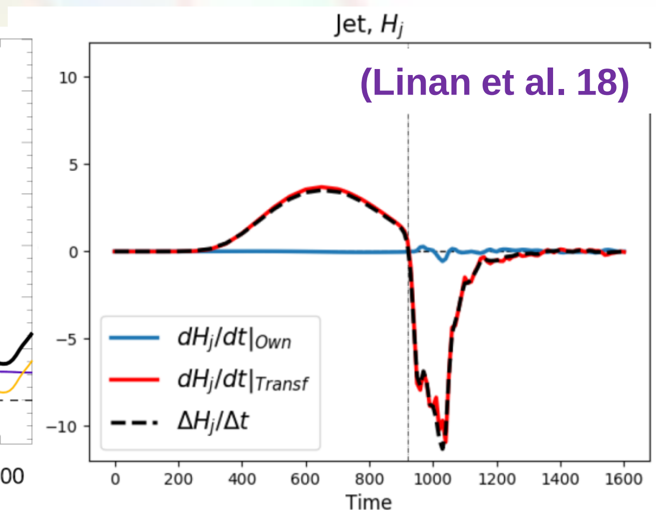
$$- 2\int_S (\mathbf{B}\cdot\mathbf{A}_j)\mathbf{v}\cdot\mathrm{d}S + 2\int_S (\mathbf{v}\cdot\mathbf{A}_j)\mathbf{B}\cdot\mathrm{d}S + \int_S \mathbf{A}_j\times\frac{\partial}{\partial t}\mathbf{A}_j\cdot\mathrm{d}S - 2\int_S \frac{\partial\phi}{\partial t}\mathbf{A}_j\cdot\mathrm{d}S$$

- ➔ Unlike magnetic helicity, the evolution (accumulation) of H_j cannot be determined solely from its flux.



(Pariat et al. 09)





Helicity eruptivity proxy estimation in observation

- The helicity-flux integration method is useless to estimate the eruptivity proxy $|H_j|/|H_V|$
- **➔ One must use the finite volume method, hence determine B in the full 3D domain**

Finite volume (FV)

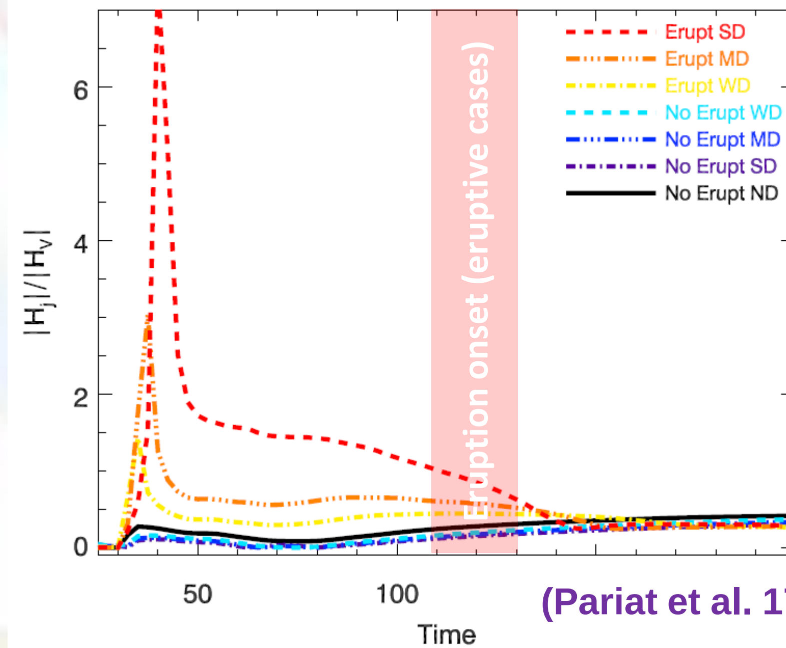
$$\mathcal{H}_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\rm p}) \cdot (\mathbf{B} - \mathbf{B}_{\rm p}) \, {\rm d}\mathcal{V}$$

– Requires **B** in $\mathcal{V}$ *e.g.*, from MHD simulations or NLFFF
– Compute $\mathcal{H}_{\mathcal{V}}$ at one time
– May employ different gauges (see Table 2)

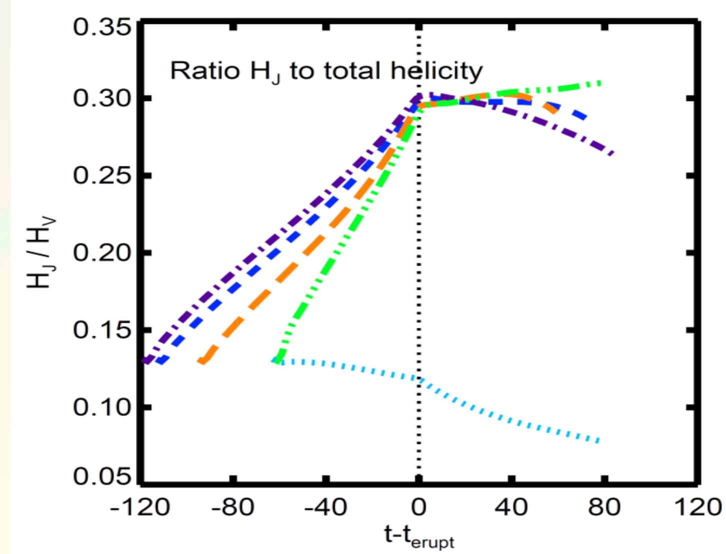
Helicity-flux integration (FI)

$$\frac{{\rm d}\mathcal{H}_{\mathcal{V}}}{{\rm d}t} = 2 \int_{\partial\mathcal{V}} [(\mathbf{A}_{\rm p} \cdot \mathbf{B}) v_n - (\mathbf{A}_{\rm p} \cdot \mathbf{v}) B_n] \, {\rm d}S$$

– Requires time evolution of vector field on $\partial\mathcal{V}$
– Requires knowledge or model of flows on $\partial\mathcal{V}$
– Valid for a specific set of gauge and assumptions,



(Pariat et al. 17)



(Zuccarello et al. tbs)

Finite volume method

Finite volume (FV)

$$\mathcal{H}_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V}$$

- Requires **B** in $\mathcal{V}$ *e.g.*, from MHD simulations or NLFFF
- Compute $\mathcal{H}_{\mathcal{V}}$ at one time
- May employ different gauges (see Table 2)

- Less commonly used for helicity studies so far: **e.g. Valori et al. 13, DeRosa et al. 15, Polito et al. 2017, Temmer et al. 17**

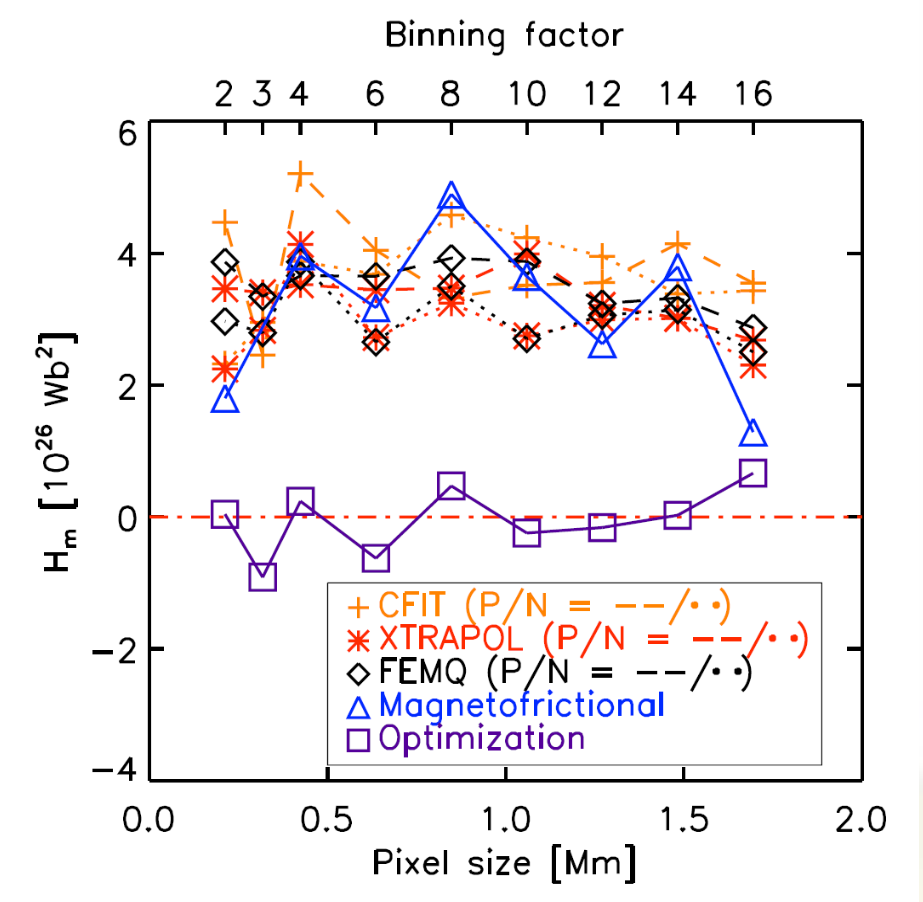
- Which extrapolation approximation?
  - Potential field extrapolation : helicity is null by definition
  - Linear force-free field extrapolation: helicity directly given by linear force free parameter: you get what you put in!

- **Helicity computation by the finite volume method requires extrapolation in the non-linear force-free field (NLFFF) approximation**
  - or MHD model, though so far either initiated by NLFFF extrapolation or more complicated to produce and less consistent.

Actual NLFFF extrapolation limitation

- Magnetic helicity estimation is highly sensitive to extrapolation method: **DeRosa et al. 2015**
  - Helicity is a non local quantity
  - Differences between extrapolation in the whole domain leads to important variation of the helicity measure

- To a very large extent magnetic extrapolation is not a well posed problem and is largely underconstrained ➔ **next talk**



- **Obtaining quality/reliable extrapolations requires additional input data, e.g. to go single view point magnetogram**

  ➔ multi-view point magnetic field measurements allowed by PHI/Solar Orbiter ➔ next talk by G. Valori

Thanks for your attention

Go Gherardo!