Calculating relative magnetic helicity in spherical wedge volumes

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Outline

- Introduction – Magnetic helicity
- Method description
- Method validation
- Application to NLFF fields - Results
- Concluding remarks

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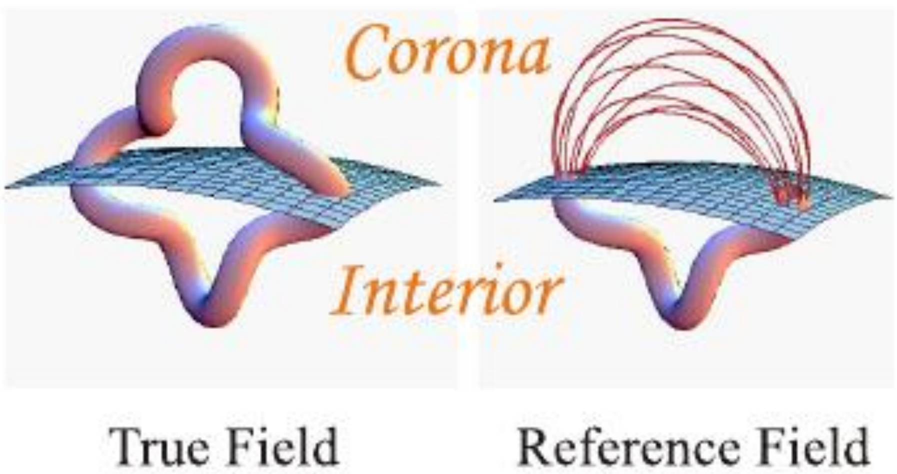
Introduction
Magnetic helicity

- Signed scalar quantity (right (+), or left (-) handded) $H = \int A \cdot B dV \qquad B = \nabla \times A$
- Helicity measures the twist and writhe of mfls, and the amount of flux linkages between pairs of lines (Gauss linking number)
- Gauge invariant only for closed ***B*** $\hat{n} \cdot B\big|_{\partial V} = 0$



Berger & Field 1984, Finn & Antonsen 1985

Relative magnetic helicity

$$H = \int (A + A_p) \cdot (B - B_p) dV$$

gauge invariant for closed (and solenoidal) $B_j = B - B_p$

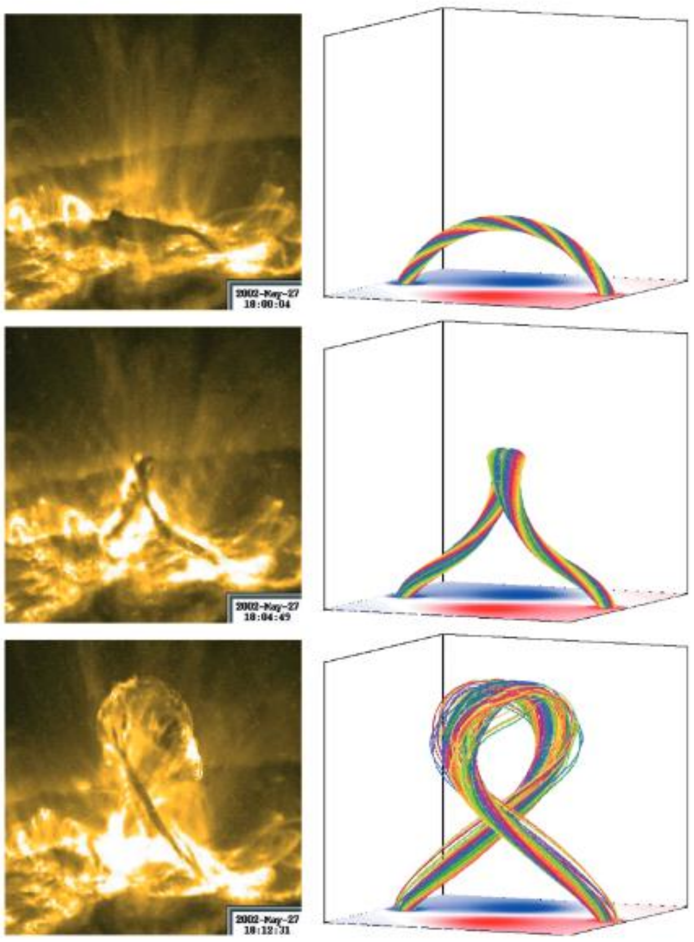
$$\hat{n} \cdot B_j\big|_{\partial V} = 0 \Rightarrow$$

$$\hat{n} \cdot B\big|_{\partial V} = \hat{n} \cdot B_p\big|_{\partial V}$$

∂V: the whole boundary

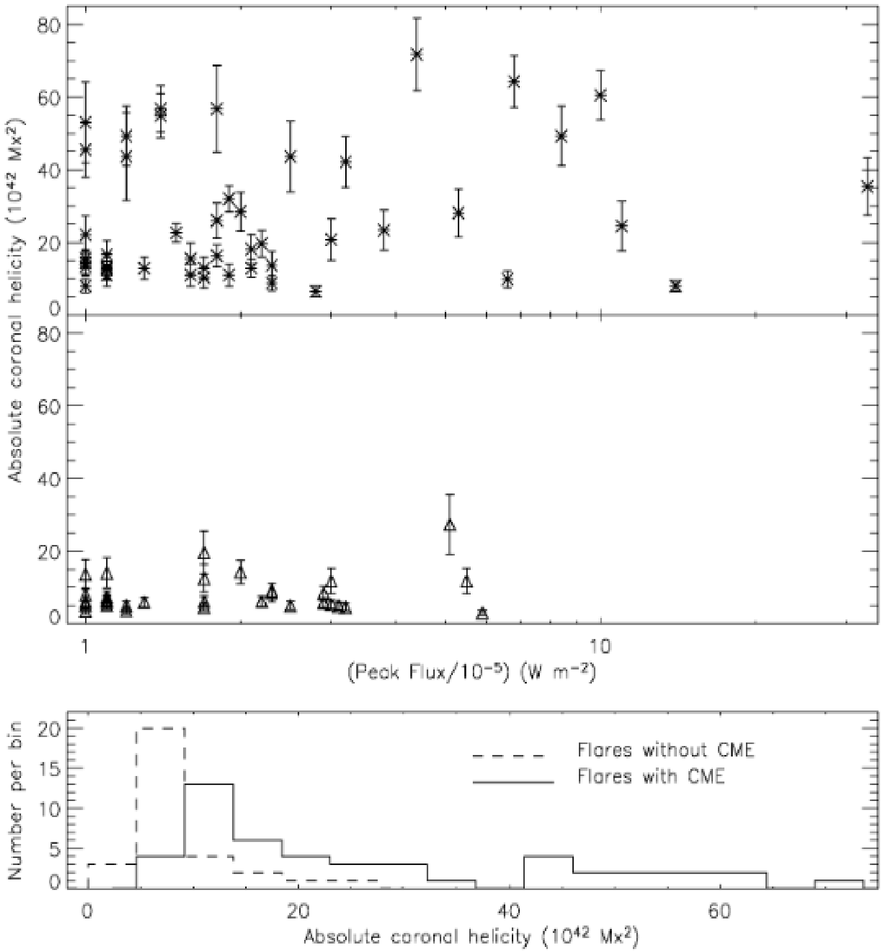
Introduction
Magnetic helicity properties

- Conserved in ideal MHD (Woltjer 1958), and approximately conserved in resistive MHD (Taylor 1974, Pariat et al. 2015); topological invariant
- Emerges via helical magnetic flux tubes and/or is generated by photospheric proper motions
- An isolated configuration with accumulated magnetic helicity cannot relax to a potential field (but to a LFF)
- If not transferred to larger scales it can only be expelled in the form of CMEs (Rust 1994)

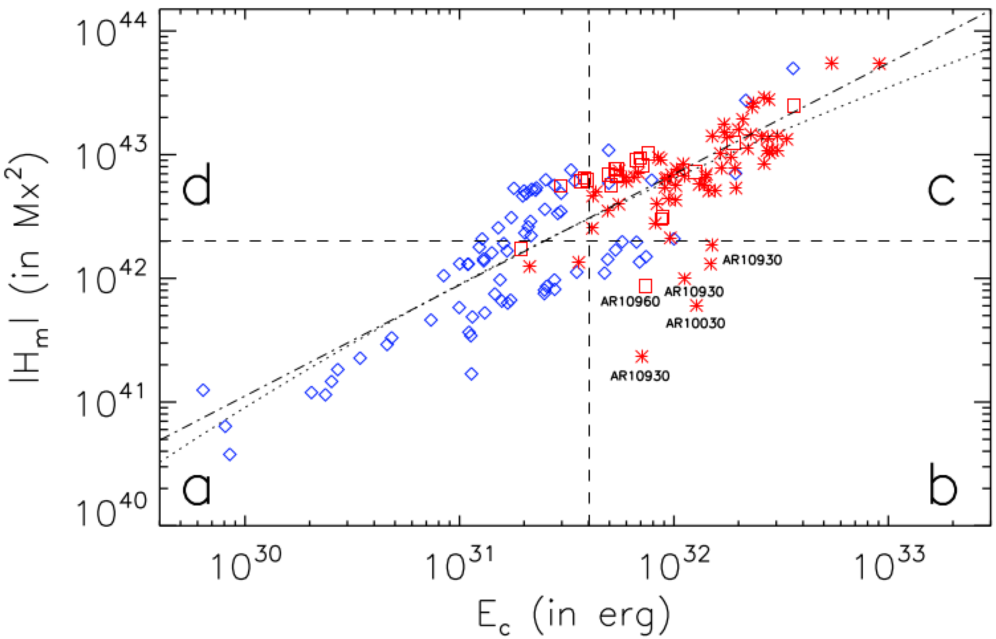


Török & Kliem 2005

Introduction
Magnetic helicity applications



Nindos & Andrews 2004



Tziotziou et al. 2012

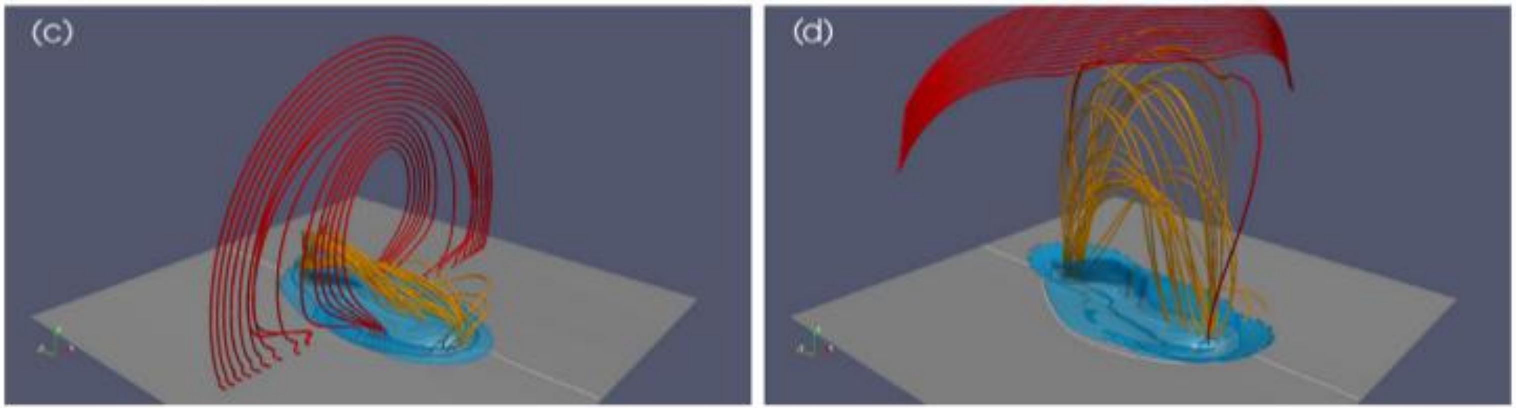
Introduction
Magnetic helicity calculations

- Finite volume methods in cartesian coordinates
- Require ***B*** in 3D volume
- Many methods developed the last years
  - DeVore gauge

    Valori et al. 2012, Moraitis et al. 2014
  - Coulomb gauge

    Thalmann et al. 2011, Yang et al. 2013, Rudenko et al. 2011
- Methods agree (ISSI team “Helicity estimations in models and observations”, Valori et al. 2016)

Introduction
Magnetic helicity calculations

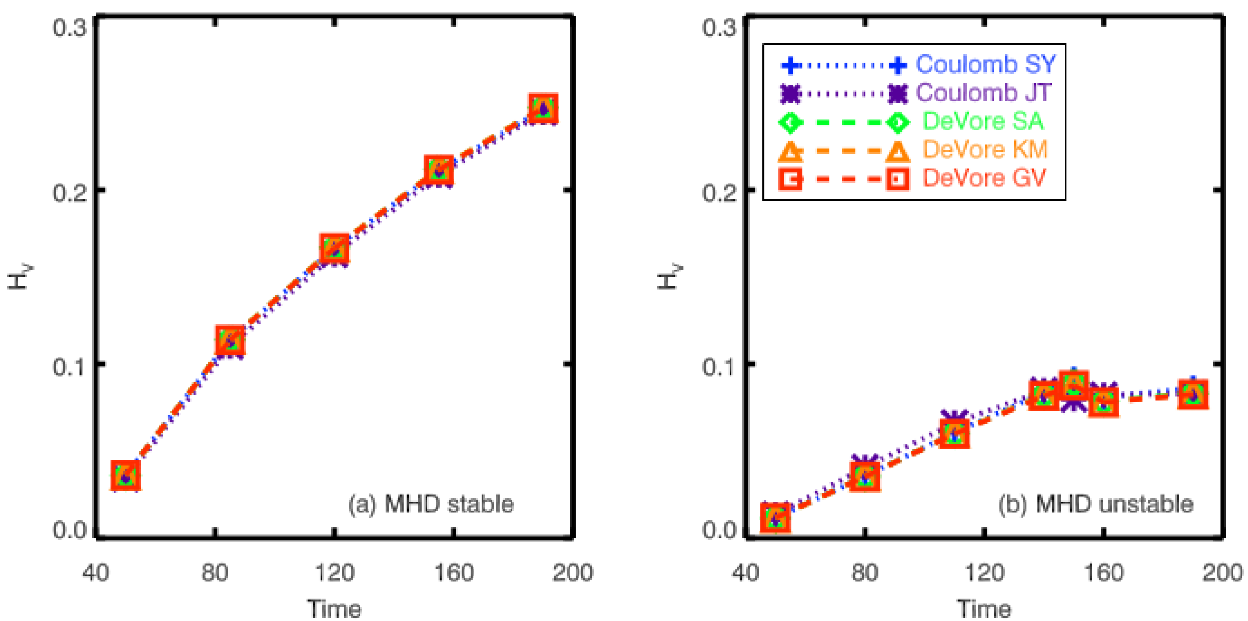


Leake et al. 2013

- Coulomb gauge

Thalmann et al. 2011, Yang et al. 2013, Rudenko et al. 2011

- Metho ... ervations”,
  Valori



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- Methods agree (ISSI team “Helicity estimations in models and observations”, Valori et al. 2016)
- Considering only the photospheric boundary can lead to incorrect helicity values, and even to incorrect sign (Valori et al. 2011)

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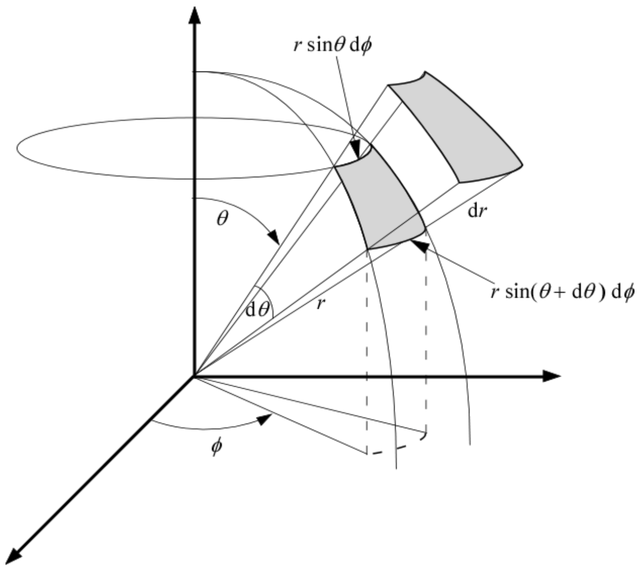
Helicity calculation method
Potential field

$$\mathbf{B}_{\rm p} = \nabla\Phi$$

numerical solution of Laplace's equation $\nabla^2\Phi = 0$



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)+\frac{1}{r^2\sin\theta^2}\frac{\partial^2\Phi}{\partial\phi^2}=0$$

in the spherical wedge

$$V = \{ (r,\theta,\varphi) : r \in [r_{min}, r_{max}], \theta \in [\theta_{min}, \theta_{max}], \varphi \in [\varphi_{min}, \varphi_{max}] \}$$

with Neumann BCs $\left.\frac{\partial\Phi}{\partial\hat{n}}\right|_{\partial V} = \hat{n}\cdot\mathbf{B}|_{\partial V}$

- Multigrid technique (MUDPACK library)
- BVP well defined only for flux-balanced 3D field $\int_{\partial V}\mathbf{B}\cdot d\mathbf{S} = 0$
  (check with 2 flags)
- Current version requires special grid size

Helicity calculation method
Vector potentials

invert $B = \nabla \times A$ for vector potential ***A*** with the method of Valori et al. 2012

(modified) DeVore gauge

$\hat{r} \cdot A$

$$\mathbf{A}(r,\theta,\phi) = \frac{1}{r}\left(r_0 \boldsymbol{\alpha}(\theta,\phi) + \hat{\mathbf{r}} \times \int_{r_0}^{r} \mathrm{d}r'\, r' \mathbf{B}(r',\theta,\phi)\right)$$

$$\nabla_\perp \times \boldsymbol{\alpha} = \frac{1}{r_0 \sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\,\alpha_\phi) - \frac{\partial \alpha_\theta}{\partial\phi}\right) = B_r(r_0,\theta,\phi)$$

DVS gauge

$$\alpha_\phi(\theta,\phi) = \frac{c r_0}{\sin\theta}\int_{\theta_0}^{\theta} \mathrm{d}\theta'\, \sin\theta'\, B_r(r_0,\theta',\phi)$$

$$\alpha_\theta(\theta,\phi) = -(1-c) r_0 \sin\theta \int_{\phi_0}^{\phi} \mathrm{d}\phi'\, B_r(r_0,\theta,\phi')$$

DVC gauge

$$\boldsymbol{\alpha} = \hat{\mathbf{r}} \times \nabla_\perp u = \frac{1}{r_0}\left(-\frac{1}{\sin\theta}\frac{\partial u}{\partial\phi}, \frac{\partial u}{\partial\theta}\right)$$

$$\nabla_\perp \boldsymbol{\alpha} = 0$$

$$\nabla_\perp^2 u = B_r(r_0,\theta,\phi)$$

Amari et al. 2013, Yeates & Hornig 2016

- Formulas valid for divergence-free fields
- Differentiation with 2nd order derivatives, integration with trapezoidal rule
- Top/bottom reference planes give different results - top is usually better
- Same formulas for ***B*** and $\boldsymbol{B_p}$

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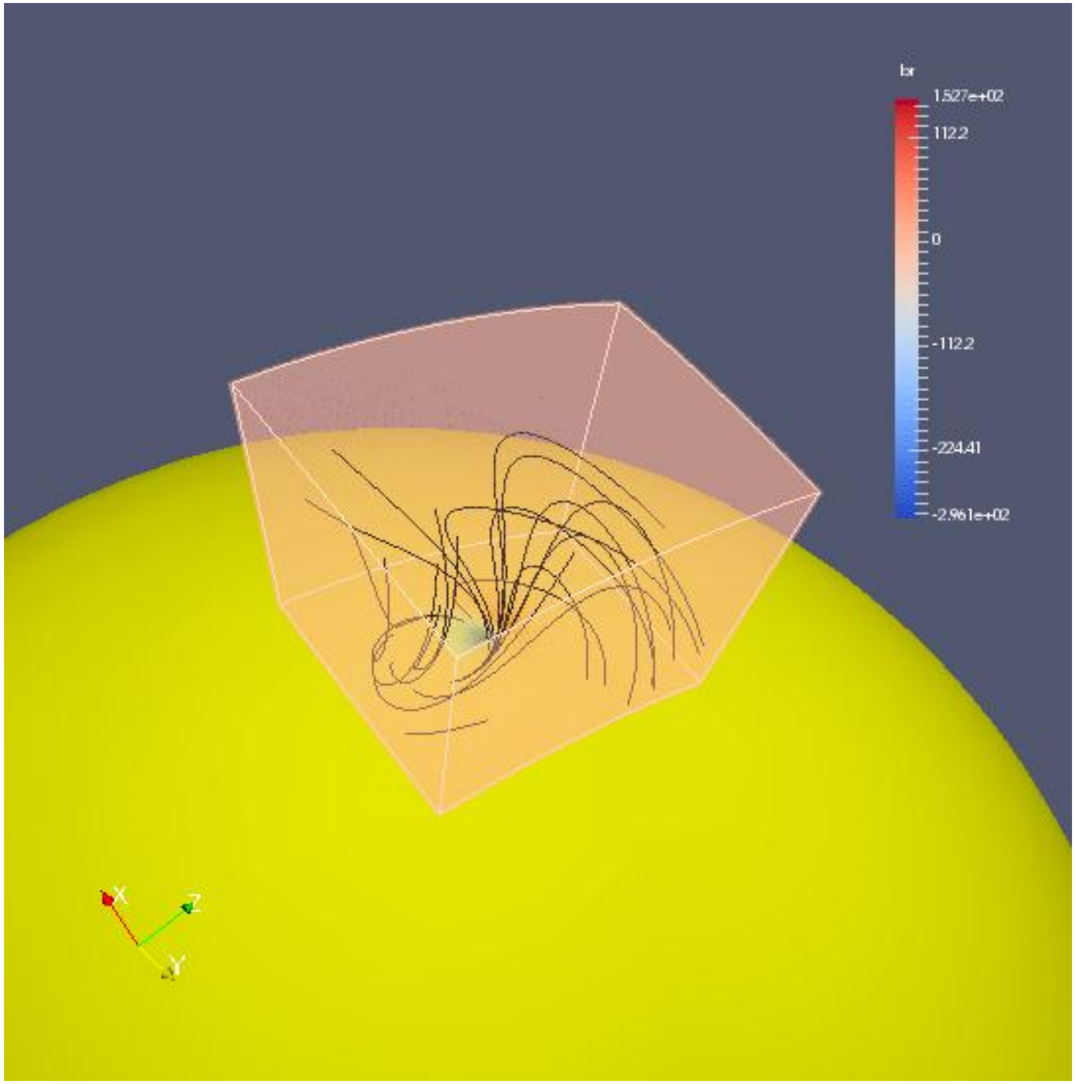
Method validation
Low & Lou field



- semi-analytical, force-free fields of Low & Lou 1990
- LL parameters:
  $n=m=1$, $l=0.3$, $\Phi=\pi/4$
- angular size:
  $20^o x 20^o$ on the Sun, or
  ~200Mm x ~200Mm
- AR height: 200Mm
- resolution:
  129x129x129 grid points
  257x257x257 grid points
- Test for:
  resolution
  reference plane
  solenoidality

Method validation
Low & Lou field – Potential field

field	grid	$\langle\|f_i\|\rangle$	$\epsilon_{\rm flux}$	ξ	E	$E_{\rm c}/E$	$E_{\rm div}/E$	$s_{\max}$
$\mathbf{B}$	129	$2.21\,10^{-4}$	$1.70\,10^{-3}$	$1.99\,10^{-2}$	45.3	0.262	$1.10\,10^{-3}$	$7.9\,10^{-3}$
$\mathbf{B_p}$		$1.15\,10^{-4}$	$1.83\,10^{-3}$	$1.81\,10^{-4}$	33.4			
$\mathbf{B}$	257	$2.16\,10^{-4}$	$2.15\,10^{-3}$	$3.67\,10^{-2}$	45.2	0.261	$2.51\,10^{-3}$	$5.1\,10^{-3}$
$\mathbf{B_p}$		$2.14\,10^{-4}$	$2.23\,10^{-3}$	$3.59\,10^{-4}$	33.4			

- average fractional flux increase
  - (Wheatland et al. 2000)

$$f_i = \frac{\int_{\Delta S_i} \boldsymbol{B} \cdot \boldsymbol{dS}}{\int_{\Delta S_i} |\boldsymbol{B}|\, dS} \approx \frac{(\nabla \cdot \boldsymbol{B})_i \Delta V_i}{B_i A_i}$$

- flux imbalance ratio

$$\epsilon_{\rm flux} = \frac{|\Phi^+ - \Phi^-|}{\Phi^+ + \Phi^-}$$

- average of the Lorentz force relative to its components
  - (Malanushenko et al. 2014)

$$\xi = \frac{1}{N} \sum_{i=1}^{N} \frac{|\mathbf{F}_{\rm L}|}{|\mathbf{F}_{\rm mp}| + |\mathbf{F}_{\rm mt}|}$$

$$\mathbf{F}_{\rm L} = -\frac{c}{8\pi} \nabla_\perp B^2 + \frac{c}{4\pi} B^2 \frac{d\hat{\mathbf{b}}}{db} \equiv \mathbf{F}_{\rm mp} + \mathbf{F}_{\rm mt}$$

- free energy

$$E_c = \frac{1}{8\pi} \int dV\, B^2 - \frac{1}{8\pi} \int dV\, B_p^2$$

- 'divergence' energy

$$E_{divg} = |E_c - E_c'|$$

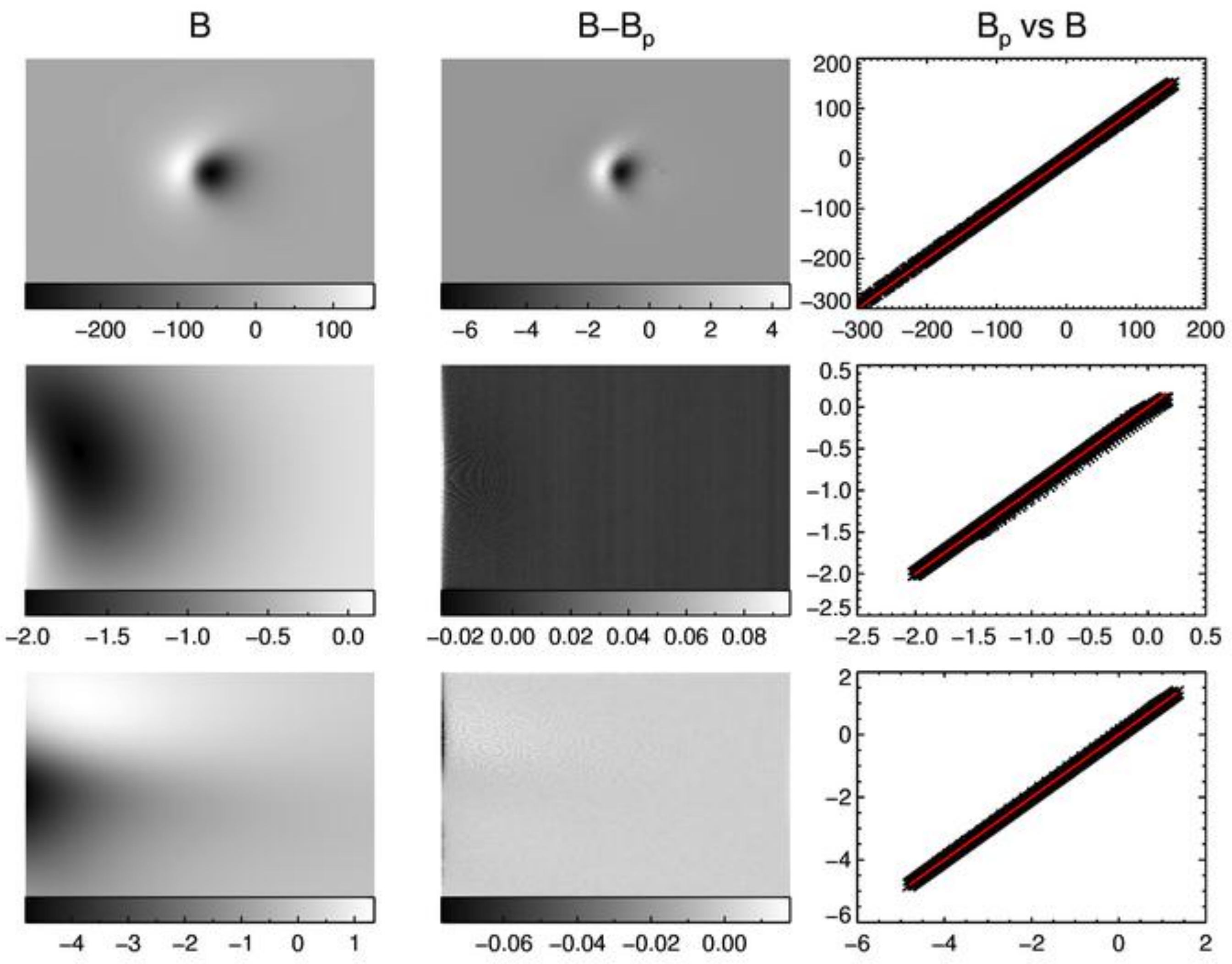
$$E_c' = \frac{1}{8\pi} \int dV \left(B - B_p\right)^2$$

Method validation
Low & Lou field – Potential field

B_r @ $r=r_{min}$

B_θ @ $\theta=\theta_{min}$

B_φ @ $\varphi=\varphi_{min}$



Method validation
Low & Lou field – Vector potential

			correlation coefficients of $\mathbf{B}$ vs $\nabla \times \mathbf{A}$			Schrijver metrics				
field	gauge	grid	B_r	B_θ	B_ϕ	C_{vec}	C_{CS}	E'_{n}	E'_{m}	ϵ
B	DVSt	129	0.9999	1.0000	1.0000	0.9999	1.0000	0.9948	0.9959	0.9980
	DVSt	257	0.9999	1.0000	1.0000	0.9999	1.0000	0.9942	0.9949	0.9986
	DVSb	129	0.9990	1.0000	1.0000	0.9995	0.9986	0.9814	0.9613	1.0025
	DVCt	129	0.9999	1.0000	1.0000	0.9999	0.9999	0.9947	0.9953	0.9980
$\mathbf{B_p}$	DVSt	129	1.0000	1.0000	1.0000	1.0000	0.9997	0.9884	0.9816	0.9998
	DVSt	257	0.9995	1.0000	1.0000	0.9997	0.9962	0.9568	0.9283	0.9993
	DVSb	129	0.9990	1.0000	1.0000	0.9995	0.9920	0.9727	0.9441	0.9901
	DVCt	129	1.0000	1.0000	1.0000	1.0000	0.9997	0.9883	0.9814	0.9998

- Successful reconstruction
- Top better than bottom
- DVS equivalent to DVC
- Small differences with resolution
- Metrics for $\boldsymbol{B}_{\mathbf{p}}$ worse than $\boldsymbol{B}$

C_{vec} vector correlation
C_{CS} Cauchy-Schwarz
E_n' complement of the normalized vector error
E_m' complement of the mean vector error
ϵ total energy normalized to that of the input field
Schrijver et al. 2006

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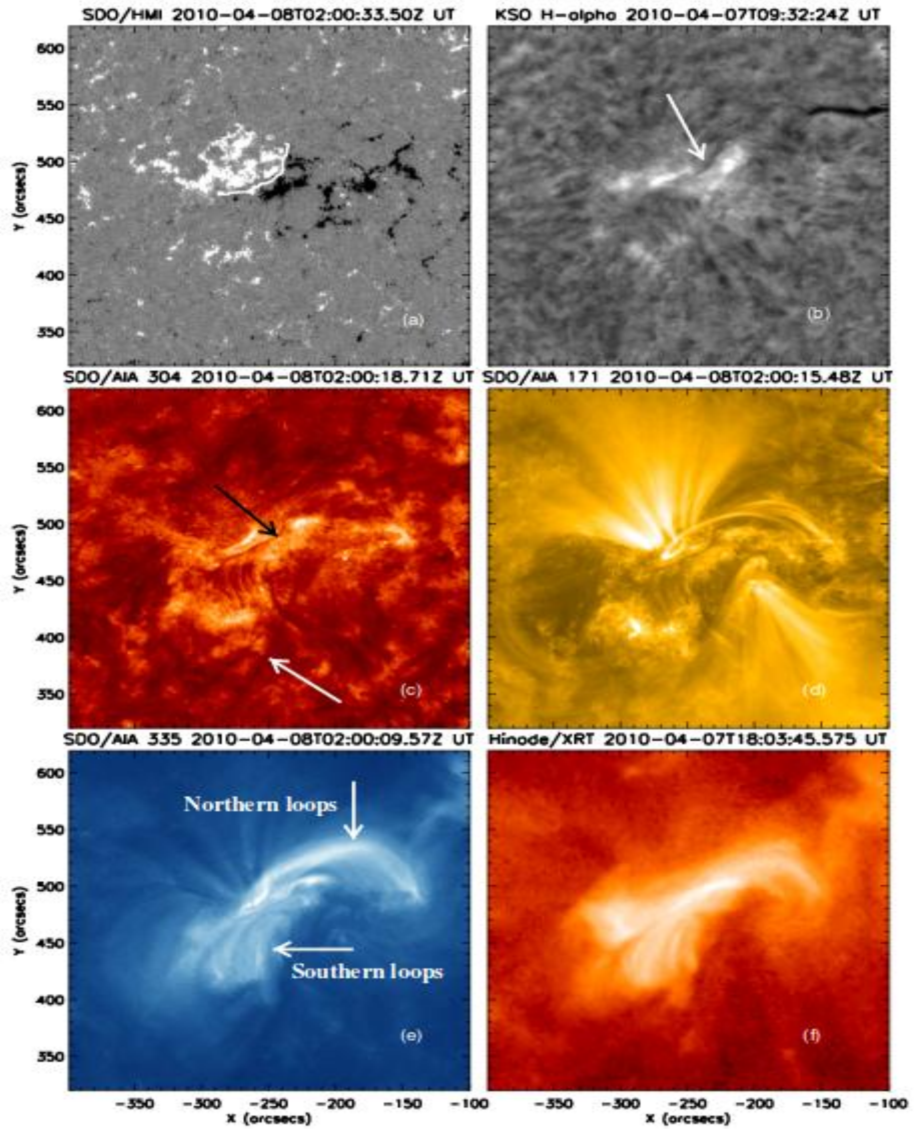
Application to NLFF fields

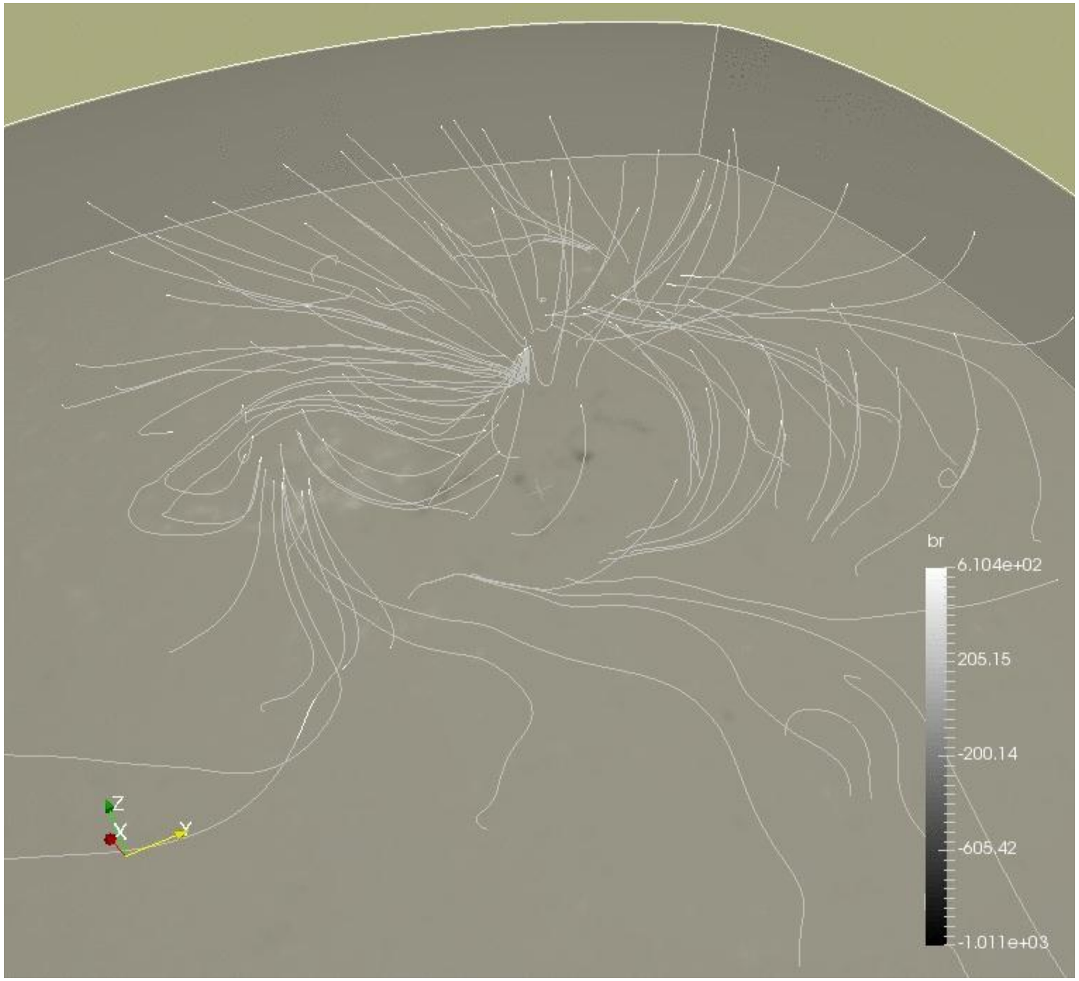


NOAA AR 11060 (SOL2010-04-08)
location: N25E16
activity: B3.7 flare (8 Apr 2010 02:30 UT)

data-driven NLFF modelling:
Su et al. 2011,
Savcheva et al. 2016

- SDO/HMI LOS magnetogram
- insert flux-rope along selected filament
- relax to force-free state with magneto-frictional method
- compare pre-flare coronal loops with disk-projected field lines

Application to NLFF fields



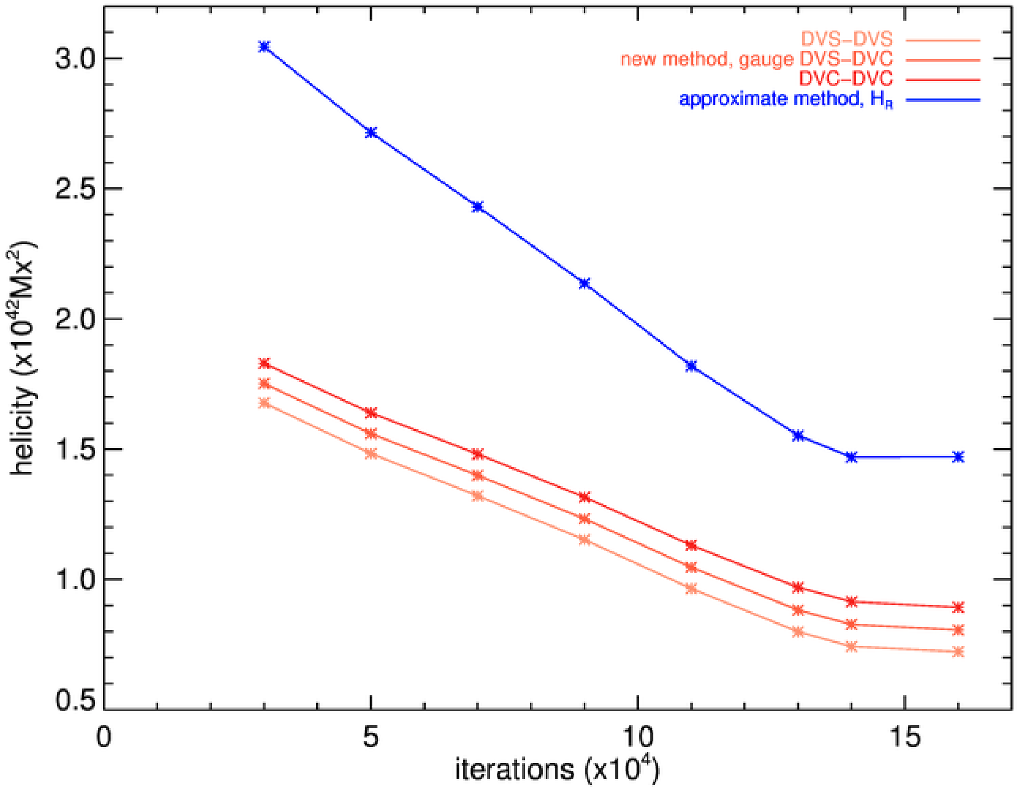
Compute instantaneous helicity during relaxation with:

- New, exact method
- Approximate method introduced in Bobra et al. 2008

$$H_R = \int_{V_c} \left(\boldsymbol{A} \cdot \boldsymbol{B} - \boldsymbol{A}_p \cdot \boldsymbol{B}_p\right) dV + \int_S \chi B_r \, dS$$

$$\boldsymbol{A}_1 - \boldsymbol{A}_2 = \nabla \chi$$

Application to NLFF fields



- Helicity relaxes to constant value
- Results for the 3 gauges very close
- Approximate method similar pattern, but up to 2 off

Conclusions

- Helicity is an important quantity in solar applications
- Helicity computations in spherical geometry were lacking, but needed
- We developed the first consistent helicity calculation method in spherical coordinates
- Testing indicates that method is working
- Preliminary results from application to NLFF fields show improvement on helicity wrt approximate methods + the importance of taking into account all boundaries