FLARECAST Science Workshop
Meudon, France, June 29th 2017

Relative magnetic helicity as a diagnostic of solar eruptivity

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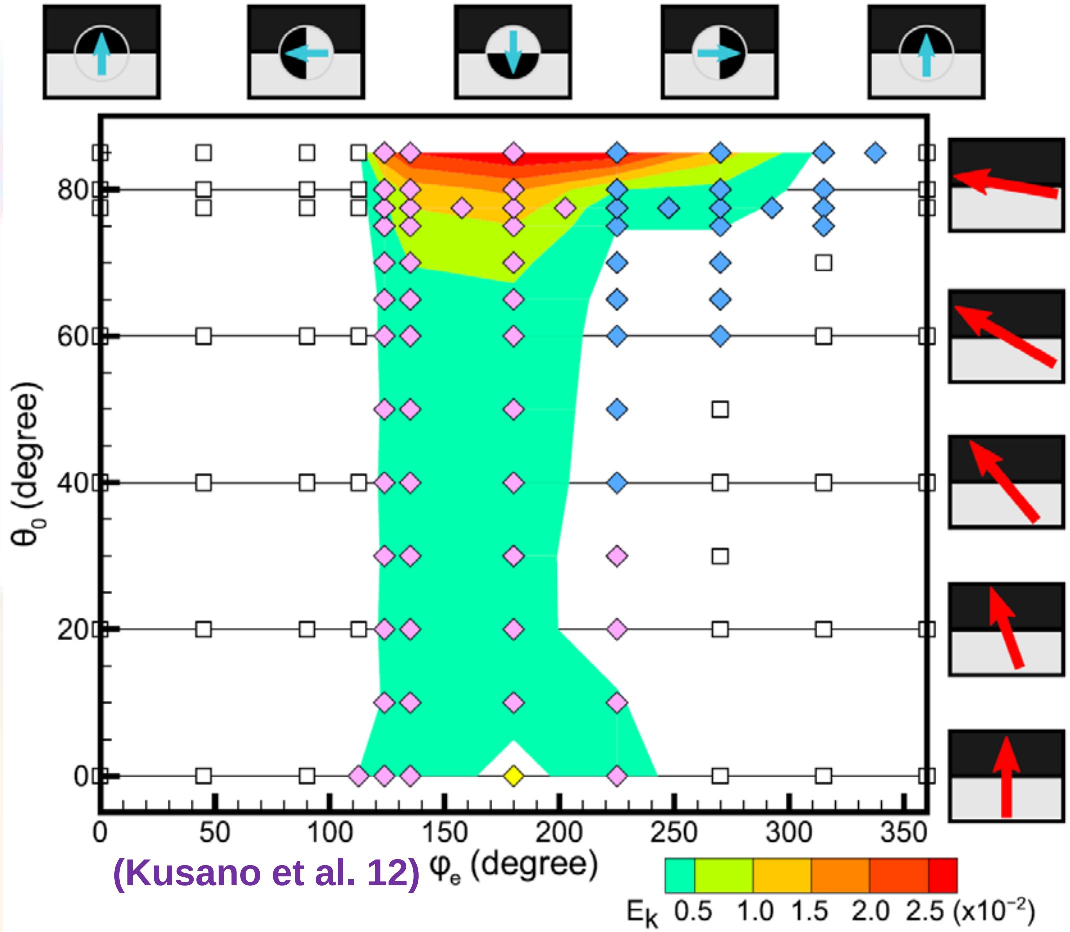
Eruptivity prediction & numerical modeling

- **Search for eruptivity criterion is almost exclusively based on observational datasets …**
- **… and barely benefits from the recent tremendous improvements in numerical modeling .**

- **Kusano et al. 2012**: parametric analysis based on relative orientation of large scale sheared polarity and small scale
  - Eruptivity criterion not directly/easily measurable in observed cases
  - Limited utility in an (automated) forecasting method



(Kusano et al. 12)

- Useful numerical models must present several cases either eruptive or stable (ideally > 2 simulations), depending on few number of parameters

Outline

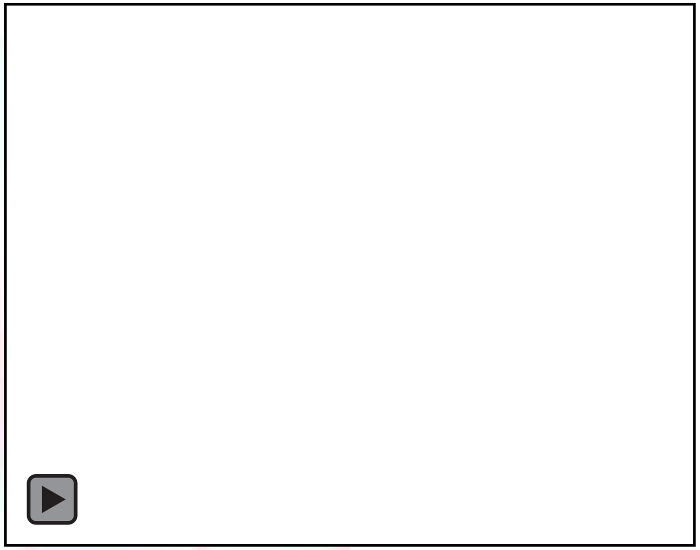
- **Flux emergence model: Leake et al. 2013 & 2014**

- Eruptivity criterion analysis
  - Magnetic flux & energy-based quantities
  - Magnetic-helicity-based quantities
    - Relative magnetic helicity
    - Current-carrying magnetic helicity

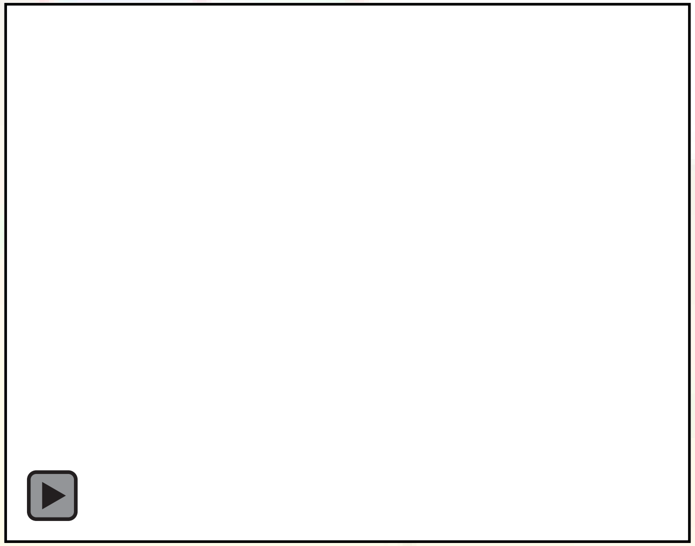
- Other models & Conclusions

Motivations & Methodology

- **Goal: use flux emergence simulations to look for efficient eruptivity criterion**
  - **Leake et al. 2013 and Leake et al. 2014**:
    - **7 flux emergence simulations**
    - 3D visco-resistive MHD eq. solved with Lagrangian-remap code **(Arber et al. 2001)**
    - **lead to eruptive and non-eruptive cases**
    - **varying only an unique initial parameter**

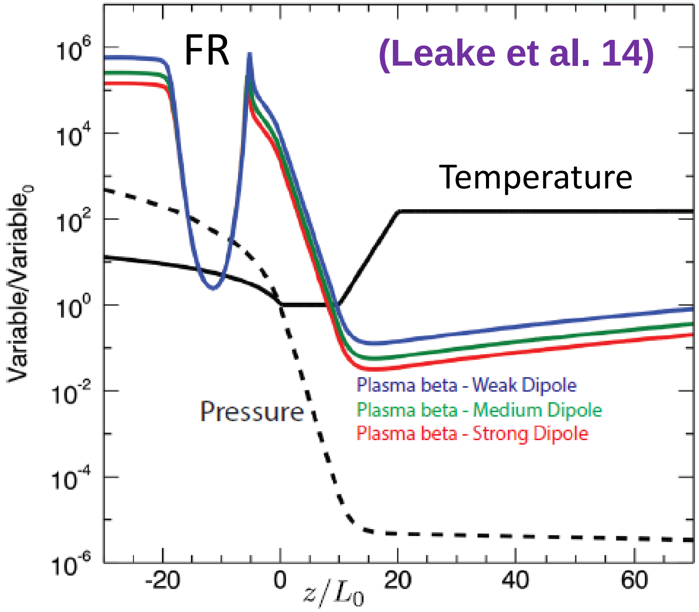


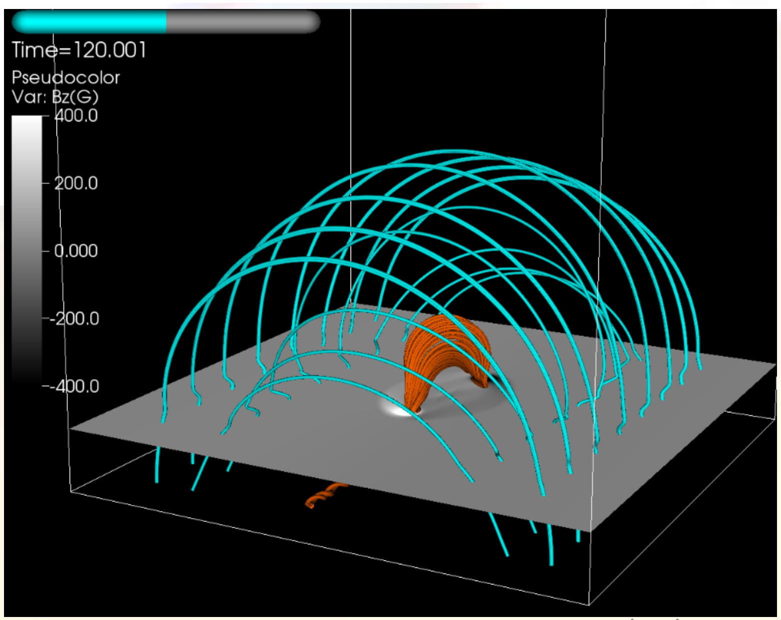
(Leake et al. 13, 14)



- Methodology: - extract part of the magnetic field,
  - compute different physical quantities,
  - search for the ones that discriminates between the eruptive and non-eruptive case
- **Guennou et al. 2017**: Analyze of the
  <u>2D photospheric magnetic field</u>
  - ➔ Cf. Talk by C. Guennou
- **This talk : Pariat et al. 2017:** Analyze of the
  <u>3D coronal magnetic field B(z>0)</u>

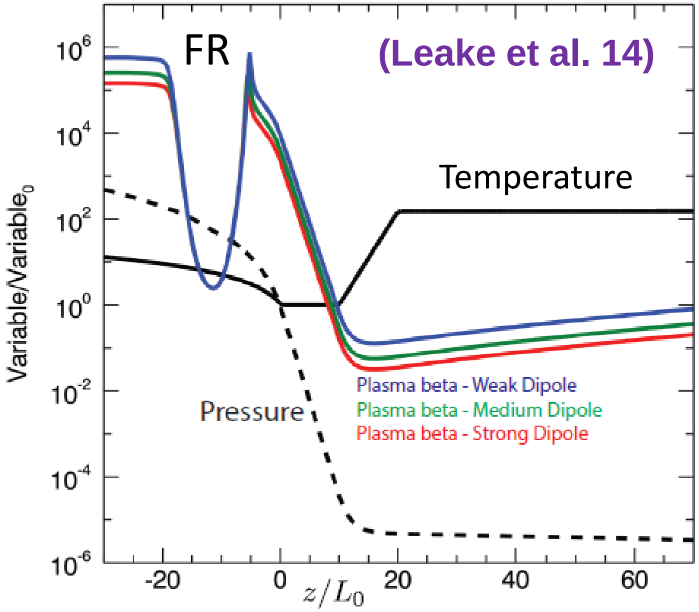
Parametric flux emergence simulations





- Twisted FR emerge in coronal arcade field
- **Emerging twisted flux rope: identical in all cases**
- **Overlying arcade field: 1 param. ➔ 7 cases**
  - Signed strength, Bd, of the surrounding arcade magnetic field

Parametric flux emergence simulations



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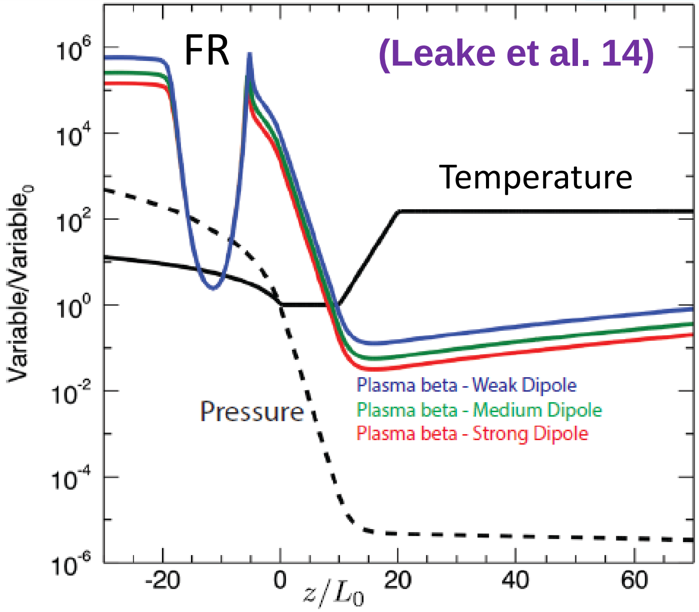
  - Bd=0: no surrounding field
    - → stable flux rope in the corona
    - No eruption
  - Bd>0: same orientation of arcade field and azimuthal part of emerging field: interaction of // fields
    - → formation of stable flux rope
    - No eruption

Parametric flux emergence simulations



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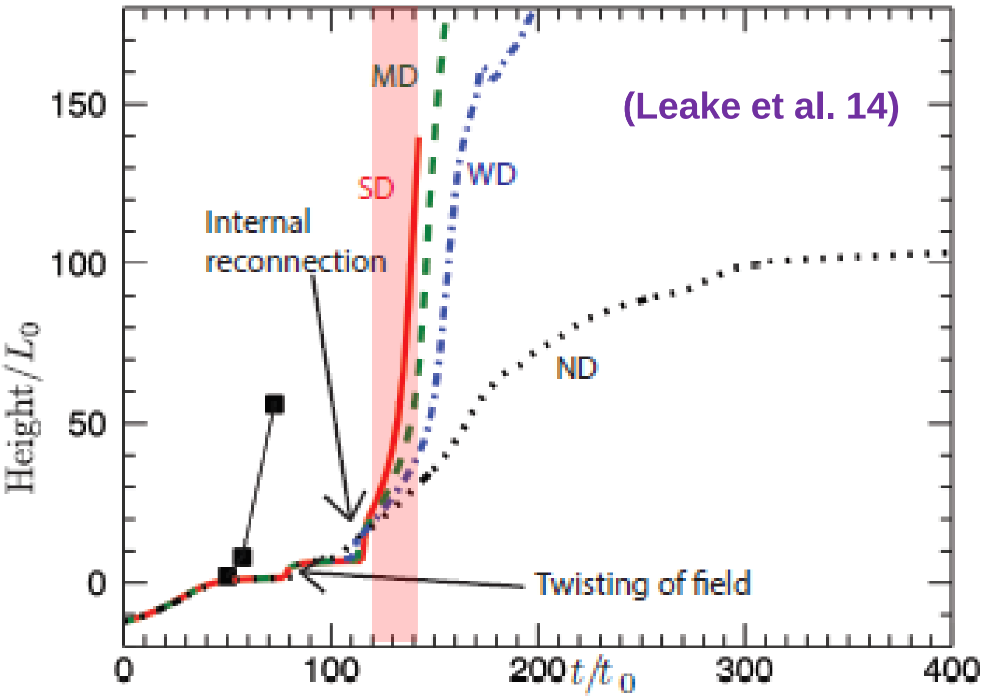
  - Bd>0: same orientation of arcade field and azimuthal part of emerging field: interaction of // fields
    - → formation of stable flux rope
    - No eruption
  - Bd<0: opposite orientation of arcade field and azimuthal part of emerging field: interaction of anti-// fields
    - → reconnection and formation of unstable flux rope
    - Eruptive behavior

Stability of the system

- **Emerging twisted flux rope: identical in all cases**
- **Overlying arcade field: 1 parameter ➔ 7 cases**

Label	No Erupt SD	No Erupt MD	No Erupt WD	No Erupt ND	Erupt WD	Erupt MD	Erupt SD
B_d	10	7.5	5	0	−5	−7.5	−10
Dipole Strength	Strong	Medium	Weak	Null	Weak	Medium	Strong
Eruption	No	No	No	No	Yes	Yes	Yes

- Eruptive simulations:
  - Onset at t ~ 120 t_0
  - With stronger dipole strength eruption occurs earlier
  - No precise determination of the system instability time.

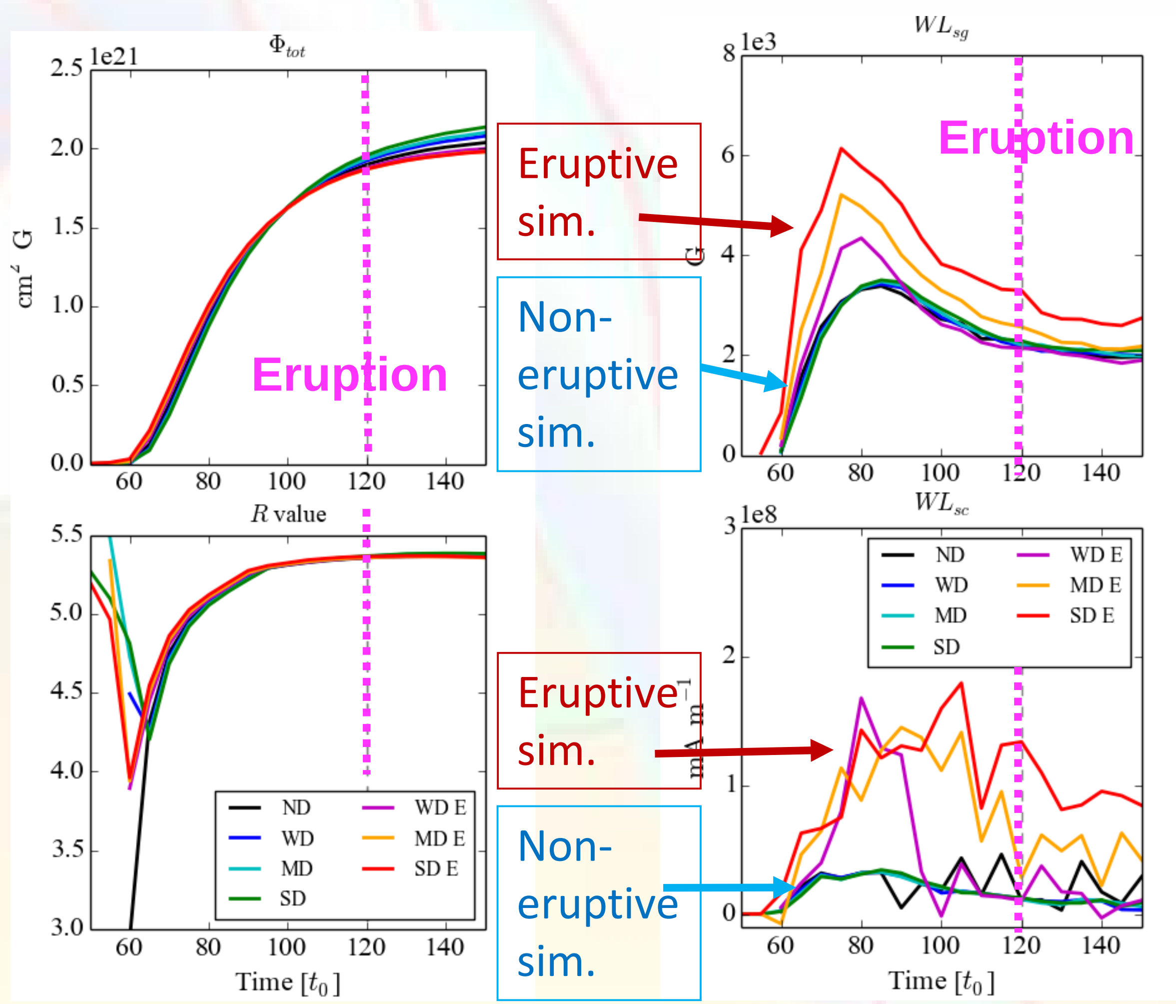


- Non-eruptive simulation:
  - stable at least until 400 t_0
  - appears completely relaxed
  - Not expected to erupt even after 400 t_0

Search for eruptivity criterion

- **Goal: search for eruptivity indicators from 3D coronal magnetic datacube**
- Good eruptivity criterion should:
  - Discriminate eruptive and non-eruptive sim. during pre-eruptive phase
  - Reach its highest value
    - for eruptive simulation only,
    - during the pre-eruptive phase only.
  - Present similar trend for eruptive and non-eruptive sim. in post-eruptive phase

Useless Criteria **Pertinent Criteria**



(Guennou et al. 17)

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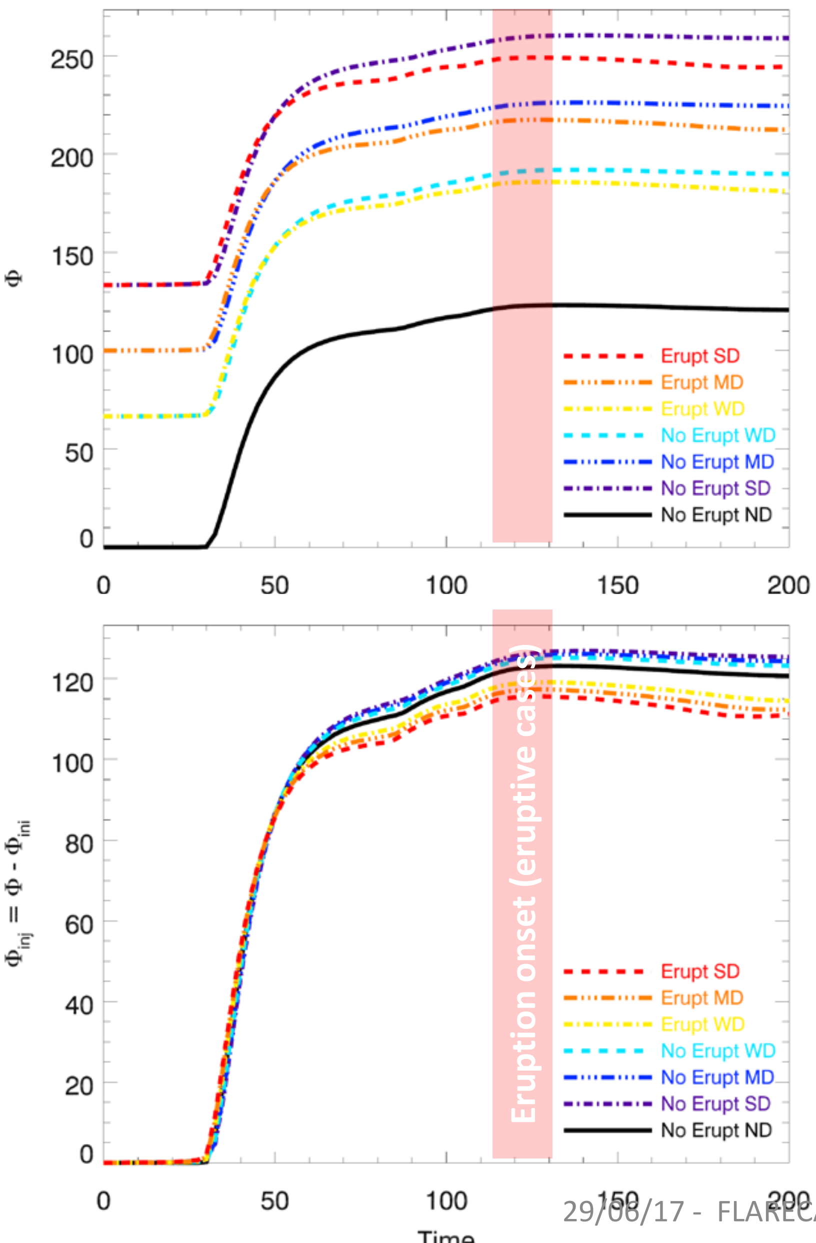
Magnetic fluxes

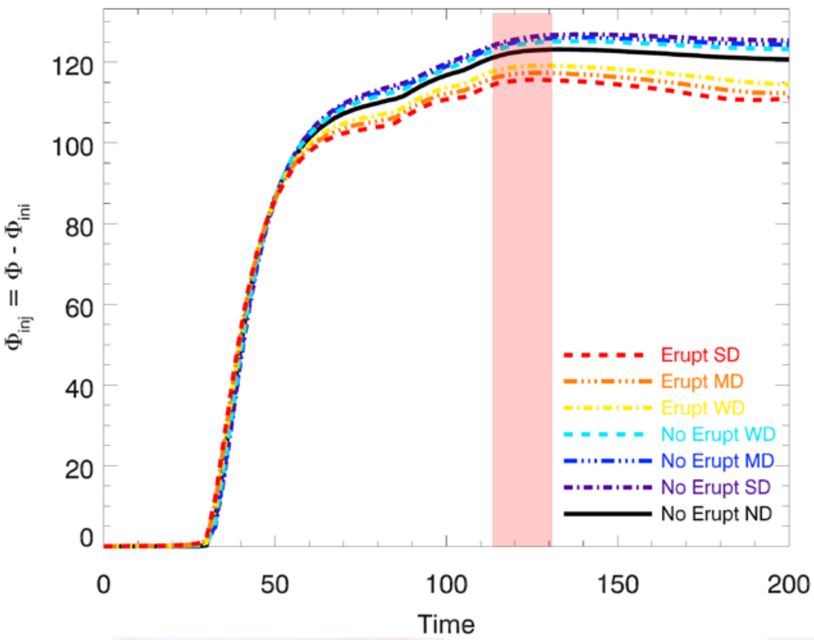
(Pariat et al. 17)



- Reference magnetic flux depends on the arcade field strength
- **Injected flux by emerging flux rope is roughly identical for all 7 simulations**

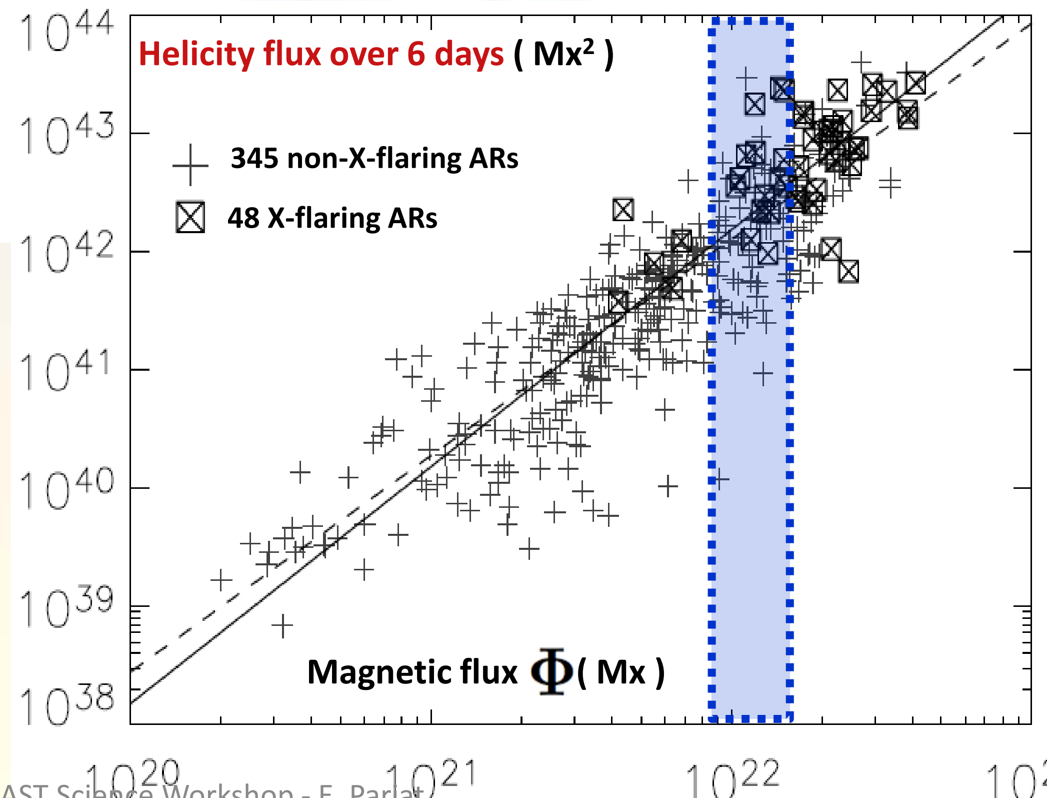
Magnetic fluxes

(Pariat et al. 17)



- Reference magnetic flux depends on the arcade field strength
- **Injected flux by emerging flux rope is roughly identical for all 7 simulations**

(Labonté et al. 07)



- Limits of the model: ARs have
  - Ratio of injected/surrounding magnetic fluxes ~ 1.
  - roughly constant injected mag. flux.
- **➔ determining why active regions with a given magnetic flux erupt and others do not.**

Potential & Non Potential

- **For a given distribution of a magnetic field on the boundary of a domain, there is an unique decomposition of the magnetic field in potential and non-potential field.** $\boldsymbol{B} = \boldsymbol{B}_{\mathrm{p}} + \boldsymbol{B}_{\mathrm{J}}$
- Potential field: $\boldsymbol{B}_{\mathrm{p}} = \boldsymbol{\nabla}\phi,$ with $\hat{\boldsymbol{n}} \cdot (\boldsymbol{B} - \boldsymbol{B}_{\mathrm{p}})|_{\partial\mathcal{V}} = 0,$
  - the potential field has the same distribution than the studied field on the whole boundary
- Non-potential field: $\nabla \times \boldsymbol{B}_j = \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}\,;$
  - The non potential field "carry" all the electric current of the studied field.
- Thomson theorem: $E_{mag} = E_{pot} + E_{free}$
  - Total magnetic energy is the sum of the mag. energy of the potential field and the "free" magnetic energy (mag. energy of the non-potential field)
- Observationally based assumption: during an eruption, the magnetic field distribution does not change ➔ Bp and Epot do not change

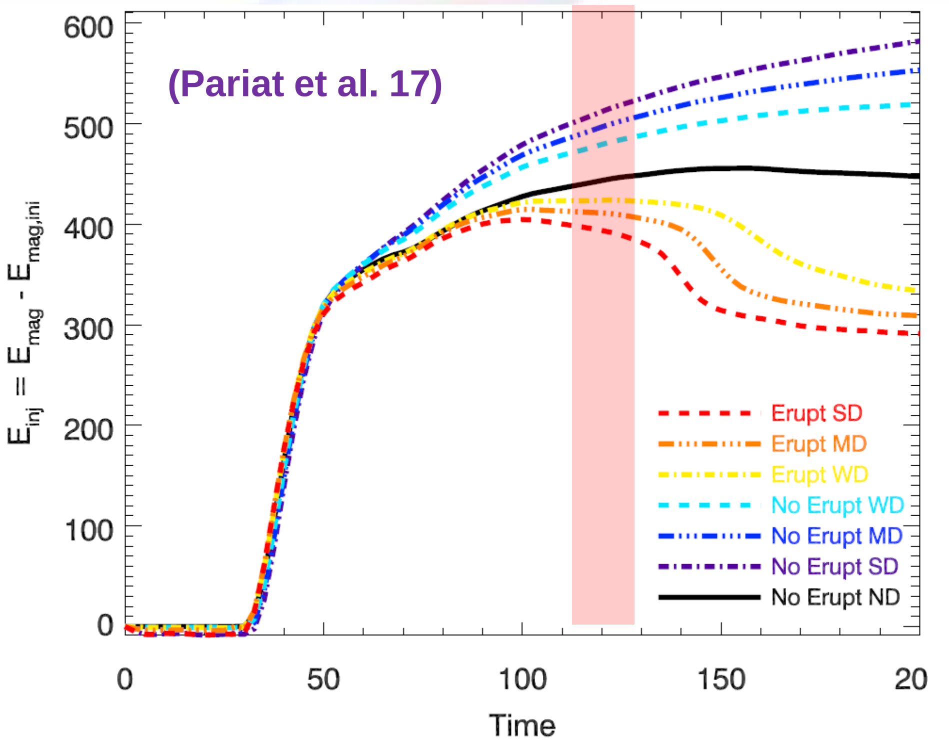
➔ **the energy source of an eruption is the free magnetic energy**

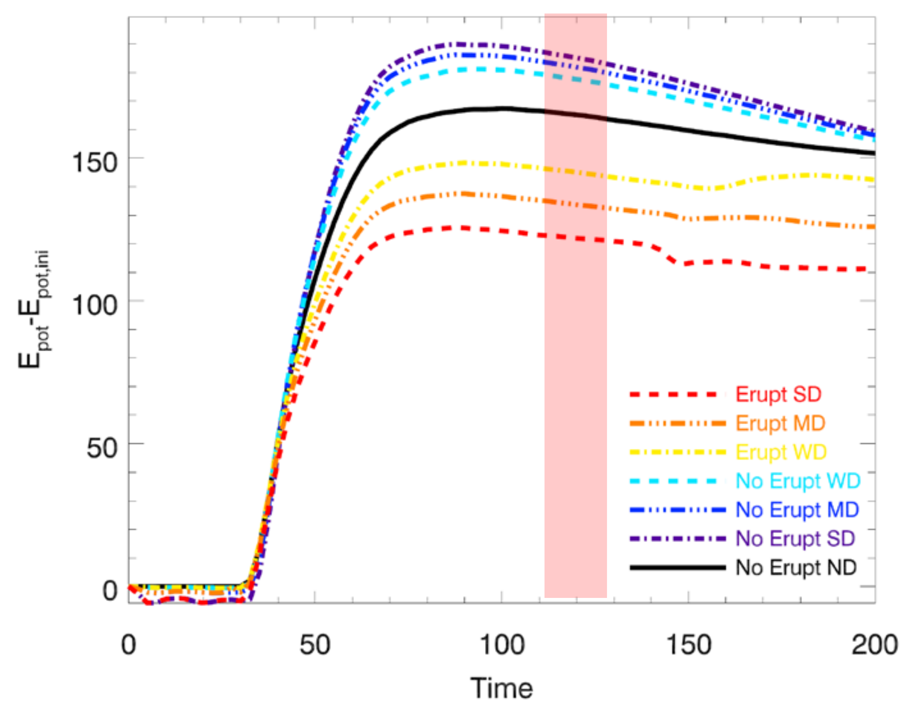
Total and potential magnetic energy

$$E_{mag} = E_{pot} + E_{free}(+ E_{ns}),$$

- Eruptive simulation have a lower injection of total magnetic energy and potential magnetic energy.

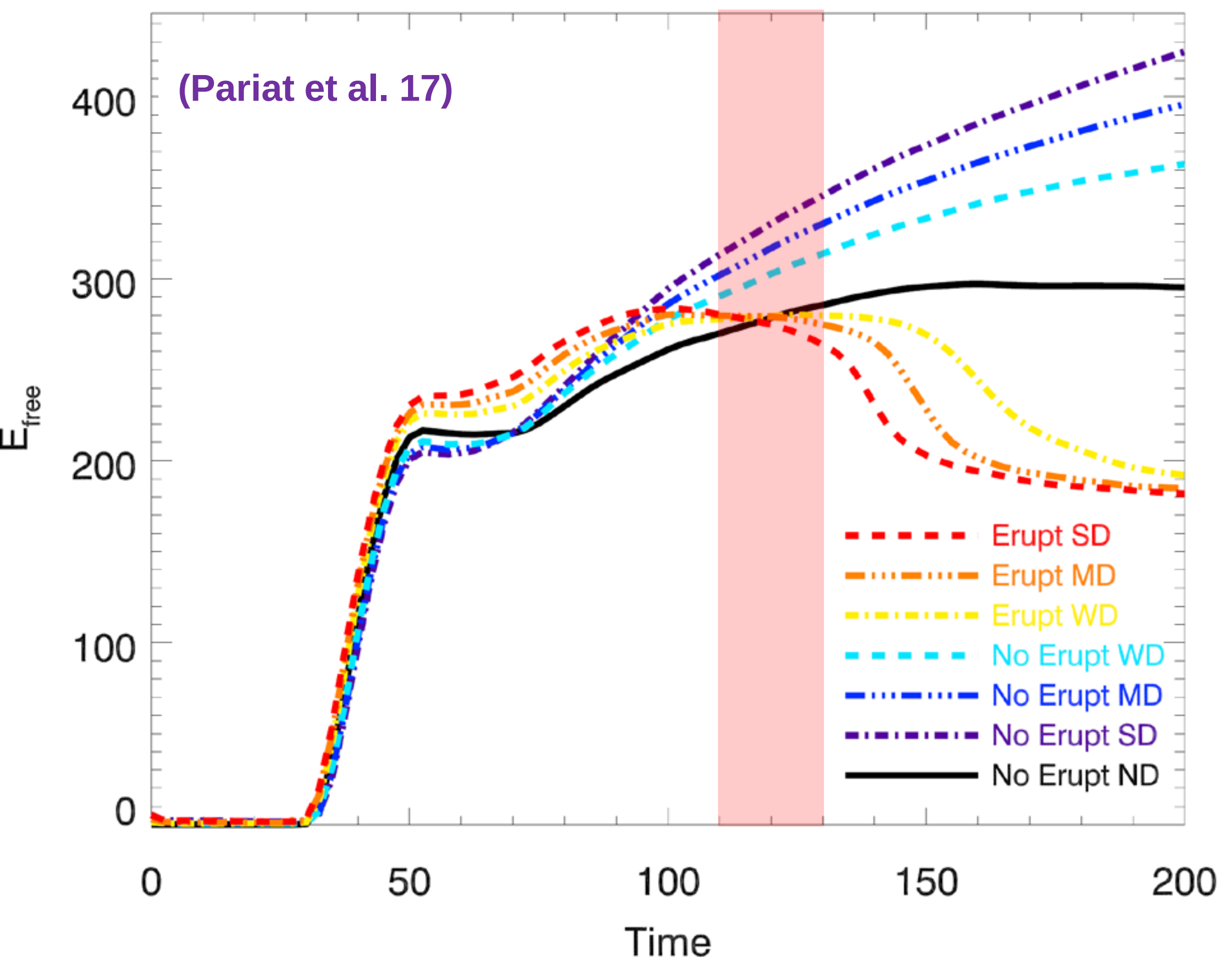
- **Both total and potential magnetic energies are not good indictors of the eruptivity of the system**





Free magnetic energy

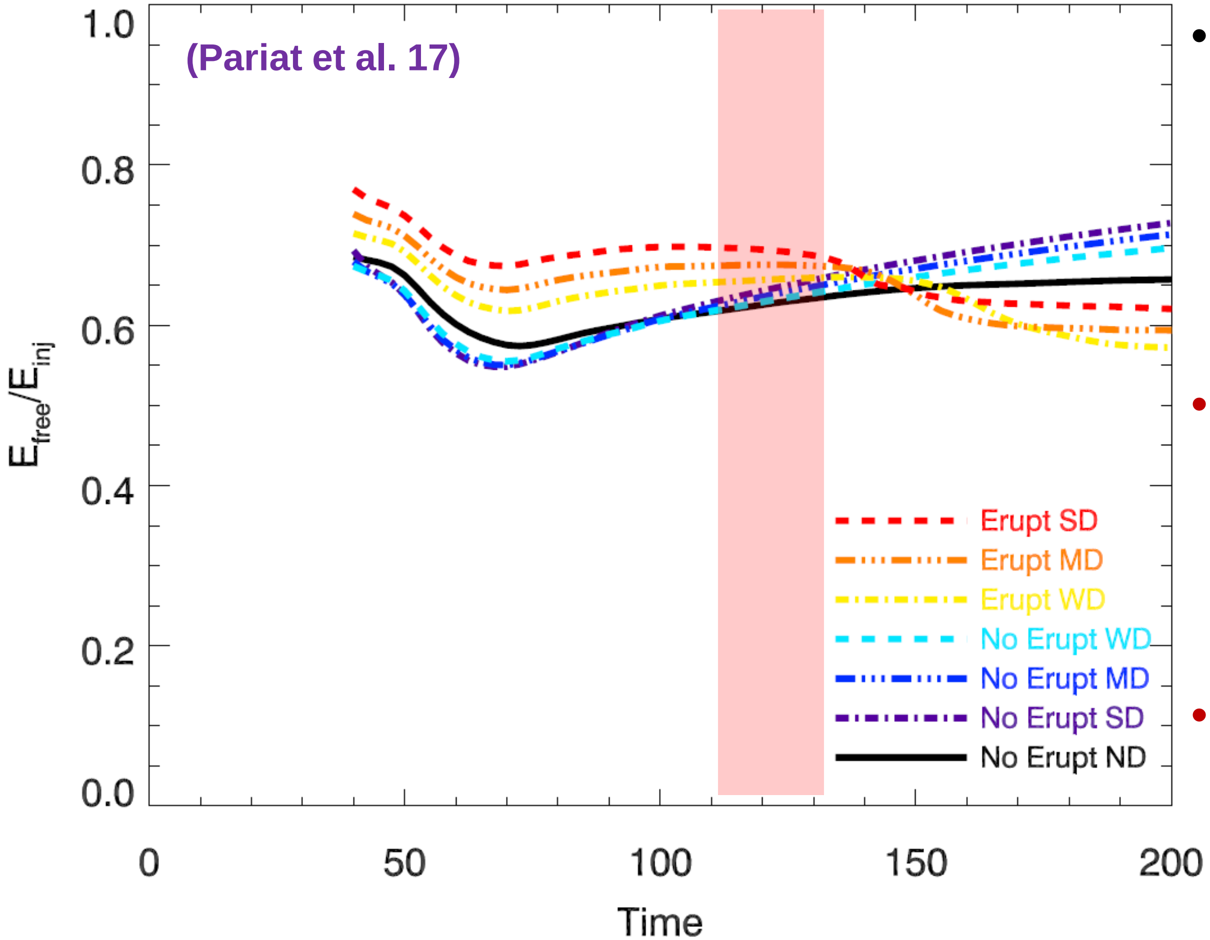
$$E_{mag} = E_{pot} + E_{free} (+ E_{ns}).$$



- Free energy is slightly higher for eruptive simulation in the pre-eruption phase.
- However highest value of E_{free} are reached by non-eruptive simulations.
- **Free magnetic energy is not a good indicator of the eruptivity state of the system**

Free magnetic energy ratio

$$E_{mag} = E_{pot} + E_{free}(+ E_{ns}),$$



- E_{free}/E_{inj} is higher for eruptive simulation vs. non eruptive in the pre-eruption phase with marginally the highest values
- **Ratio of free magnetic energy to injected energy may be a proxy of eruptivity of the system**
- **However, E_{free}/E_{inj} not strongly discriminative: maximum value for eruptive flare are only marginally above non-eruptive ones.**

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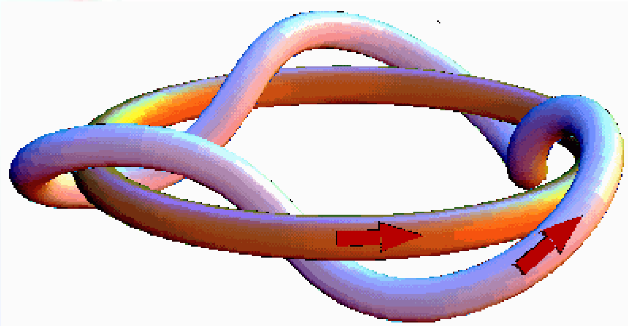
Definition of Magnetic Helicity

- **Helicity of the magnetic field in MHD plasmas**
  (Elsasser 56)

$$H = \int_{\mathcal{V}} \vec{A} \cdot \vec{B} \, \mathrm{d}V \quad , \quad \vec{B} = \vec{\nabla} \times \vec{A}$$ ← Magnetic vector potential

  – Unique signed scalar value for volume considered



(Berger 00)

- **Magnetic helicity: signed level of knotedness and twist of magnetic field lines**
  – Magnetic flux weighted **Gauss Linking Number** of pairs of magnetic field lines (Moffatt 1968)

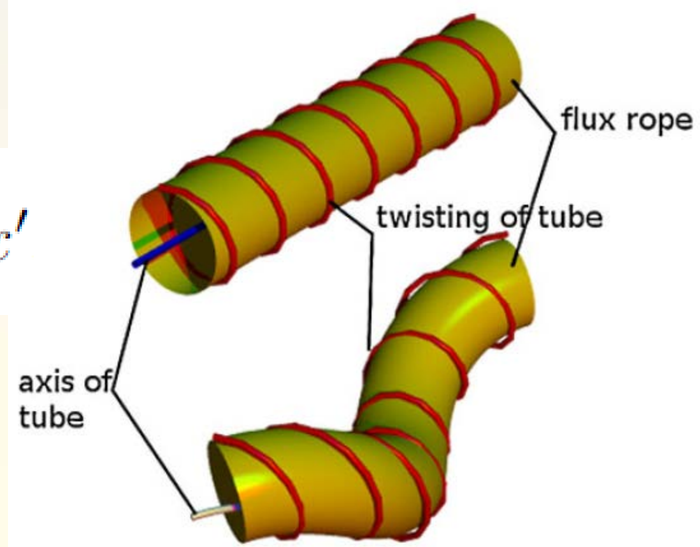
$$L_{12} = -\frac{1}{4\pi} \oint_1 \oint_2 \frac{d\mathbf{x}}{d\sigma} \cdot \frac{\mathbf{r}}{r^3} \times \frac{d\mathbf{y}}{d\tau} \, d\tau \, d\sigma$$

$$H = -\frac{1}{4\pi} \int \int \mathbf{B}(\mathbf{x}) \cdot \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{x}') \, d^3x \, d^3x'$$

  – For a uniformly twisted flux tube

$$H = N \, \Phi_{ax}^2$$

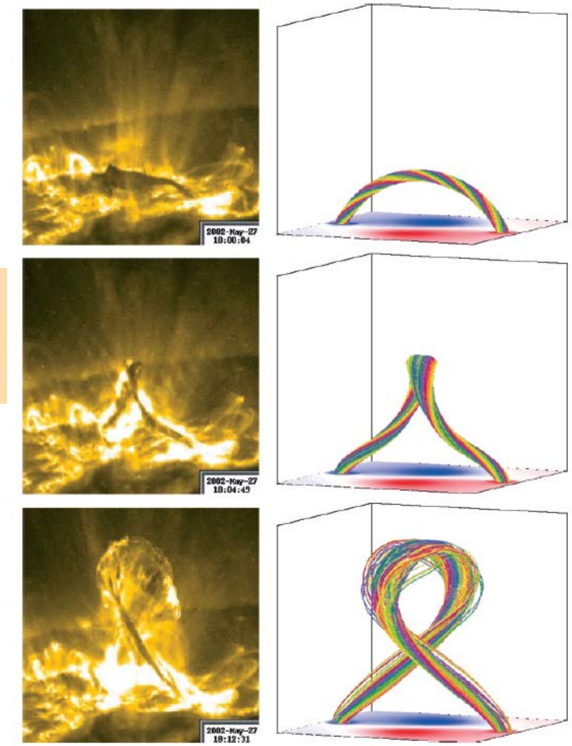
N:nbr of turns, Φ_{ax}: axial flux



(Prior & Berger 12)

Magnetic helicity properties

(Török et al. 05)



- **Magnetic helicity is an ideal MHD invariant. For E⊥B: no dissipation ➔ magnetic helicity is conserved (Woltjer 1958).**

Time variations **Surface Flux** **Dissipation**

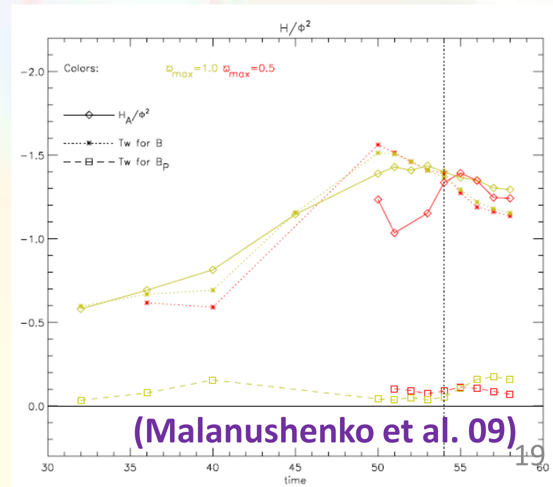
$$\frac{dH_m}{dt} = \int_{\partial\mathcal{V}} \left(\mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t}\right) \cdot \mathrm{d}\mathbf{S} - 2\int_{\partial\mathcal{V}} (\mathbf{E} \times \mathbf{A}) \cdot \mathrm{d}\mathbf{S} - 2\int_{\mathcal{V}} \mathbf{E} \cdot \mathbf{B} \;\mathrm{d}\mathcal{V}$$

- **Taylor 1974: hypothesis helicity conservation true even in non-ideal MHD**
  - **Pariat et al. 16 : verified for a solar like active event**

- Magnetic helicity bounds the system E distribution:

$$\mu_0 \hat{E}(k) > k \hat{H}(k)$$ **(Frisch et al. 1975)**

- **Inverse helicity cascade:** Helicity goes from small to large spatial scales. **(Frisch et al. 1975, Alexakis et al. 2006)**
  - e.g. kink instability **(Malanushenko et al. 2009)**

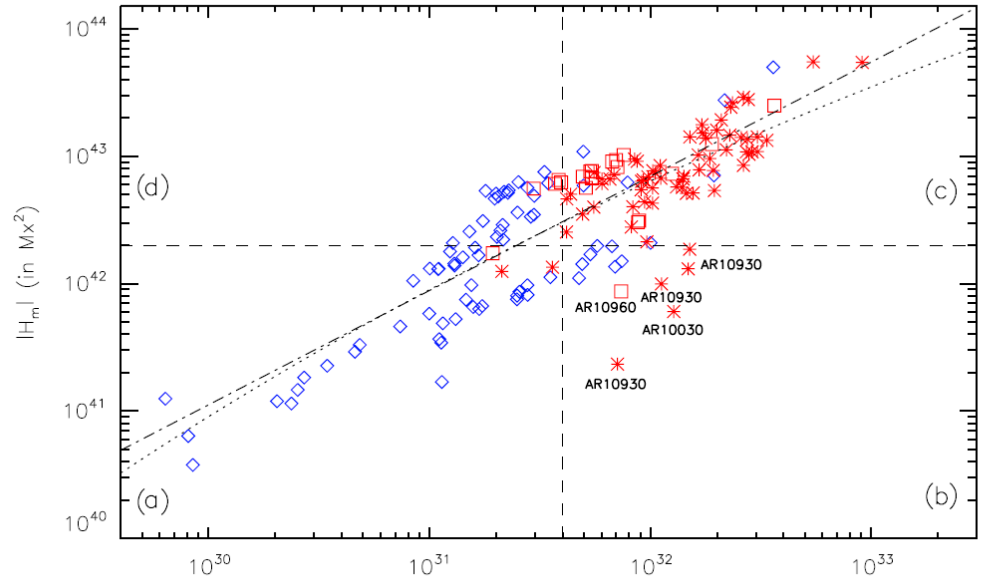


(Malanushenko et al. 09)

- Impact on dynamic of magnetic reconnection:
  e.g. Linton et al. 2001, Del Soro et al. 2010

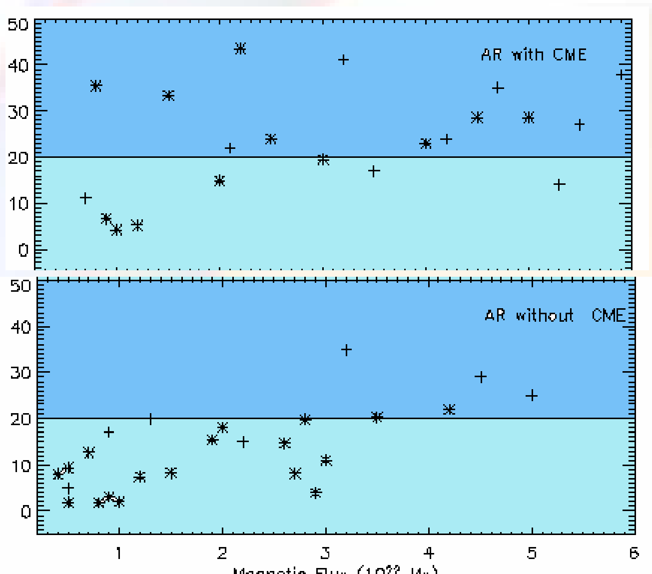
Helicity and solar eruption

- Helicity conservation could be the “raison d’être” of coronal mass ejections **(Rust 94, Low 96).**
- Several observational studies have shown diverse indications that magnetic helicity can be tightly linked with enhanced eruptivity: **(Nindos et al. 04, Labonte et al. 07, Park et al. 08, 10, Tziotziou et al. 12)**

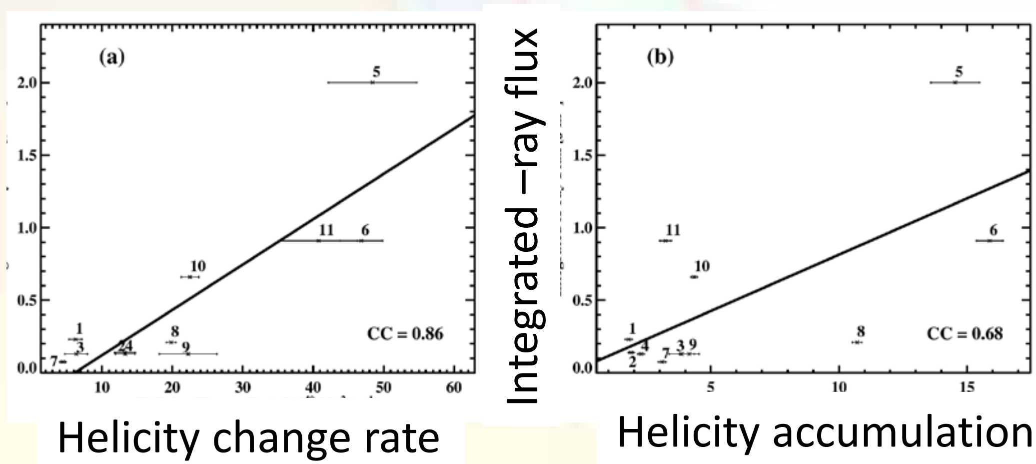


(Tziotziou et al. 12)

Total Coronal Helicity (10^{42} Mx^2)



(Nindos 04)



(Park et al. 08)

Gauge invariance of magnetic helicity

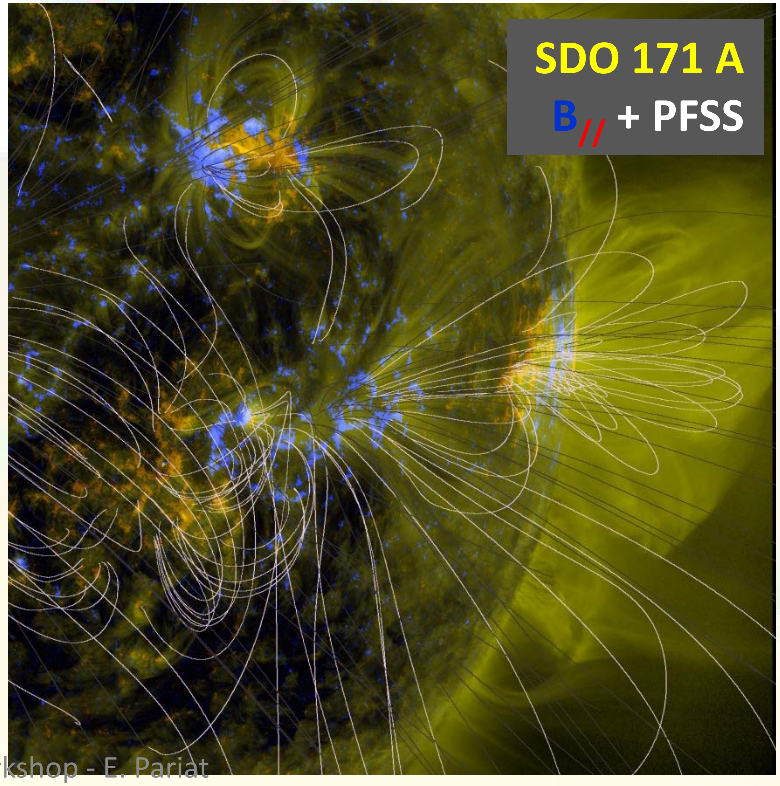
- Gauge transformation of magnetic helicity: $H = \int_{\mathcal{V}} \vec{A} \cdot \vec{B} \, \mathrm{d}V$

$$\vec{A}' \mapsto \vec{A} + \vec{\nabla}\varphi \qquad H_{A'} = H_A + \oint_S \varphi \vec{B} \cdot \overrightarrow{\mathrm{dS}}$$

- Magnetic helicity is gauge invariant only for magnetically bounded systems:

$$\mathbf{B} \cdot \mathbf{dS} \big|_S = 0$$

- Strict definition of magnetic helicity useless for numerous applications:
  - e.g. natural plasmas, like the solar corona have boundaries threaded by magnetic fields



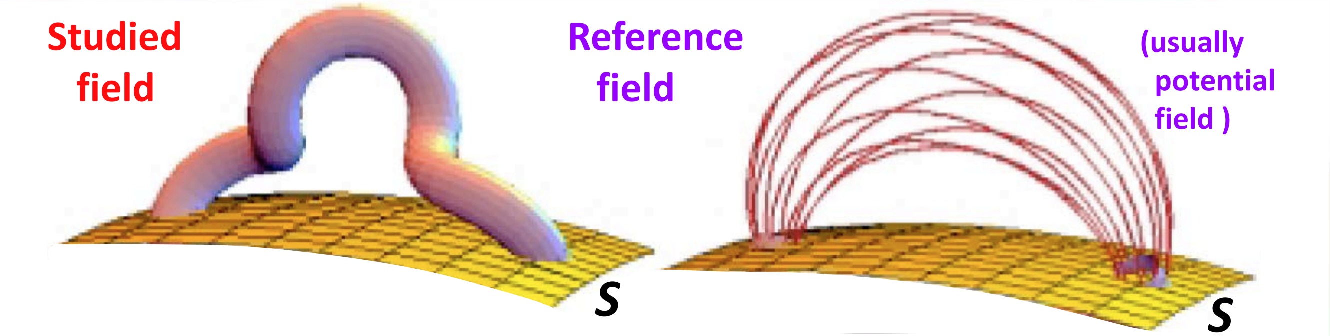
Relative Magnetic Helicity

➔ Useful quantity: **Relative Magnetic Helicity:** helicity of a studied field relative to a reference field **(Berger 84, Finn & Antonsen 85).**

$$H_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\ \mathrm{d}\mathcal{V}$$ **(Finn & Antonsen 85)**

with boundary condition : $(\mathbf{B}_{\mathrm{p}} \cdot \mathrm{d}\mathbf{S})\,|_{\partial\mathcal{V}} = (\mathbf{B} \cdot \mathrm{d}\mathbf{S})\,|_{\partial\mathcal{V}}$ $\qquad \nabla \times \mathbf{A} = \mathbf{B}$

- **Gauge invariant provided that studied and reference fields share the same magnetic-flux distribution <u>on the whole boundary</u>.**



Relative Magnetic Helicity Estimations

- The computation of relative magnetic helicity is not straightforward:
  - **Computation of reference field must be done imposing boundary conditions on the whole domain boundary.**
  - Many previous methods assumed semi-infinite volumes while all existing datasets are bounded volumes: could lead to incorrect results **(Valori et al. 2011, 2012)**, error in intensity, even in sign!

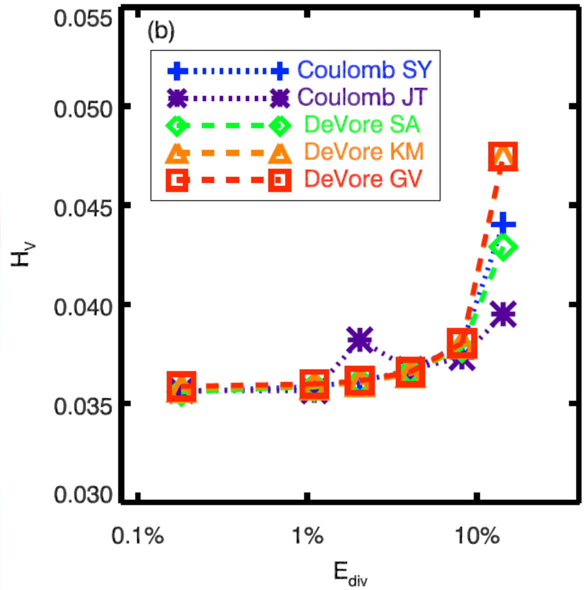
- **Several methods recently developed on 3D cuboid system (Valori et al. 2016)**
  - Using Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

    Thalmann et al. 2011, **Rudenko & Myshyakov 2011, Yang et al. 2013**
    - Simpler theoretical formulation
    - Harder to implement numerically
  - Using DeVore gauge **(DeVore et al. 2000)** : $A_z = 0$

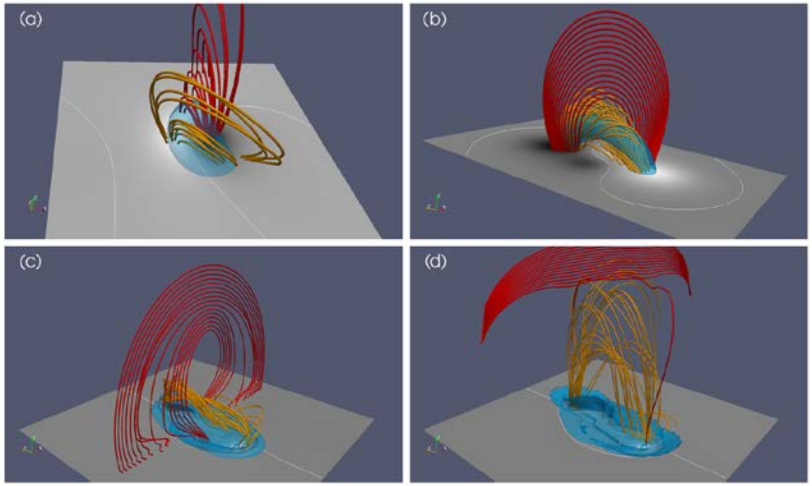
    Valori, Démoulin & Pariat 2012, Moraitis et al. 2014
    - More complex theoretical formulation
    - Simpler to implement numerically: more precise
- **New method to compute relative magnetic helicity in spherical wedge domains. (Moraitis et al. in prep.)**

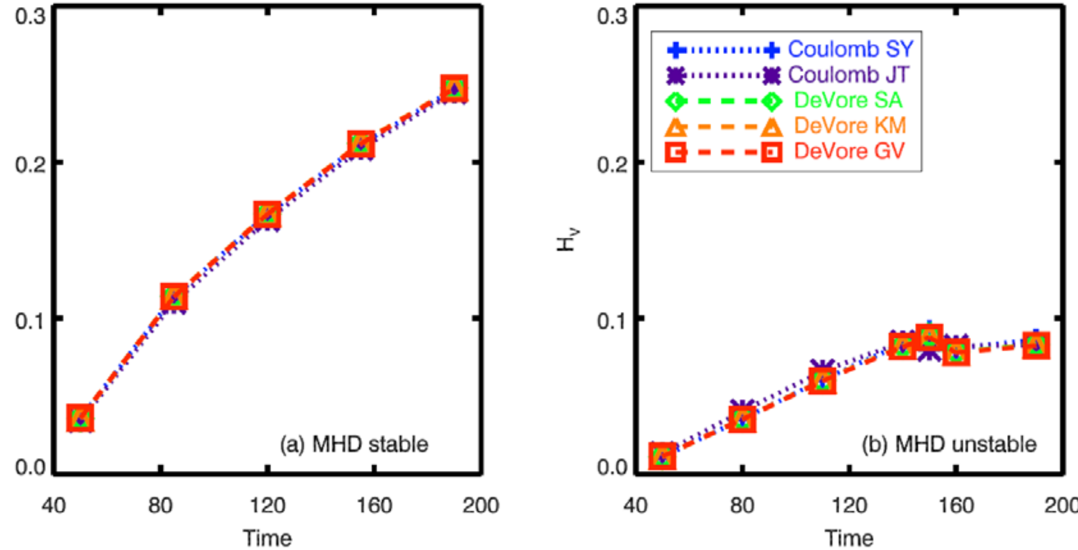
Relative magnetic helicity estimations

- **Benchmarking of these methods performed by ISSI team on "Helicity estimations in models and observations“: Valori et al. 2016**
- Numerous tests: sensibility to resolution, twist, solenoidality using various types of data.
  - Force free fields **(Low & Lou 1990)**
  - Stable flux rope (**Titov & Démoulin 1999**, data fromT. Török)
  - Flux emergence simulations **(Leake et al. 2013, 2014)**
- **Methods perform very consistently when B sufficiently solenoidal**



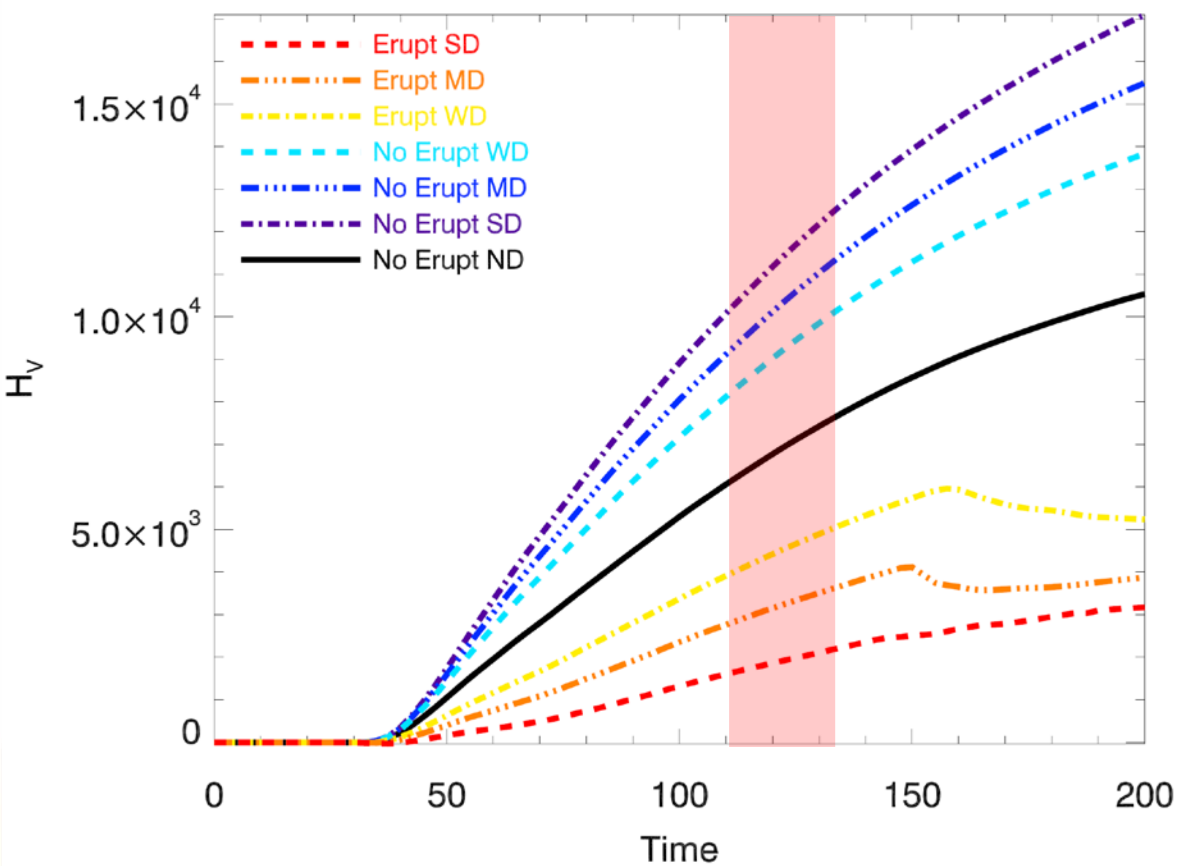
(Valori et al. 16)





Relative magnetic helicity evolution

(Pariat et al. 17)



- **Unlike with magnetic flux & free energy, helicity discriminates strongly the cases**
  - Total helicity depends
    - on dipole strength
    - on dipole orientation
- The surrounding (potential) field influences the helicity content!
- **Magnetic helicity is a non-local quantity!**

- **Unlike what is commonly believed/expected, large total helicity is not a sufficient condition of eruptivity.**

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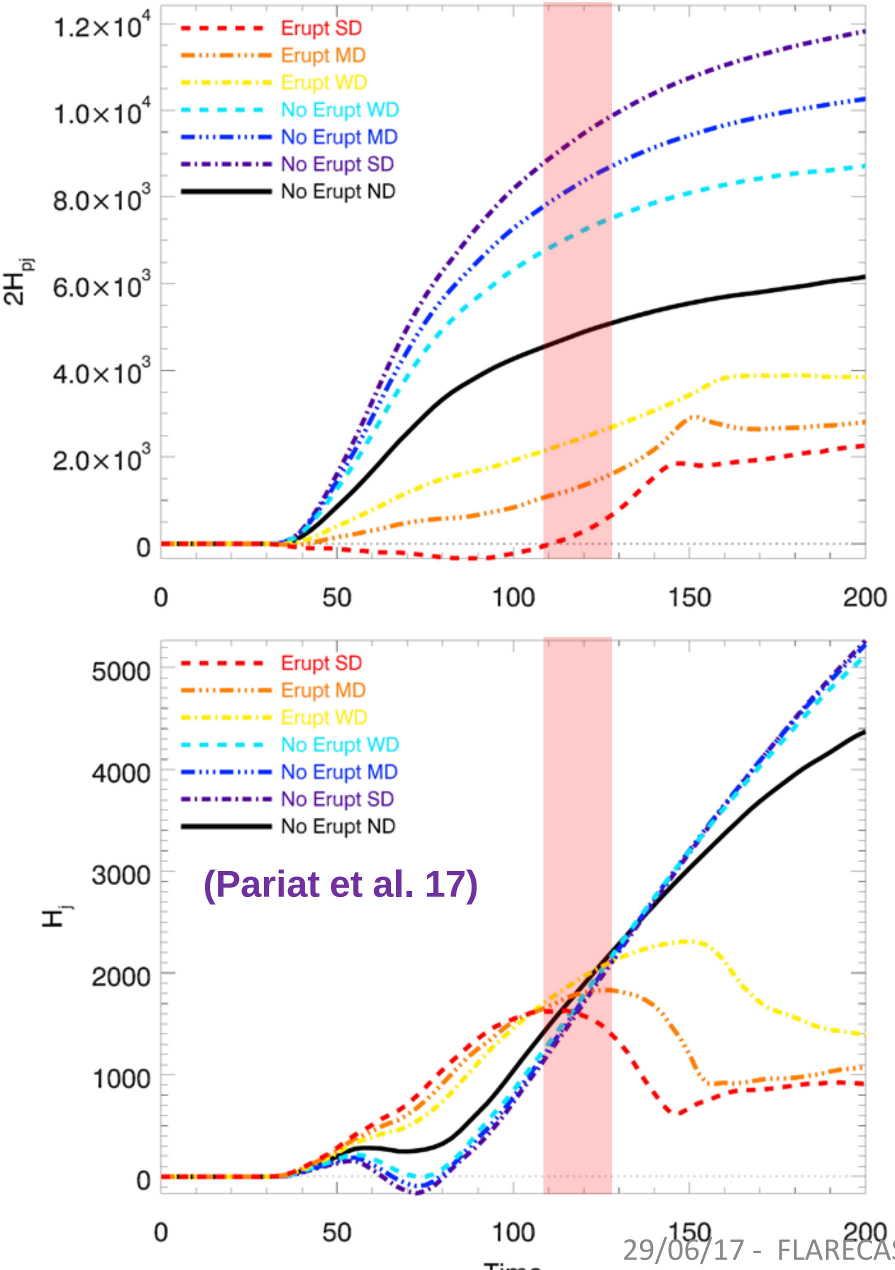
Relative magnetic helicity decomposition

$$\begin{aligned} H_V &= H_{\mathrm{j}} + 2H_{\mathrm{pj}} \quad \text{with} \\ H_{\mathrm{j}} &= \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V} \\ H_{\mathrm{pj}} &= \int_{\mathcal{V}} \mathbf{A}_{\mathrm{p}} \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V} \end{aligned}$$

- **Berger et al. 2003 :** relative magnetic helicity can be decomposed in 2 quantities:
  - H_j = magnetic helicity of the current-carrying/non-potential field B_j
  - H_{pj} = intra-helicity between potential and current carrying fields
- H_V, $H_{j,}$ & H_{pj} are all gauge invariant.
- Remark for the heli-aware: $H_{j,}$ & H_{pj} are different from the “self” and “mutual” helicities

Helicity decomposition evolution

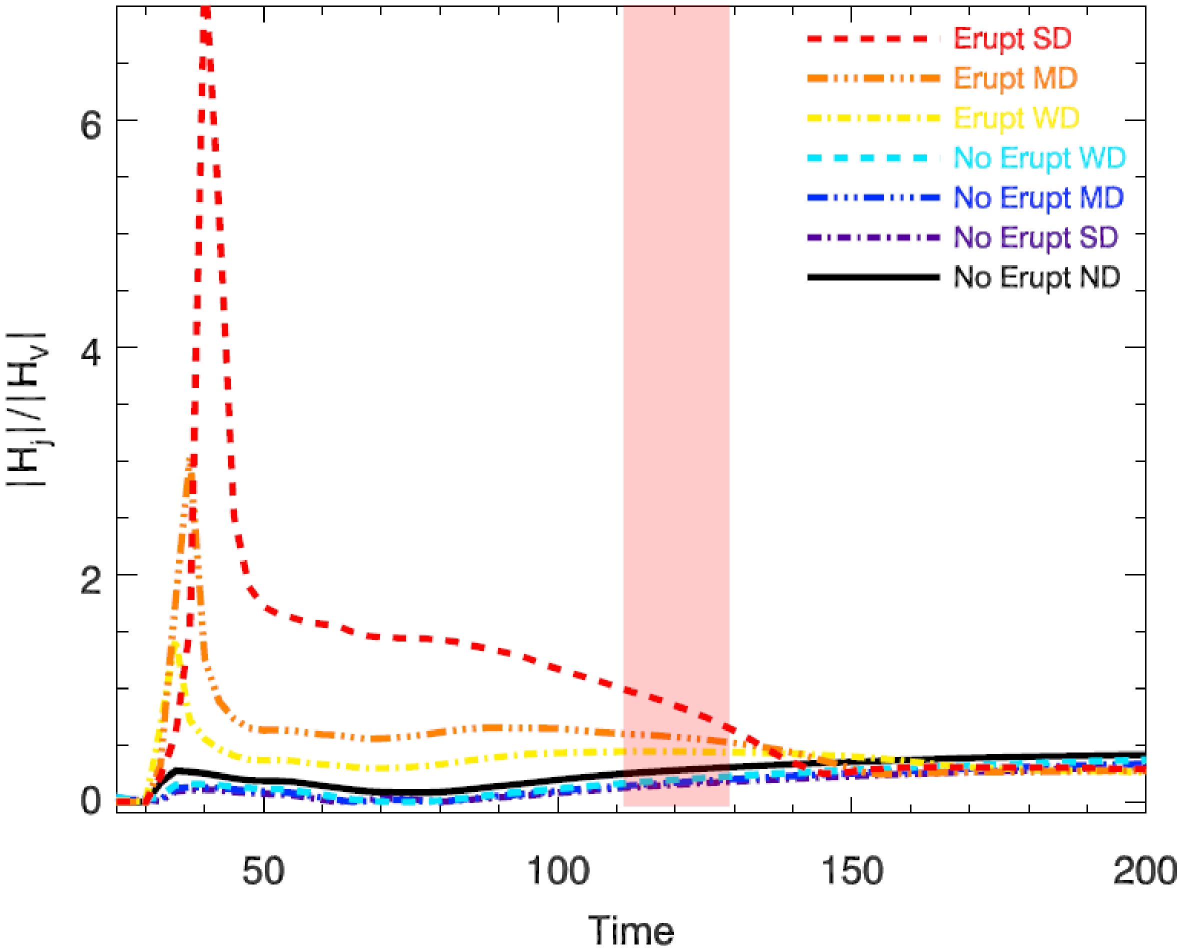


$$
\begin{aligned}
H_{\mathcal{V}} &= H_{\mathrm{j}} + 2H_{\mathrm{pj}} \quad \text{with} \\
H_{\mathrm{j}} &= \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V} \\
H_{\mathrm{pj}} &= \int_{\mathcal{V}} \mathbf{A}_{\mathrm{p}} \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V}
\end{aligned}
$$

- Total helicity is overall dominated by $2H_{pj}$
- $2H_{pj}$ has same properties than total helicity ➔ not a good eruptivity proxy
- **H_j behaves similarly to E_{free}**
  - **higher for the eruptive simulations in the pre-eruptive phase**
  - **however higest values reached by non-eruptive simulations**
- **H_j is not a good eruptivity proxy.**

$|H_j|/|H_V|$: excellent eruptivity indicators

(Pariat et al. 17)



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H_V &= H_{\mathrm{j}} + 2H_{\mathrm{pj}} \quad \text{with} \\
H_{\mathrm{j}} &= \int_{\mathcal{V}} (\mathbf{A} - \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V} \\
H_{\mathrm{pj}} &= \int_{\mathcal{V}} \mathbf{A}_{\mathrm{p}} \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\, \mathrm{d}\mathcal{V}
\end{aligned}
$$

- **$|H_j|/|H_V|$ appears as an excellent eruptivity predictor of these sims.**
  - **Highest value for the eruptive simulations in the pre-eruptive phase**
  - **Eruptive and non-eruptive simulations have similar values in post-eruption phase**
- $|H_j|/|H_V|$ is also sensitive to dipole strength which fits with promptness to erupt

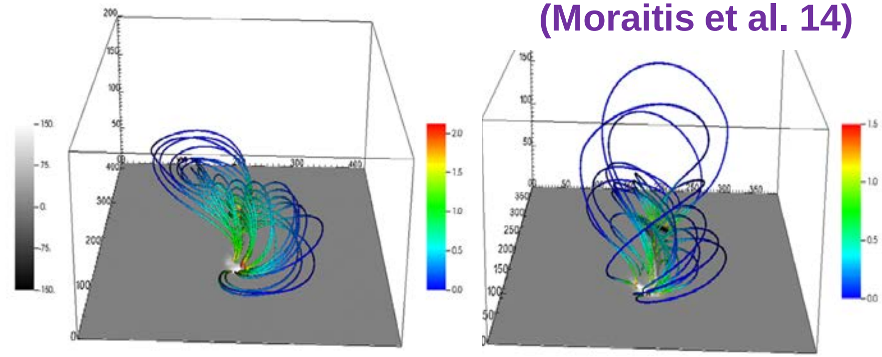
Outline

- Flux emergence model: Leake et al. 2013 & 2014
- Eruptivity criterion analysis
  - Magnetic flux & energy-based quantities
  - Magnetic-helicity-based quantities
    - Relative magnetic helicity
    - Current-carrying magnetic helicity
- **Other models & conclusions**

More evidences : other flux emerg. simulations

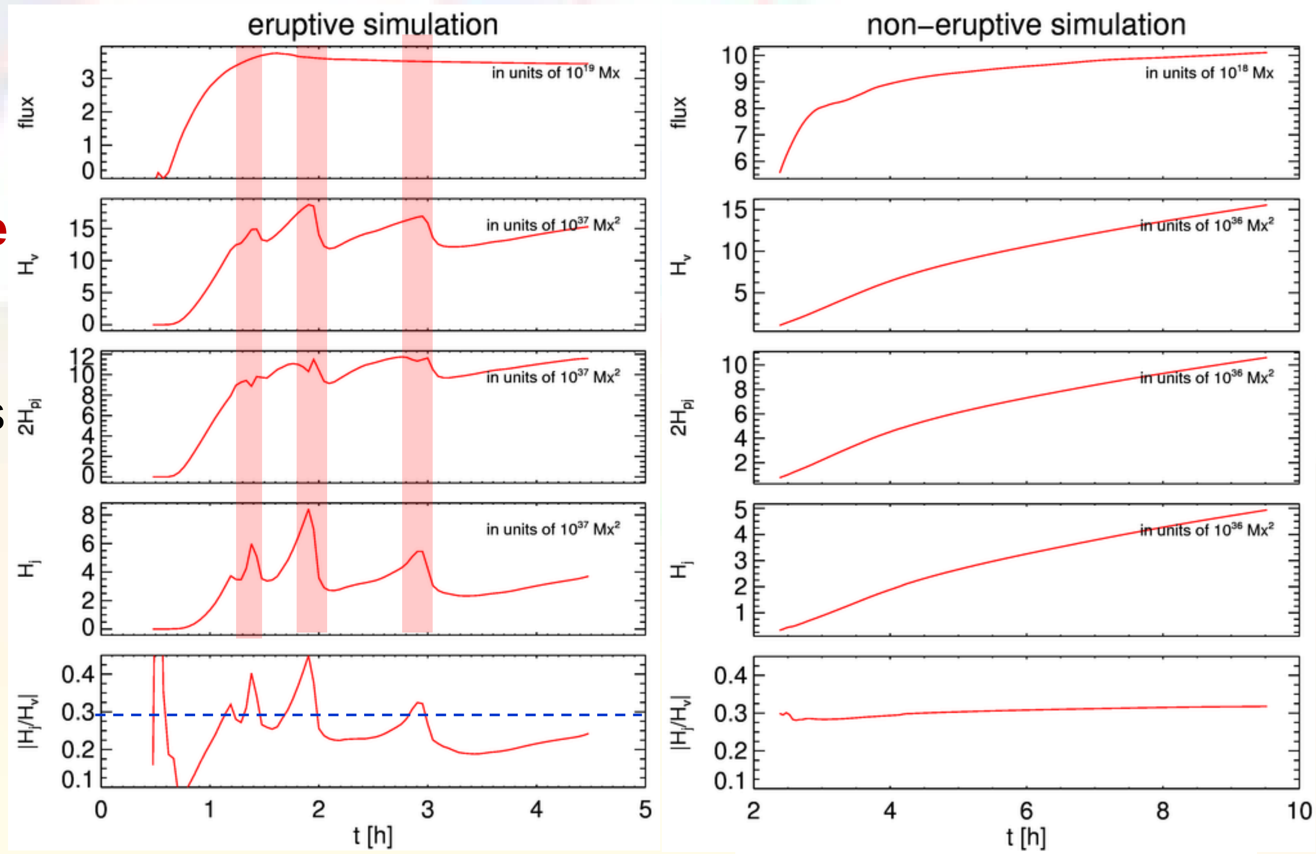
- **Moraitis et al. 2014**: analyze of the helicity content of 2 flux emergence simulations (not directly comparable) :
  - Non-eruptive **(e.g. Archontis et al. 2004)**
  - Multi-eruptive **(e.g Archontis et al. 2014)**

(Moraitis et al. 14)



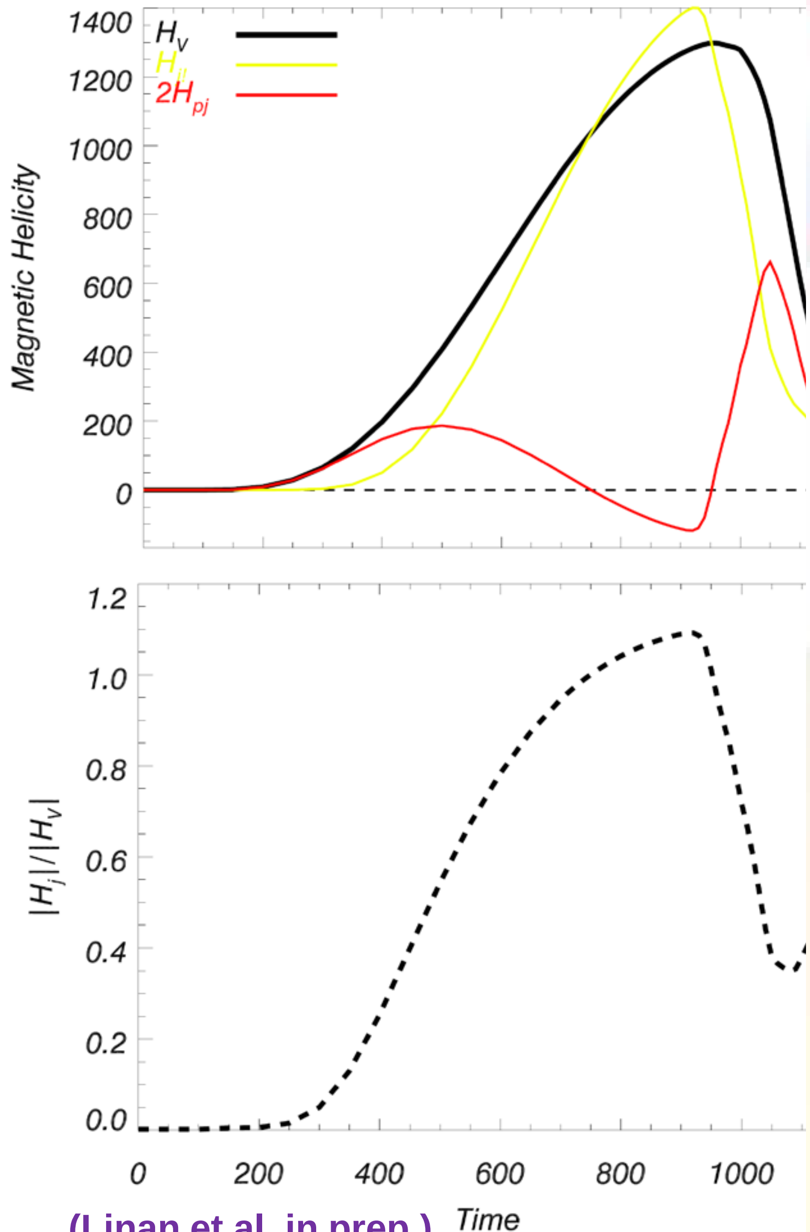
- **Multi-eruptive case:**
  - **Systematic high values of $|H_j|/|H_V|$ some time before the eruptions onset.**
  - $|H_j|/|H_V|$ decreases after eruptions

- **Non-eruptive case: constant and relatively lower values of $|H_j|/|H_V|$**

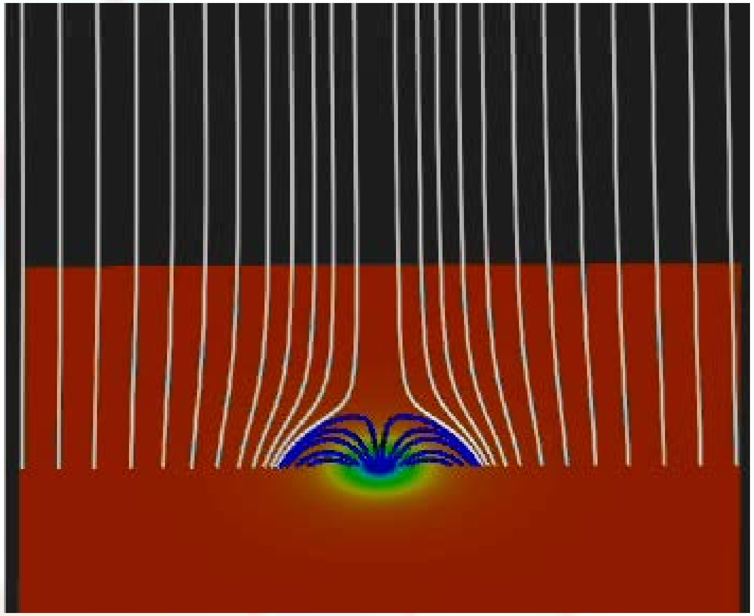


(Figure by K. Moraitis)

More evidences: jet simulation



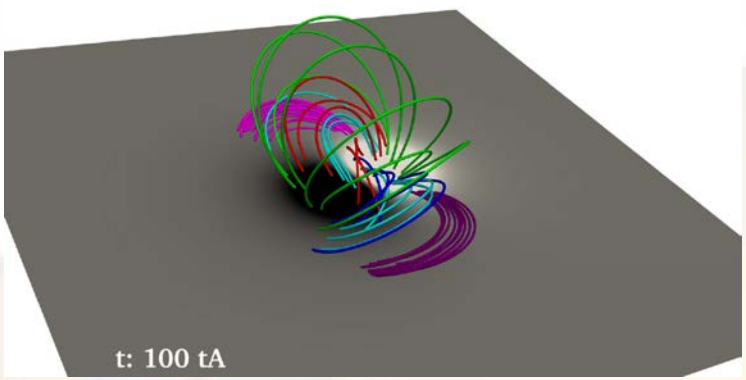
(Linan et al. in prep.)

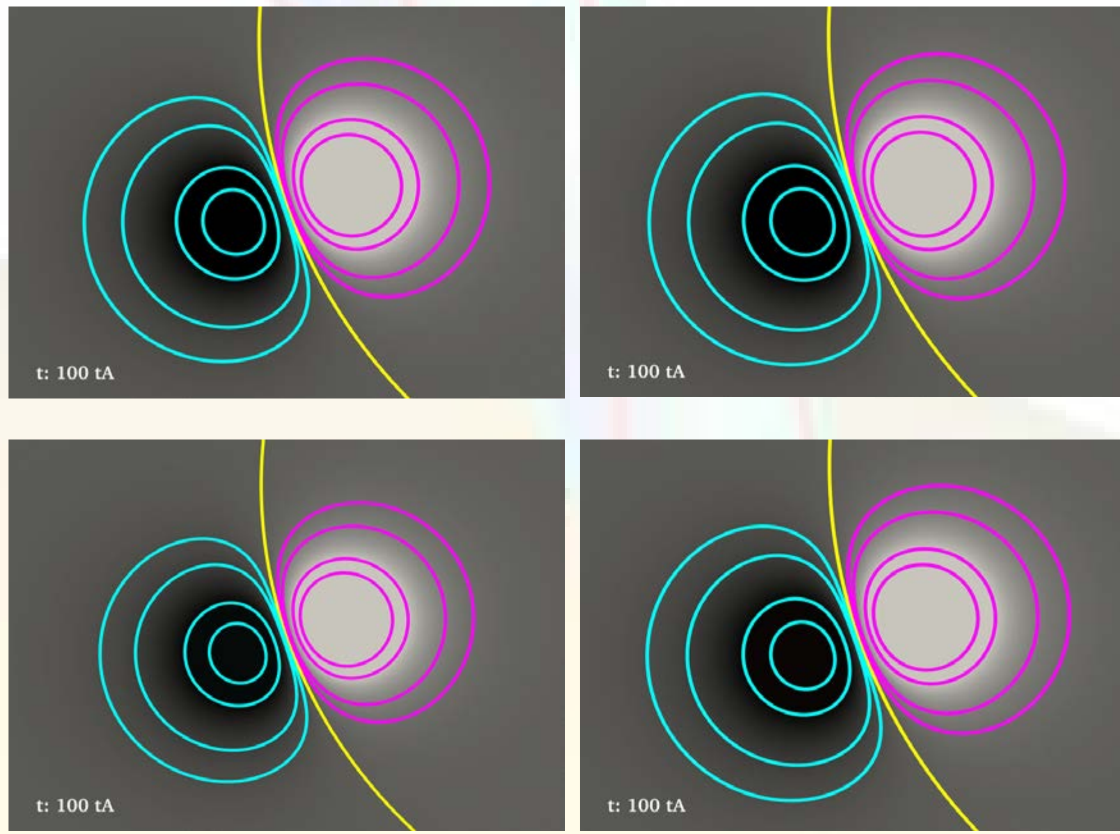


- Coronal jet simulations: **Pariat et al 09, 15**
- **Helicity initially dominated by H_{pj} but H_j become dominant after t~500**
- **Very high value of $|H_j|/|H_V|$ at jet onset.**
  - Remark: system "over" eruptive due to topological constraints
- $|H_j|/|H_V|$ returns to low value once the system has relaxed.

Further evidences : torus-instability triggered eruptive simulations

- **Zuccarello et al. 2015**: parametric eruptive simulations
- **4 different line-tied boundary driving patterns** with different: shear around the PIL magnetic flux dispersion + 1 non-eruptive control case (diffusion)
- **Precise determination of the onset time, t_{erupt}, thanks to numerous relaxation runs initiated at regular stage of the simulations**





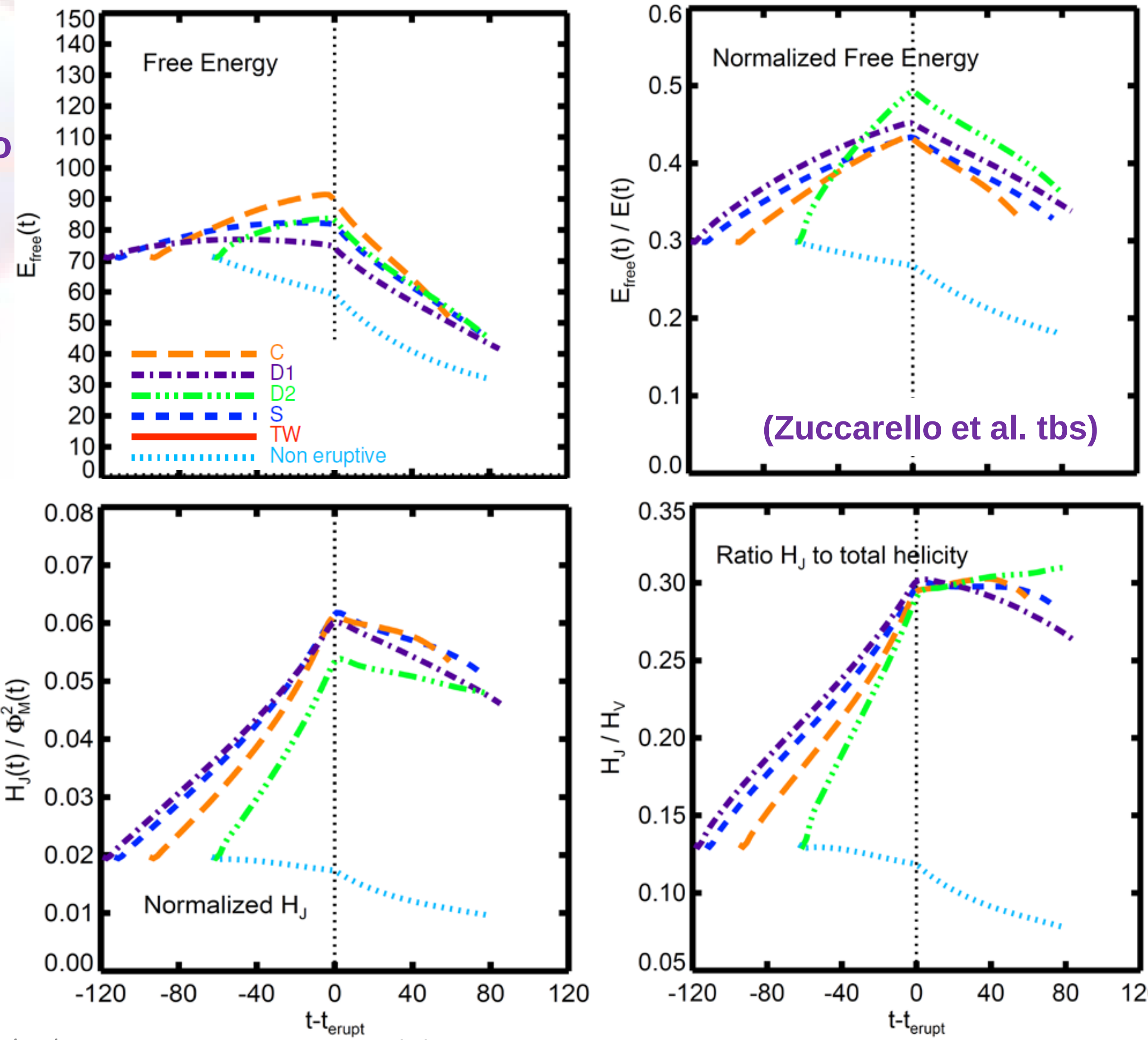
(Aulanier et al. 10, Zuccarello et al. 16)

Further evidences : torus-instability triggered eruptive simulations

- Computation of several quantities at the sim. respective t_{erupt} : **Zuccarello et al. to be submitted.**

- **Despites different boundary drivers and t_{erupt}, eruptions are triggered when $|H_j|/|H_V|$ reaches the same value:**
  - <4% dispersion
  - within measurement precision of helicity

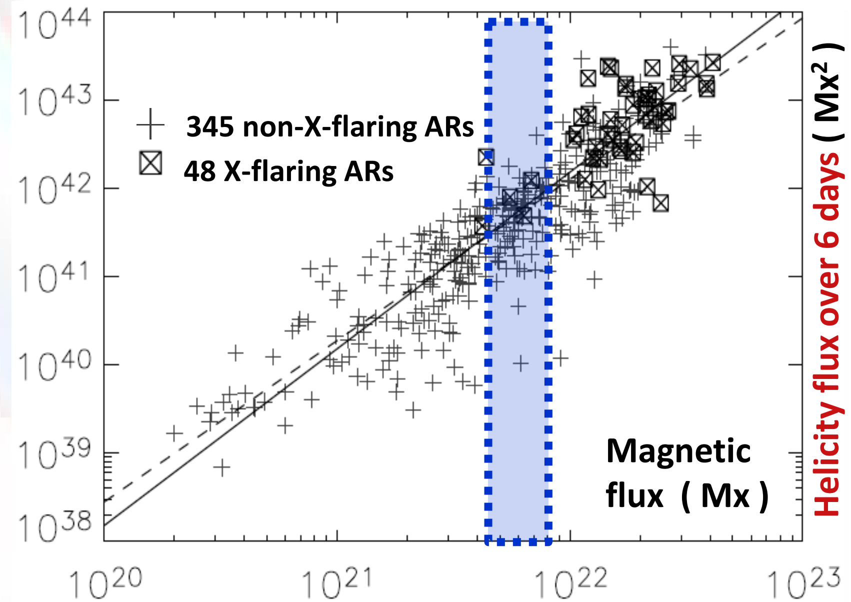
- All other quantities have dispertions of values above 8 % at t_{erupt} , including torus instability criteria

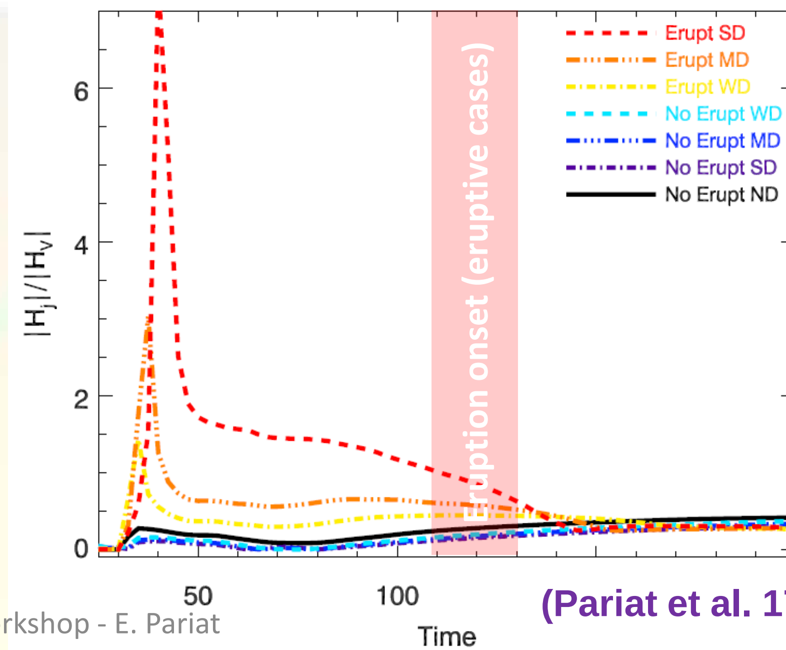


Conclusions

- (too) Rare attempts to use parametric numerical simulation to study eruptivity proxy of solar active events.
- **The ratio |Hj|/|Hv| is an excellent indicator of the eruptivity state in several numerical models**
  - 15 different numerical simulations
  - inducing 11 eruptions & 6 stable systems
  - in 4 very different magnetic configuration
  - performed by 3 different MHD numerical codes
- **BUT: our understanding of relative magnetic helicity is not “mature” enough**
  - not simply/directly measurable from standard observations
  - actual def. of relative helicity may not be optimal: e.g. not simply additive quantity.

(Labonté et al. 07)





(Pariat et al. 17)

Thanks for your attention
Thanks for your participation