Relative magnetic helicity as a diagnostic of solar eruptivity in flux emergence simulations (and others)

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Outline

• **Introduction: flare & eruption previsions**

• Flux emergence model: Leake et al. 2013 & 2014

• Eruptivity criterion analysis
  – Magnetic flux & energy-based quantities
  – Magnetic-helicity-based quantities
    • Relative magnetic helicity
    • Current-carrying magnetic helicity

• Other models & Conclusions
Efficiency of flares & eruptions forecasting

- Multiplication of daily forecasts centers and methods: MET Office, SWPC, SIDC, …

- Barnes et al. 2016: comparison of a large number of forecasting methods with a common dataset:
  - “[…] none of the methods achieves a particularly high skill score. […] Thus there is considerable room for improvement in flare forecasting.”

![Table 4](image-url)
Flares & eruptions forecasting approach

- **Prediction are not based on determinist approach but on an empirical one:**
- **Correlations between:**
  - Characteristics of an active region: McIntosh class, Mt Wilson magnetic class, PIL length, magnetic properties, …
  - Observed probability for a region with a given characteristic to flare

(Falconer et al. 11)
Flaring/eruptivity criterion

- Single criteria alone always gives very poor prediction
  - Combination of several criterion improves prediction.

- Prediction criterion are only based on necessary conditions for eruption
  - e.g. based on the energy build-up of active region

- No clear physical criterion of sufficient conditions for eruption trigger
Eruptivity prediction & numerical modeling

- Search for eruptivity criterion is almost exclusively based on observational datasets …
- … and barely benefits from the recent tremendous improvements in numerical modeling.
- Useful numerical models must present several cases either eruptive or stable, ideally
  - > 2 cases
  - depending on few number of parameters
- Kusano et al. 2012: parametric analysis based on relative orientation of large scale sheared polarity and small scale
Motivations & Methodology

• Goal: use flux emergence simulations to look for efficient eruptivity criterion
  – Leake et al. 2013 and Leake et al. 2014:
    • 7 flux emergence simulations
    • 3D visco-resistive MHD eq. solved with Lagrangian-remap code (Arber et al. 2001)
    • lead to eruptive and non-eruptive cases
    • varying only an unique initial parameter

• Methodology: - extract part of the magnetic field,
  – compute different physical quantities,
  – search for the ones that discriminates between the eruptive and non-eruptive case

• Guennou et al. 17: 2D photospheric mag. field
  – similarly to observed data
  – 99 physical quantities studied.

• This talk: 3D coronal magnetic field $B(z>0)$
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Parametric flux emergence simulations

- Twisted FR emerge in coronal arcade field
- Emerging twisted flux rope: identical in all cases
- Overlying arcade field: 1 param. $\Rightarrow$ 7 cases
  - Signed strength, $B_d$, of the surrounding arcade magnetic field

(Leake et al. 14)
Parametric flux emergence simulations

- Twisted FR emerge in coronal arcade field
- **Emerging twisted flux rope: identical in all cases**
- **Overlying arcade field: 1 param. ➔ 7 cases**
  - Signed strength, $B_d$, of the surrounding arcade magnetic field
  - $B_d=0$: no surrounding field
    - ➔ stable flux rope in the corona
    - No eruption
  - $B_d>0$: same orientation of arcade field and azimuthal part of emerging field: interaction of // fields
    - ➔ formation of stable flux rope
    - No eruption
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  - $B_d>0$: same orientation of arcade field and azimuthal part of emerging field: interaction of // fields
    - ➔ formation of stable flux rope
    - No eruption
  - $B_d<0$: opposite orientation of arcade field and azimuthal part of emerging field: interaction of anti- // fields
    - ➔ reconnection and formation of unstable flux rope
    - Eruptive behavior

(Leake et al. 14)
Search for eruptivity criterion

- **Emerging twisted flux rope:** identical in all cases
- **Overlying arcade field:** 1 parameter \( \Rightarrow \) 7 cases

<table>
<thead>
<tr>
<th>Label</th>
<th>( B_d )</th>
<th>No Erupt SD</th>
<th>No Erupt MD</th>
<th>No Erupt WD</th>
<th>No Erupt ND</th>
<th>Erupt WD</th>
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<th>Erupt SD</th>
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<tbody>
<tr>
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<td>Medium</td>
<td>Weak</td>
<td>Null</td>
<td>–5</td>
<td>Weak</td>
<td>Medium</td>
<td>Strong</td>
</tr>
<tr>
<td>Eruption</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

- **Eruptive simulations:** onset at \( t \sim 120 \ t_0 \)
- **Non-eruptive simulation** stable \( > 400 \ t_0 \)

- **Goal:** search for eruptivity indicators from 3D coronal magnetic datacube
- **Good eruptivity criterion should:**
  - Discriminate eruptive and non-eruptive sim. during pre-eruptive phase
  - Reach its highest value
    - for eruptive simulation only,
    - during the pre-eruptive phase only.
  - Present similar trend for eruptive and non-eruptive sim. in post-eruptive phase

(Leake et al. 14)
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• Conclusions
Magnetic fluxes

- Reference magnetic flux depends on the arcade field strength
- Injected flux by emerging flux rope is roughly identical for all 7 simulations

(Pariat et al. 17)
Magnetic fluxes

- Reference magnetic flux depends on the arcade field strength
- Injected flux by emerging flux rope is roughly identical for all 7 simulations

![Graph showing magnetic flux over time]

- Limits of the model: eruptivity criterion valid given a roughly constant injected magnetic flux.
- ➡️ determining why active regions with a given magnetic flux erupt and others do not.

![Graph showing helicity flux over 6 days]
Total and potential magnetic energy

\[ E_{\text{mag}} = E_{\text{pot}} + E_{\text{free}} + E_{\text{ns}} \cdot \]

- Eruptive simulation have a lower injection of total magnetic energy and potential magnetic energy.

- Both total and potential magnetic energies are not good indicators of the eruptivity of the system.
Free magnetic energy

\[ E_{mag} = E_{pot} + E_{free} + E_{ns} \]

- Free energy is slightly higher for eruptive simulation in the pre-eruption phase.
- However, the highest value of \( E_{free} \) are reached by non-eruptive simulations.
- Free magnetic energy is not a good indicator of the eruptivity state of the system.

(Pariat et al. 17)
Free magnetic energy ratio

\[ E_{mag} = E_{pot} + E_{free} + E_{ns} \]

- \( E_{free}/E_{inj} \) is higher for eruptive simulation vs. non-eruptive in the pre-eruption phase with marginally the highest values.

- Ratio of free magnetic energy to injected energy may be a proxy of eruptivity of the system.

- However, \( E_{free}/E_{inj} \) not strongly discriminative: maximum value for eruptive flare are only marginally above non-eruptive ones.
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Relative magnetic helicity

- **Magnetic helicity of MHD plasmas** (Elsasser 1956)
  - unique signed scalar value for volume considered
  - magnetic flux weighted Gauss Linking Number of pairs of magnetic field lines (Moffatt 1968): signed level of entanglement & twist of field lines
  - **Useful quantity for natural plasmas: Relative Magnetic Helicity**: helicity of a studied field relative to a reference field (Berger 1984, Finn & Antonsen 1985).

\[
H = \int_{\mathcal{V}} \mathbf{A} \cdot \mathbf{B} \, d\mathcal{V}
\]

\[
H_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_p) \cdot (\mathbf{B} - \mathbf{B}_p) \, d\mathcal{V}
\]

with boundary condition:
\[
(\mathbf{B}_p \cdot d\mathbf{S}) |_{\partial \mathcal{V}} = (\mathbf{B} \cdot d\mathbf{S}) |_{\partial \mathcal{V}}
\]

- Gauge invariant provided that studied and reference fields share the same magnetic-flux distribution on the whole boundary.
Magnetic helicity properties

- **Magnetic helicity is an ideal MHD invariant.** For \( E \perp B \): no dissipation \( \Rightarrow \) magnetic helicity is conserved (Woltjer 1958).

\[
\frac{dH_m}{dt} = \int_{\partial V} \left( A \times \frac{\partial A}{\partial t} \right) \cdot dS - 2 \int_{\partial V} (E \times A) \cdot dS - 2 \int_V E \cdot B \ dV
\]

- **Taylor 1974: hypothesis helicity conservation true even in non-ideal MHD**
  - Pariat et al. 16: verified for a solar like active event

- **Magnetic helicity bounds the system E distribution:**
  \[ \mu_0 \dot{E}(k) > k \dot{H}(k) \]  (Frisch et al. 1975)

- **Inverse helicity cascade:** Helicity goes from small to large spatial scales. (Frisch et al. 1975, Alexakis et al. 2006)
  - e.g. kink instability (Malanushenko et al. 2009)

- **Impact on dynamic of magnetic reconnection:**
  e.g. Linton et al. 2001, Del Soro et al. 2010
Relative Magnetic Helicity Estimations

• The computation of relative magnetic helicity is not straightforward:
  – **Computation of reference field must be done imposing boundary conditions on the whole domain boundary.**
  – Many previous methods assumed semi-infinite volumes while all existing datasets are bounded volumes: could lead to incorrect results (Valori et al. 2011, 2012), error in intensity, even in sign!

• **Several methods recently developed on 3D cuboid system** (Valori et al. 2016)
  – Using Coulomb gauge: \( \nabla \cdot \mathbf{A} = 0 \)
    
    Thalmann et al. 2011, Rudenko & Myshyakov 2011, Yang et al. 2013
    • Simpler theoretical formulation
    • Harder to implement numerically
  – Using DeVore gauge (**DeVore et al. 2000**): \( A_z = 0 \)
    
    Valori, Démoulin & Pariat 2012, Moraitis et al. 2014
    • More complex theoretical formulation
    • Simpler to implement numerically: more precise

• **New method to compute relative magnetic helicity in spherical wedge domains**. (Moraitis et al. in prep.)
Relative magnetic helicity estimations

- Benchmarking of these methods performed by ISSI team on "Helicity estimations in models and observations": Valori et al. 2016
- Numerous tests: sensibility to resolution, twist, solenoidality using various types of data.
  - Force free fields (Low & Lou 1990)
  - Stable flux rope (Titov & Démoulin 1999, data from Török)
  - Flux emergence simulations (Leake et al. 2013, 2014)
- Methods perform very consistently when $B$ sufficiently solenoidal
Relative magnetic helicity evolution

- Unlike with magnetic flux & free energy, helicity discriminates strongly the cases
  - Total helicity depends
    - on dipole strength
    - on dipole orientation
- The surrounding (potential) field influences the helicity content!
- Magnetic helicity is a non-local quantity!

- Unlike what is commonly believed/expected, large total helicity is not a sufficient condition of eruptivity.
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Relative magnetic helicity decomposition

\[ H_V = H_j + 2H_{pj} \quad \text{with} \]
\[ H_j = \int_V (A - A_p) \cdot (B - B_p) \, d\mathcal{V} \]
\[ H_{pj} = \int_V A_p \cdot (B - B_p) \, d\mathcal{V} \]

- Berger et al. 2003: relative magnetic helicity can be decomposed in 2 quantities:
  - \( H_j \) = magnetic helicity of the current-carrying/non-potential field \( B_j \)
  - \( H_{pj} \) = intra-helicity between potential and current carrying fields

- \( H_V, H_j, \) & \( H_{pj} \) are all gauge invariant.

- Remark for the heli-aware: \( H_j, \) & \( H_{pj} \) are different from the “self” and “mutual” helicities.
Helicity decomposition evolution

\[ H_V = H_j + 2H_{pj} \]

\[ H_j = \int_V (A - A_p) \cdot (B - B_p) \, dV \]

\[ H_{pj} = \int_V A_p \cdot (B - B_p) \, dV \]

- Total helicity is overall dominated by \( 2H_{pj} \)
- \( 2H_{pj} \) has same properties than total helicity \( \Rightarrow \) not a good eruptivity proxy

- \( H_j \) behaves similarly to \( E_{\text{free}} \)
  - higher for the eruptive simulations in the pre-eruptive phase
  - however highest values reached by non-eruptive simulations
- \( H_j \) is not a good eruptivity proxy.

(Pariat et al. 17)
\[ |H_j|/|H_V| : \text{excellent eruptivity indicators} \]

\( H_V = H_j + 2H_{pj} \) \quad \text{with}\]

\[ H_j = \int_V (A - A_p) \cdot (B - B_p) \, dV \]

\[ H_{pj} = \int_V A_p \cdot (B - B_p) \, dV \]

- \(|H_j|/|H_V|\) appears as an excellent eruptivity predictor of these sims.
  - Highest value for the eruptive simulations in the pre-eruptive phase
  - Eruptive and non-eruptive simulations have similar values in post-eruption phase
- \(|H_j|/|H_V|\) is also sensitive to dipole strength which fits with promptness to erupt
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More evidences: other flux emergence simulations

- **Moraitis et al. 2014**: analyze of the helicity content of 2 flux emergence simulations (not directly comparable):
  - Non-eruptive (e.g. Archontis et al. 2004)
  - Multi-eruptive (e.g. Archontis et al. 2014)

- **Multi-eruptive case:**
  - Systematic high values of $|H_j|/|H_V|$ some time before the eruptions onset.
  - $|H_j|/|H_V|$ decreases after eruptions

- **Non-eruptive case:**
  - Constant and relatively lower values of $|H_j|/|H_V|$
More evidences: jet simulation

- Coronal jet simulations: Pariat et al 09, 15
- Helicity initially dominated by $H_{pj}$ but $H_j$ become dominant after $t \sim 500$
- Very high value of $|H_j| / |H_V|$ at jet onset.
  - Remark: system “over” eruptive due to topological constraints
- $|H_j| / |H_V|$ returns to low value once the system has relaxed.

(Linan et al. in prep.)
Further evidences: torus-instability triggered eruptive simulations

- **Zuccarello et al. 2015**: parametric eruptive simulations
- **4 different line-tied boundary driving patterns** with different: shear around the PIL, magnetic flux dispersion + 1 non-eruptive control case (diffusion)
- **Precise determination of the onset time, \( t_{\text{erupt}} \)**, thanks to numerous relaxation runs initiated at regular stage of the simulations

(Aulanier et al. 10, Zuccarello et al. 16)
Further evidences:

- torus-instability triggered eruptive simulations

- Computation of several quantities at the sim. respective $t_{\text{erupt}}$: Zuccarello et al. to be submitted.

- Despites different boundary drivers and $t_{\text{erupt}}$, eruptions are triggered when $|H_j|/|H_V|$ reaches the same value:
  - $<4\%$ dispersion
  - within measurement precision of helicity

- All other quantities have dispersions of values above $8\%$ at $t_{\text{erupt}}$, including torus instability criteria.
Conclusions

- (too) Rare attempts to use parametric numerical simulation to study eruptivity proxy of solar active events.

- Flux and energy-based quantities are poor discriminant and poor eruptivity proxies in these models.

- Magnetic helicity based quantities allow to easily discriminate between the different parametric simulations.

- The ratio $|H_j|/|H_v|$ is an excellent indicator of the eruptivity state in several numerical models:
  - 4 different magnetic systems
  - 3 different MHD numerical codes
Thanks for your attention

I hope that this talk was worth a Havana
Relative magnetic helicity evolution

- Helicity of the stable cases is larger than the eruptive cases!
- Helicity increases with arcade strength for non-eruptive cases
- Helicity decreases with arcade strength for eruptive cases
Self and Mutual helicity

- Helicity decomposition in **self** and **mutual** helicity of flux rope and arcade
  \[ H = H_{\text{self,fr}} + H_{\text{mutual}} + H_{\text{self,arc}} \]

- **H_{\text{self,fr}}** = \( H(\text{No Erupt ND}) \)
  \[ \propto \Phi_{\text{fr}}^2 \]

- **H_{\text{self,arc}}** = 0

- **H_{\text{mutual}}** \( \propto \Phi_{\text{fr}} \Phi_{\text{arc}} \);
  - sign depends on relative orientation

• Non-eruptive cases: FR & arcade have same orientation: \( H = H_{\text{self,fr}} + |H_{\text{mutual}}| \)

• Eruptive cases: FR & arcade have opposite orientation: \( H = H_{\text{self,fr}} - |H_{\text{mutual}}| \)

• With increasing dipole strength \(|H_{\text{mutual}}|\) increases
  - Qualitatively & quantitative match
  - \( H \) increases for stable cases
  - \( H \) decreases for unstable
Self and Mutual helicity

- Helicity decomposition in **self** and **mutual** helicity of flux rope and arcade

\[ H = H_{\text{self,fr}} + H_{\text{mutual}} + H_{\text{self,arc}} \]

- \( H_{\text{self,fr}} = H(\text{No Erupt ND}) \)
- \( H_{\text{self,arc}} = 0 \)
- \( H_{\text{mutual}} \propto \Phi_{fr} \Phi_{arc} \); sign depends on relative orientation

- Very good quantitative match of this toy model
- Computation of HD:
- Toy model predict that ratio of HD shall be equal to magnetic flux ratios
- Good fit with expected values: \( \Phi_{\text{ini, MD}} / \Phi_{\text{ini, WD}} = 1.5 \) & \( \Phi_{\text{arcini, SD}} / \Phi_{\text{ini,MD}} = 1.33 \)
- Problem: here self and mutual helicity can only be roughly estimated because we have a parametric dataset. Not the case with real data.

\[ H_D = H_V - H_V, \text{No Erupt ND} \sim \pm L \Phi_{Arc} \]