EWASS 2016, Symposium 17 :
Magnetic Helicity in Sun and Stars: From Dynamo Action to Eruptive Phenomena
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Magnetic helicity conservation in a solar active event

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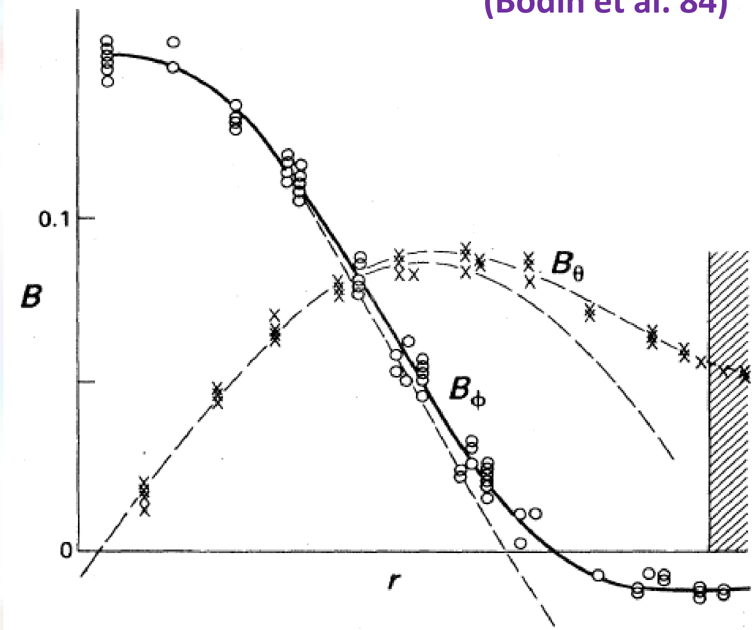
Taylor conjecture

(Bodin et al. 84)



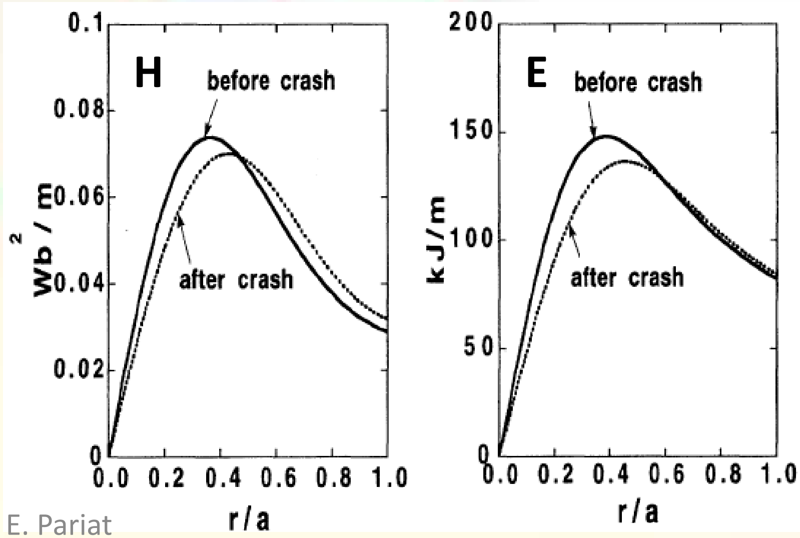
- Magnetic helicity is an ideal MHD invariant when field lines cannot reconnect. For E⊥B: no dissipation ➔ magnetic helicity is conserved **(Woltjer 58)**.

Time variations **Surface Flux** **Dissipation**

$$\frac{dH_m}{dt} = \int_{\partial\mathcal{V}} \left(\mathbf{A} \times \frac{\partial \mathbf{A}}{\partial t}\right) \cdot \mathrm{d}\mathbf{S} - 2\int_{\partial\mathcal{V}} (\mathbf{E} \times \mathbf{A}) \cdot \mathrm{d}\mathbf{S} - 2\int_{\mathcal{V}} \mathbf{E} \cdot \mathbf{B} \;\; \mathrm{d}\mathcal{V}$$

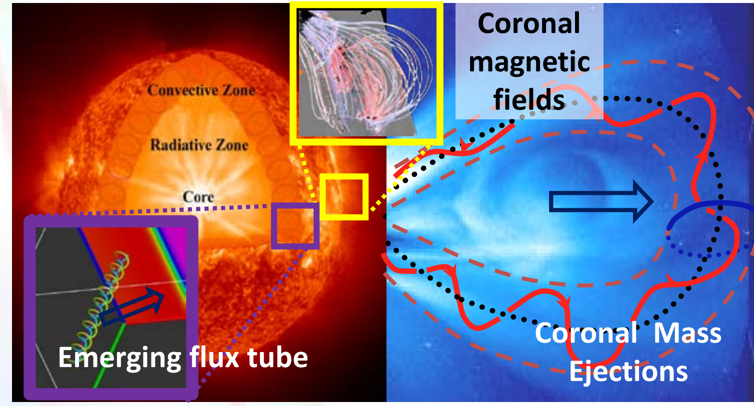
- From laboratory plasma experiment, **Taylor (1974) conjectured: even in non-ideal MHD magnetic helicity should be well conserved.**
  - Magnetic energy cascades to small scales where it is dissipated vs helicity cascades to large scales **(Ji et al. 95, Heidbrink & Dang 00)**.
  - In resistive MHD, helicity dissipation is bounded and slow compared to energy dissipation **(Berger 84, Berger 99)**

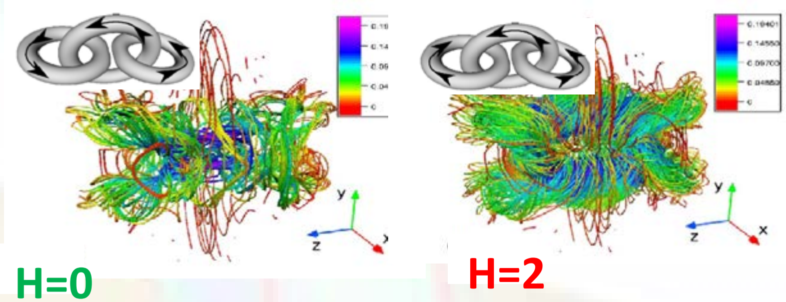
(Ji et al. 95)



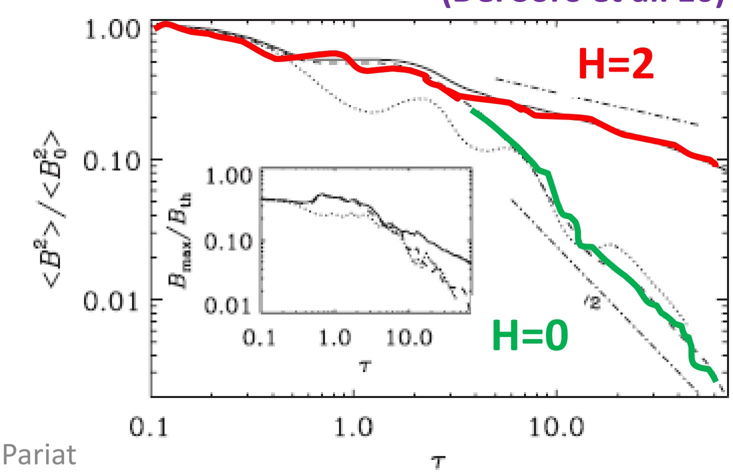
Consequence of Taylor's conjecture

- Relaxation in lab. experiments: plasma relax to minimum energy state, i.e. linear force free field (LFFF) **e.g. Bodin et al. 84, Taylor et al. 86, Yamada et al. 99**
- Impact on dynamic of magnetic reconnection: **e.g. Linton et al. 01, Del Soro et al. 10**
- Helicity conservation plays role in:
  - Saturation of stellar dynamos: **e.g. Brandenburg et al. 05, Blackman 14**
  - Coronal heating models: **Heyvaerts & Priest 84**
  - Coronal mass ejection generation
- **Magnetic helicity conservation is the "raison d'être" of CMEs (Rust 94, Low 96)**
  - Conjecture : to limit the buildup of magnetic helicity in the corona, magnetic helicity has to be ejected via CMEs.



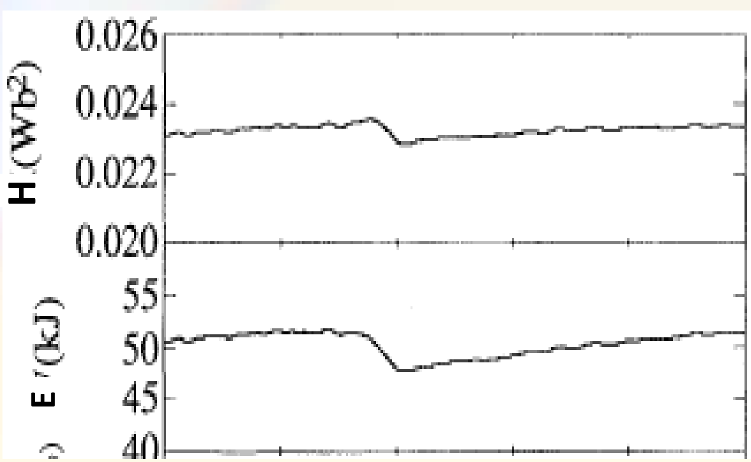


(Del Soro et al. 10)

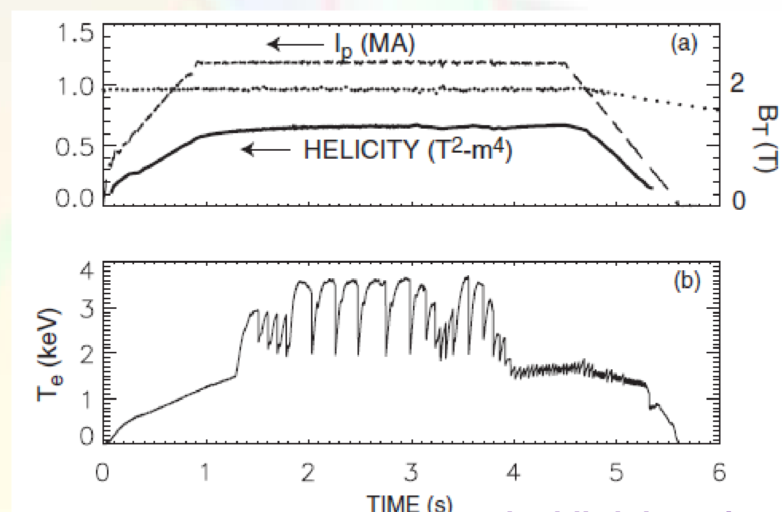


Tests on Magnetic Helicity Conservation

- **Despites its potential importance, tests on Taylor's conjecture have been very limited!**
  - Test on "relaxation" toward minimum energy state (LFFF): mixed results
    ➔ not direct test of magnetic helicity conservation, but of relaxation dynamics
- Laboratory experiments: difficult sampling of the full 3D magnetic field ; axisymmetric assumption **(Ji et al. 95, Barnes et al. 86, Heidbrink et al. 00, Gray et al. 10)**
  - Sawtooth relaxation: $\Delta H/H$=1-5% ; $\Delta E/E$=5-10%
  - Sawtooth crash: $\Delta H/H$=1%
- **Numerical simulation: no test in general conditions**, i.e. in 3D, active-like conditions, no periodicity …



(Ji et al. 95)



(Heidbrink et al. 00)

Relative Magnetic Helicity

- Magnetic helicity is gauge invariant only for magnetically bounded systems:
- Strict definition of magnetic helicity useless for numerous applications:
  - e.g. natural plasmas, like the solar corona have boundaries threaded by magnetic fields

➔ Useful quantity: **Relative Magnetic Helicity:** helicity of a studied field relative to a reference field **(Berger 84)**.

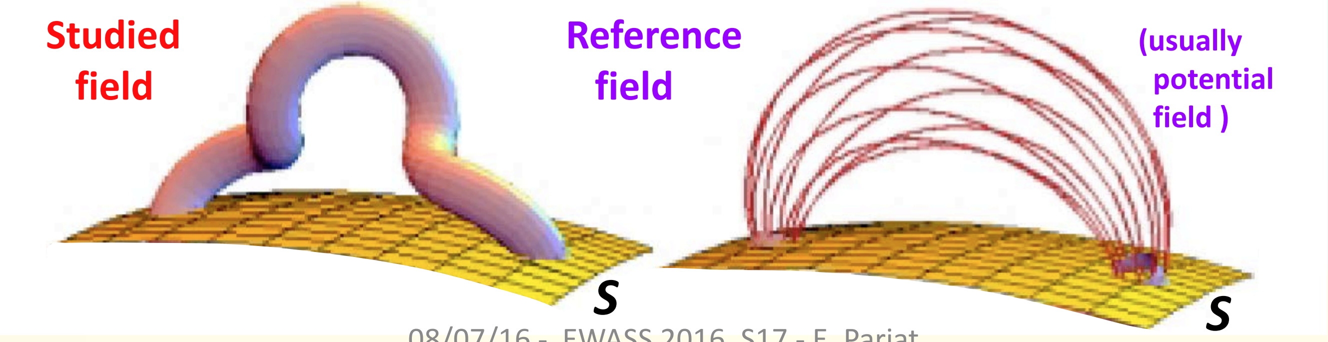
$$H_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\mathrm{p}}) \cdot (\mathbf{B} - \mathbf{B}_{\mathrm{p}})\ \mathrm{d}\mathcal{V}$$

(Finn & Antonsen 85)

with boundary condition : $(\mathbf{B}_{\mathrm{p}} \cdot \mathrm{d}\mathbf{S})|_{\partial\mathcal{V}} = (\mathbf{B} \cdot \mathrm{d}\mathbf{S})|_{\partial\mathcal{V}}$

$$\nabla \times \mathbf{A} = \mathbf{B}$$

- **Gauge invariant provided that studied and reference fields share the same magnetic-flux distribution on the boundary.**



Relative Magnetic Helicity Estimations

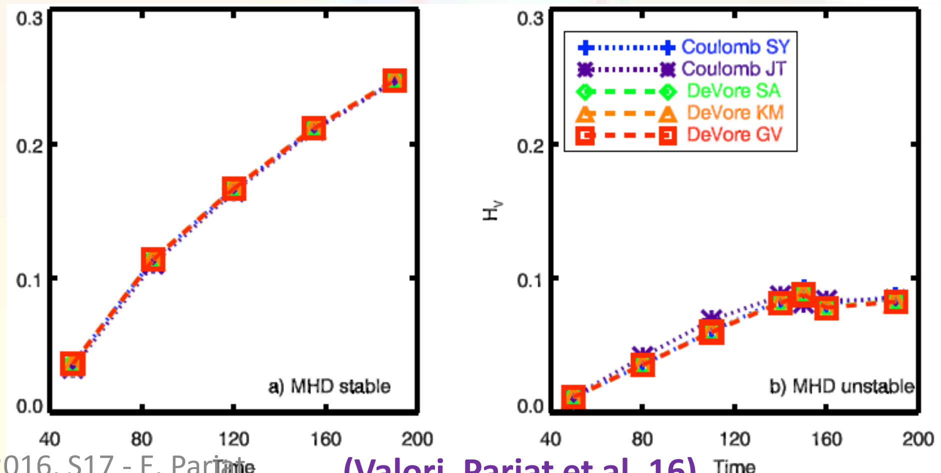
- The computation of relative magnetic helicity on a 3D cuboid system has not been straightforward
  - Volume computation of consistent gauges for the studied and reference **B**
  - Impose boundary conditions simultaneously on all 6 faces
- Methods recently developed:
  - Using Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

    Thalmann et al. 11, Rudenko & Myshyakov 11, Yang et al. 13
    - Simpler theoretical formulation but harder to implement numerically
  - Using DeVore gauge: $A_z = 0$

    Valori, Démoulin & Pariat 12
    - More complex theoretical formulation but simpler numerical implement.

- **Benchmarking of these methods performed by ISSI team on "Helicity estimations in models and observations"**

  ➔ **Cf. Talk of G. Valori**
- **Methods perform very consistently when B sufficiently solenoidal**



(Valori, Pariat et al. 16)

Magnetic helicity dissipation estimation

- General formulation of the time variation of the relative magnetic helicity:

Magnetic helicity dissipation

Time variation of relative magnetic helicity

$$\frac{dH}{dt} = -2\int_{\mathcal{V}} \mathbf{B}\cdot\mathbf{E}\, d\mathcal{V} + 2\int_{\mathcal{V}} \frac{\partial\phi}{\partial t}\nabla\cdot\mathbf{A}_{\rm p}\, d\mathcal{V}$$
$$+ \int_{\partial\mathcal{V}} \left((\mathbf{A}-\mathbf{A}_{\rm p}) \times \frac{\partial(\mathbf{A}+\mathbf{A}_{\rm p})}{\partial t} \right)\cdot d\mathbf{S} - 2\int_{\partial\mathcal{V}} \frac{\partial\phi}{\partial t}\mathbf{A}_{\rm p}\cdot d\mathbf{S}$$
$$+ 2\int_{\partial\mathcal{V}} (\mathbf{B}\cdot\mathbf{A}_{\rm p})\mathbf{v}\cdot d\mathbf{S} - 2\int_{\partial\mathcal{V}} (\mathbf{v}\cdot\mathbf{A}_{\rm p})\mathbf{B}\cdot d\mathbf{S}$$

Helicity variation and flux of the reference field

(Pariat et al. 15)

Flux of helicity of the studied field

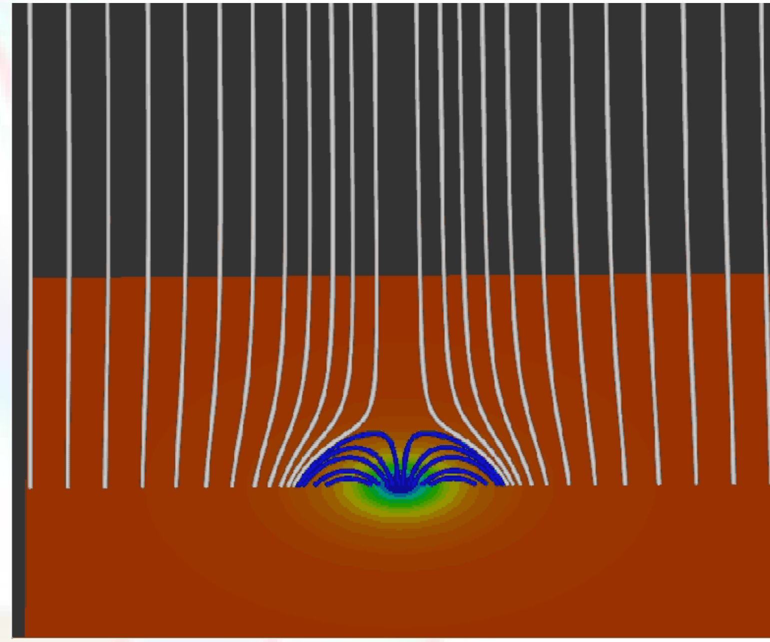
- **Helicity-conservation estimation:** measure the difference between
  - helicity variations in $\mathcal{V}$
  - **helicity flux through the boundary sides $\mathcal{S}$.**
  - Method independent of the non-ideal processes, i.e. reconnection-model

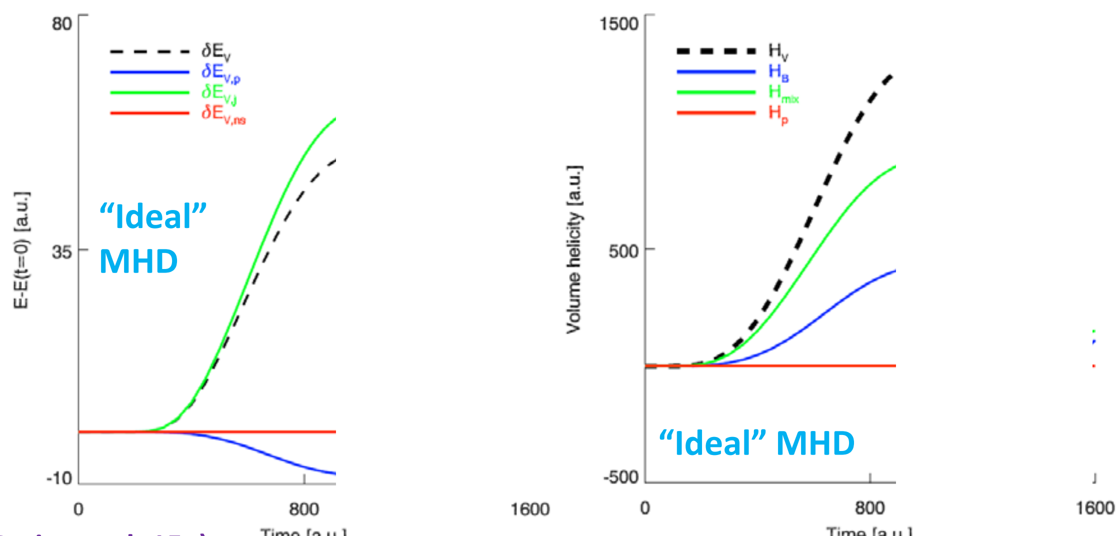
$$C_m = -2\int_{\mathcal{V}} \mathbf{E}\cdot\mathbf{B}\, d\mathcal{V} = \frac{dH}{dt} - F_{tot}$$

Test Case

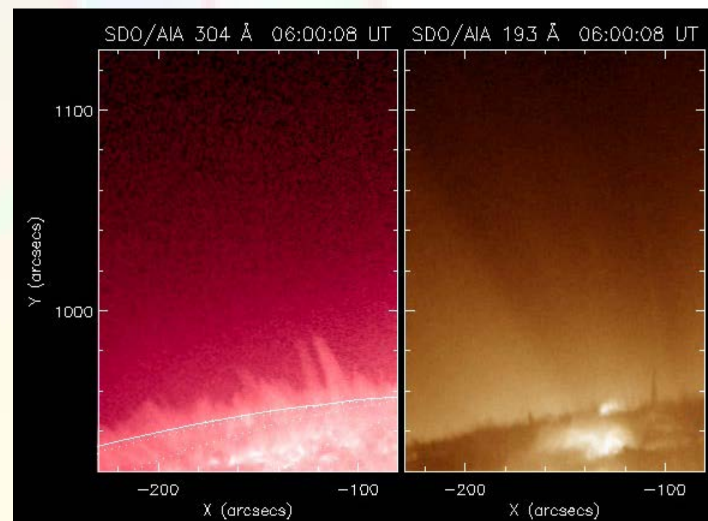
- **3D MHD simulation of a solar coronal jet:**
  Pariat et al. 09,10,15b ; Dalmasse et al. 12
  - Magnetic helicity/energy injected by bottom boundary motions
- **First phase: helicity/energy storage.**
  - Quasi-ideal MHD: reconnection inhibited.
- **Second phase: Jet generation**
  - Very impulsive energy release by recon.
  - Ejection of helicity.

(Pariat et al. 10)





(Pariat et al. 15a)



(Shen et al. 11)

Helicity conservation

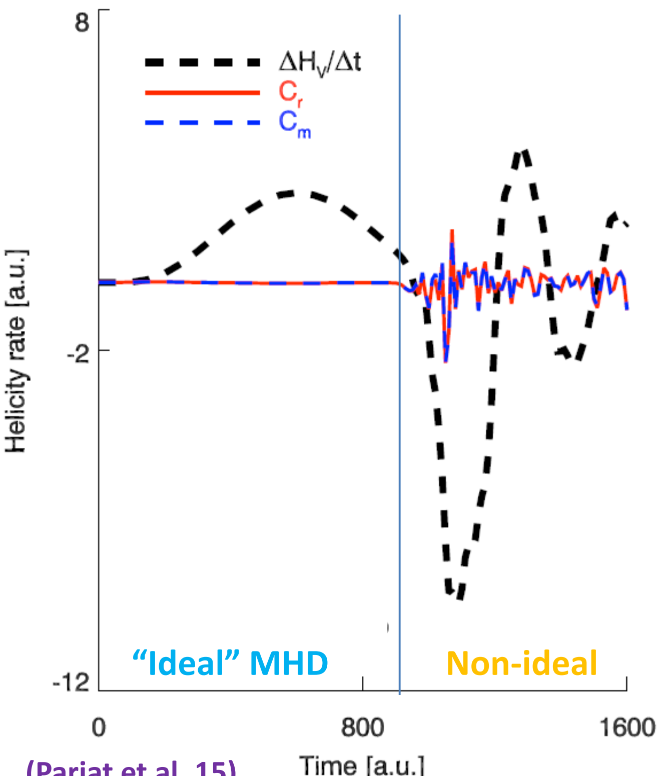
- Helicity and its flux are estimated independently
  - Direct volume helicity computation (Valori et al. 12) : **B** in $\mathcal{V}$
  - Helicity flux computation: **B**, **v** on $\mathcal{S}$

$$H_{\mathcal{V}} = \int_{\mathcal{V}} (\mathbf{A} + \mathbf{A}_{\rm p}) \cdot (\mathbf{B} - \mathbf{B}_{\rm p}) \, {\rm d}\mathcal{V}$$

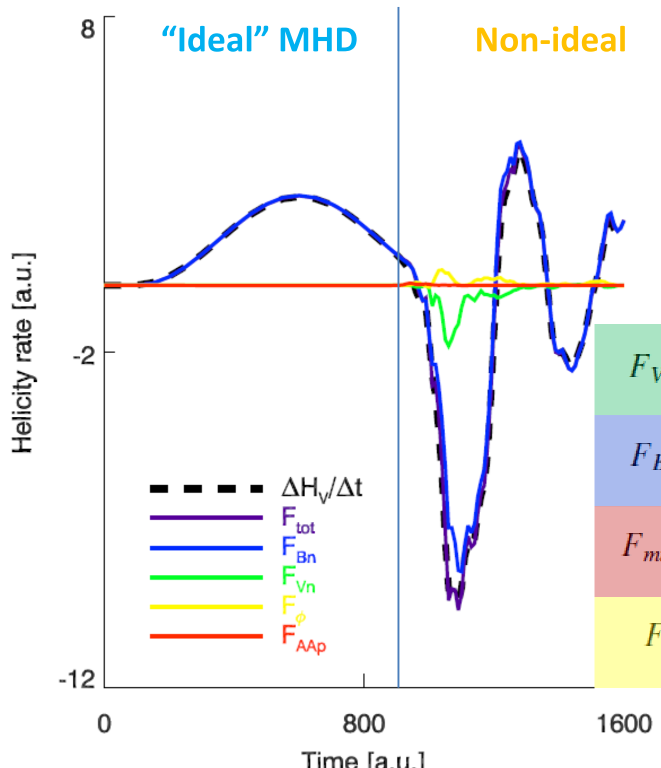
- **➔ Magnetic helicity is very well conserved both during the quasi-ideal MHD and non-ideal phases.**

$$F_{tot} = F_{Vn} + F_{Bn} + F_{mix} + F_{\phi}$$

$$C_m = \frac{dH}{dt} - F_{tot}$$



(Pariat et al. 15)



$$F_{Vn} = 2 \int_{\partial \mathcal{V}} (\mathbf{B} \cdot \mathbf{A}) \mathbf{v} \cdot {\rm d}\mathbf{S}$$

$$F_{Bn} = -2 \int_{\partial \mathcal{V}} (\mathbf{v} \cdot \mathbf{A}) \mathbf{B} \cdot {\rm d}\mathbf{S}$$

$$F_{mix} = \int_{\partial \mathcal{V}} \left((\mathbf{A} - \mathbf{A}_{\rm p}) \times \frac{\partial (\mathbf{A} + \mathbf{A}_{\rm p})}{\partial t} \right) \cdot {\rm d}\mathbf{S}$$

$$F_{\phi} = -2 \int_{\partial \mathcal{V}} \frac{\partial \phi}{\partial t} \mathbf{A}_{\rm p} \cdot {\rm d}\mathbf{S}$$

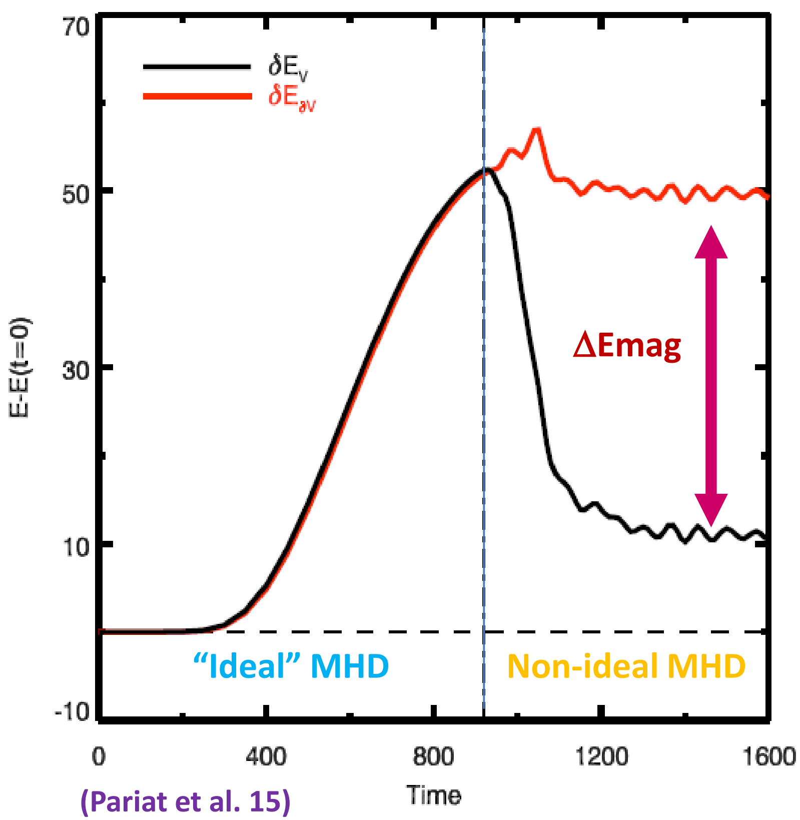
Helicity conservation

- **Magnetic helicity is very well conserved.**
  - Dissipated helicity is very small compared to the helicity injected in the system.
  - The dissipated helicity is very small compared to the amount of magnetic energy dissipated.

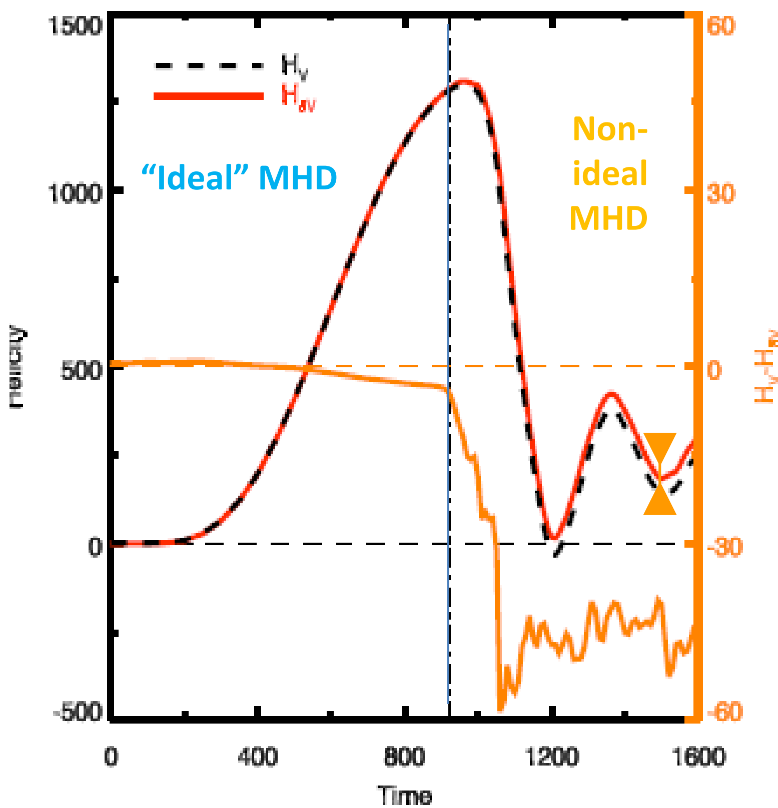
$$\frac{\Delta H_{\mathrm{diss,Ideal}}}{\Delta H_{\mathrm{inj}}} < 3 \times 10^{-3}$$

$$\frac{\Delta H_{\mathrm{diss,\ Non\text{-}ideal}}}{\Delta H_{\mathrm{ini}}} < 0.02$$

$$\frac{\Delta E_{\mathrm{diss,\ Non\text{-}ideal}}}{\Delta E_{\mathrm{inj}}} \sim 0.6$$



(Pariat et al. 15)



“Vn” and “Bn” flux terms

- The helicity flux decomposition presents two terms directly linked with the transport of helicity of the studied field
  - “emergence” term, depends on Vn
  - “shear” term, depends on Bn

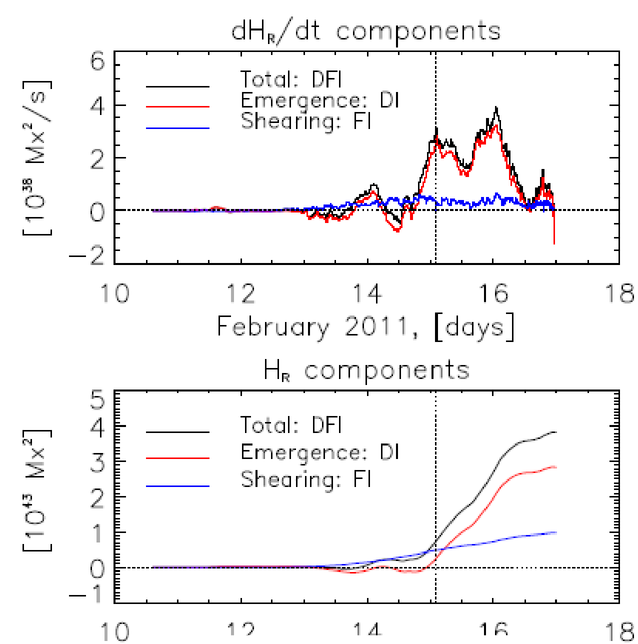
$$F_{Vn} = 2\int_{\partial V} (\mathbf{B}\cdot\mathbf{A})\mathbf{v}\cdot d\mathbf{S}$$

$$F_{Bn} = -2\int_{\partial V} (\mathbf{v}\cdot\mathbf{A})\mathbf{B}\cdot d\mathbf{S}$$

$$F_{tot} = F_{Vn} + F_{Bn} + F_{mix} + F_{\phi}$$

- Frequently used in the literature
  - Berger et al. 84, Maeshiro et al. 04, Pariat et al. 05, Démoulin & Pariat 09, Liu et al. 14a,14b; Kazachenko et al. 15, …

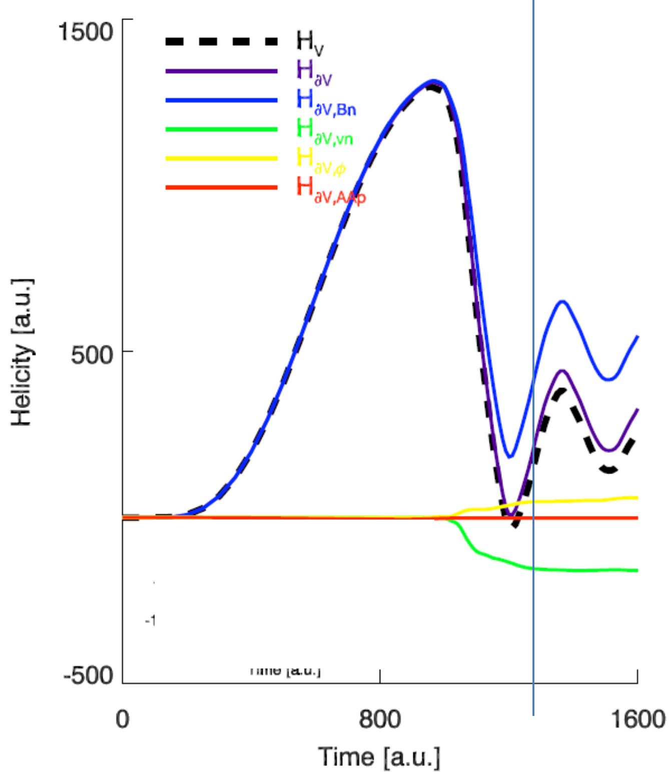
- 1st problem: helicity is not a local quantity!
  - Emergence term is different from self term of emerging structure
  - Emergence term is influenced by surrounding coronal field



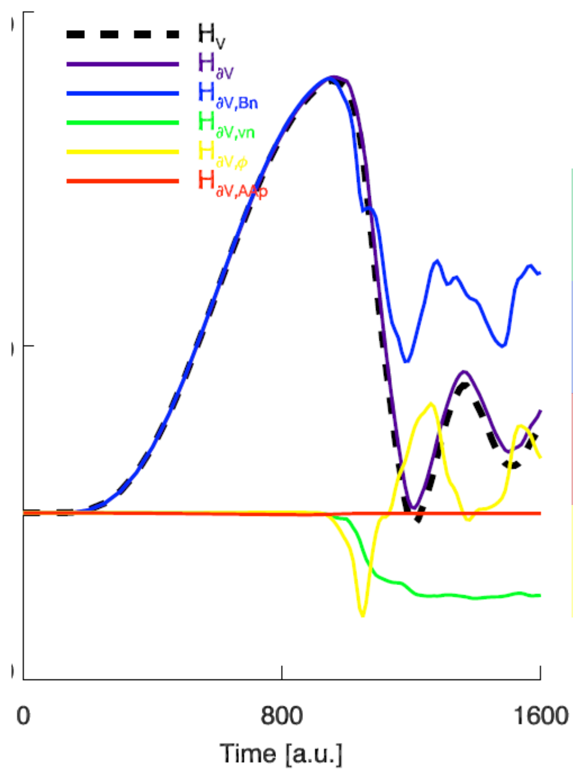
Kazachenko et al. 15

"Vn" and "Bn" flux terms

- The **"Vn"** and **"Bn"** terms are not independently gauge invariant terms!
  - Their ratio can vary when computed with different gauge set (**A**, **Ap**)
  - Their sum also varies depending on the gauges
- ➔ **the « Vn » and « Bn » terms do not convey any physical meaning**



Gauges: A: DeVore ; A_p: DeVore + Coulomb



Gauges : A & A_p: DeVore

$$F_{Vn} = 2\int_{\partial\mathcal{V}} (\mathbf{B}\cdot\mathbf{A})\mathbf{v}\cdot d\mathbf{S}$$

$$F_{Bn} = -2\int_{\partial\mathcal{V}} (\mathbf{v}\cdot\mathbf{A})\mathbf{B}\cdot d\mathbf{S}$$

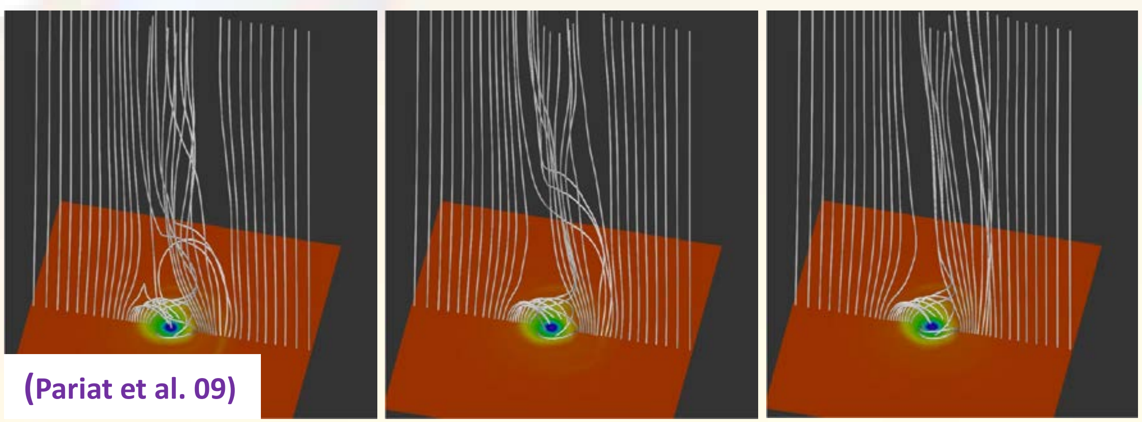
$$F_{mix} = \int_{\partial\mathcal{V}} \left((\mathbf{A}-\mathbf{A}_p)\times\frac{\partial(\mathbf{A}+\mathbf{A}_p)}{\partial t}\right)\cdot d\mathbf{S}$$

$$F_{\phi} = -2\int_{\partial\mathcal{V}} \frac{\partial\phi}{\partial t}\mathbf{A}_p\cdot d\mathbf{S}$$

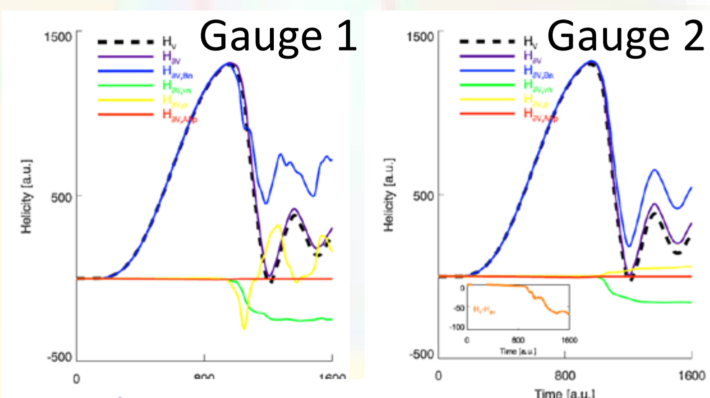
(Pariat et al. 15)

Conclusion

- Recent theoretical & numerical advances now allow the correct estimation of helicity in 3D numerical data sets.
- Estimations of the helicity conservation on an impulsive solar active like events (solar coronal jet).
  - Independent of reconnection models
  - Using several general gauges.
  - **As conjectured, magnetic helicity is very well conserved**
- **Forty years after, the Taylor conjecture can now be numerically tested in general configurations, using typical numerical data sets**
- ➔ Study and characterization of magnetic helicity in solar atmosphere: HELISOL



(Pariat et al. 09)



(Pariat et al. 15b)

Thanks for your attention