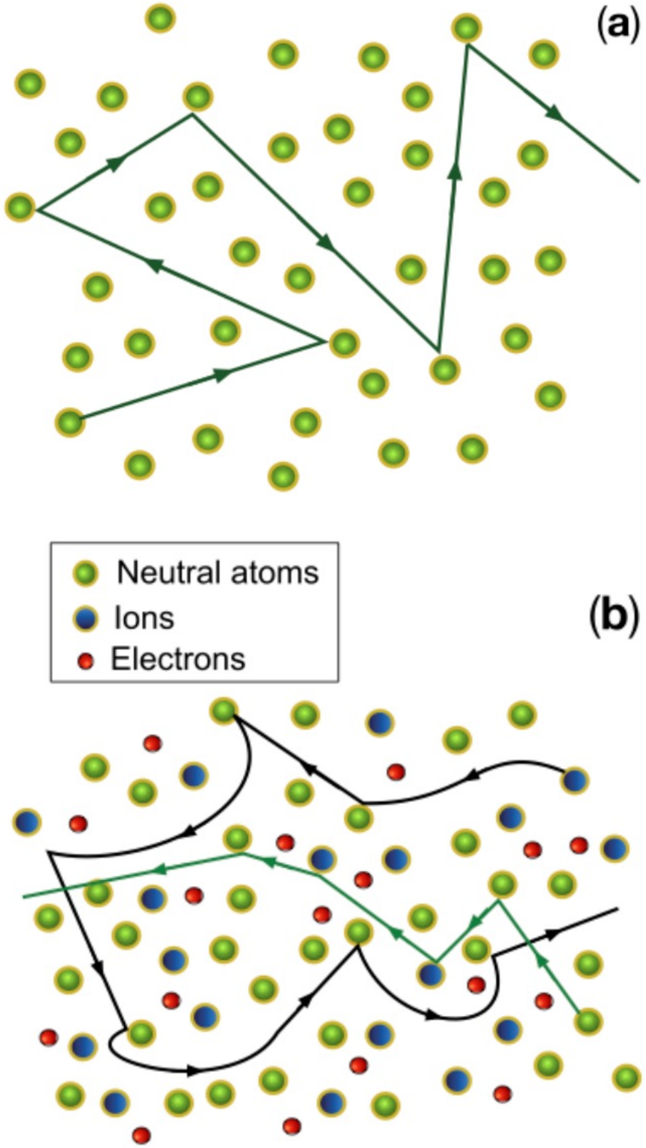
Collisions : short vs long range potentials



Short range interaction (e.g. neutral gas) :

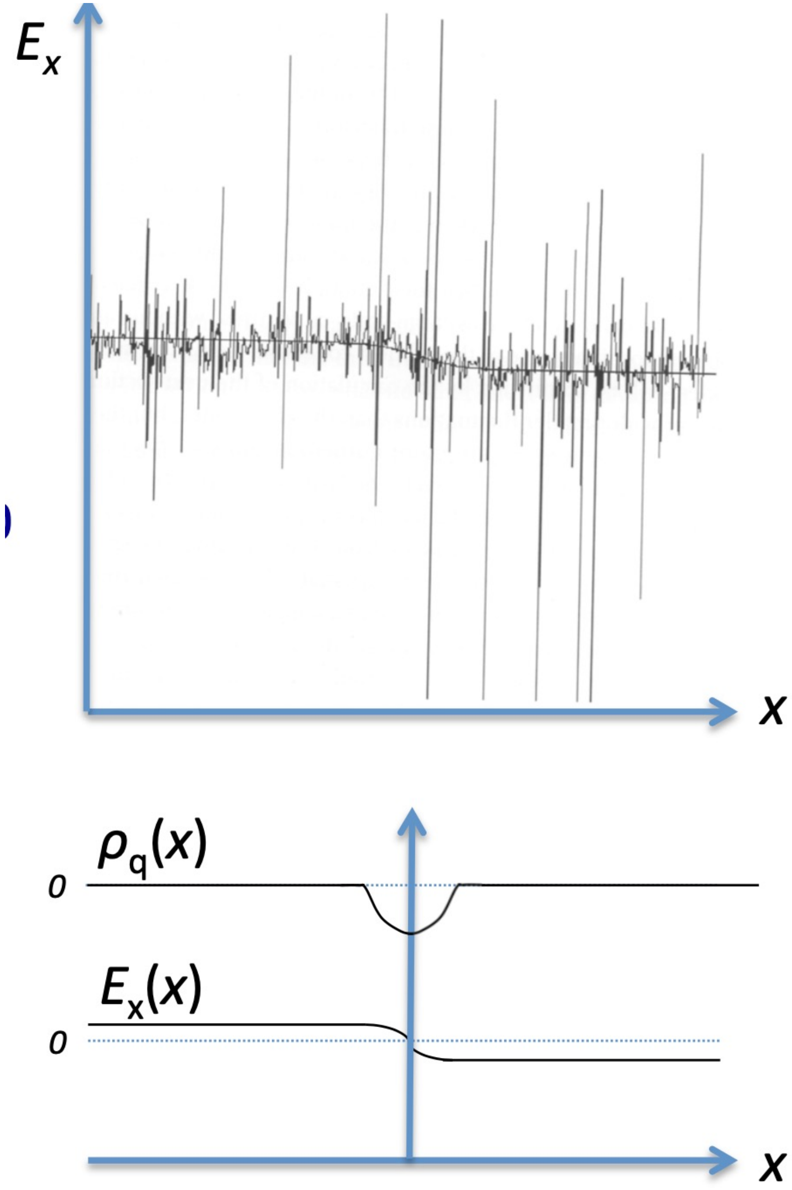
Hard sphere model, binary collisions with large angle deflections.

In a plasma : continuous interaction of the charged particle with the electromagnetic field produced by the background charge distribution

Continuous deflections…

Condition for a large angle deflection ?

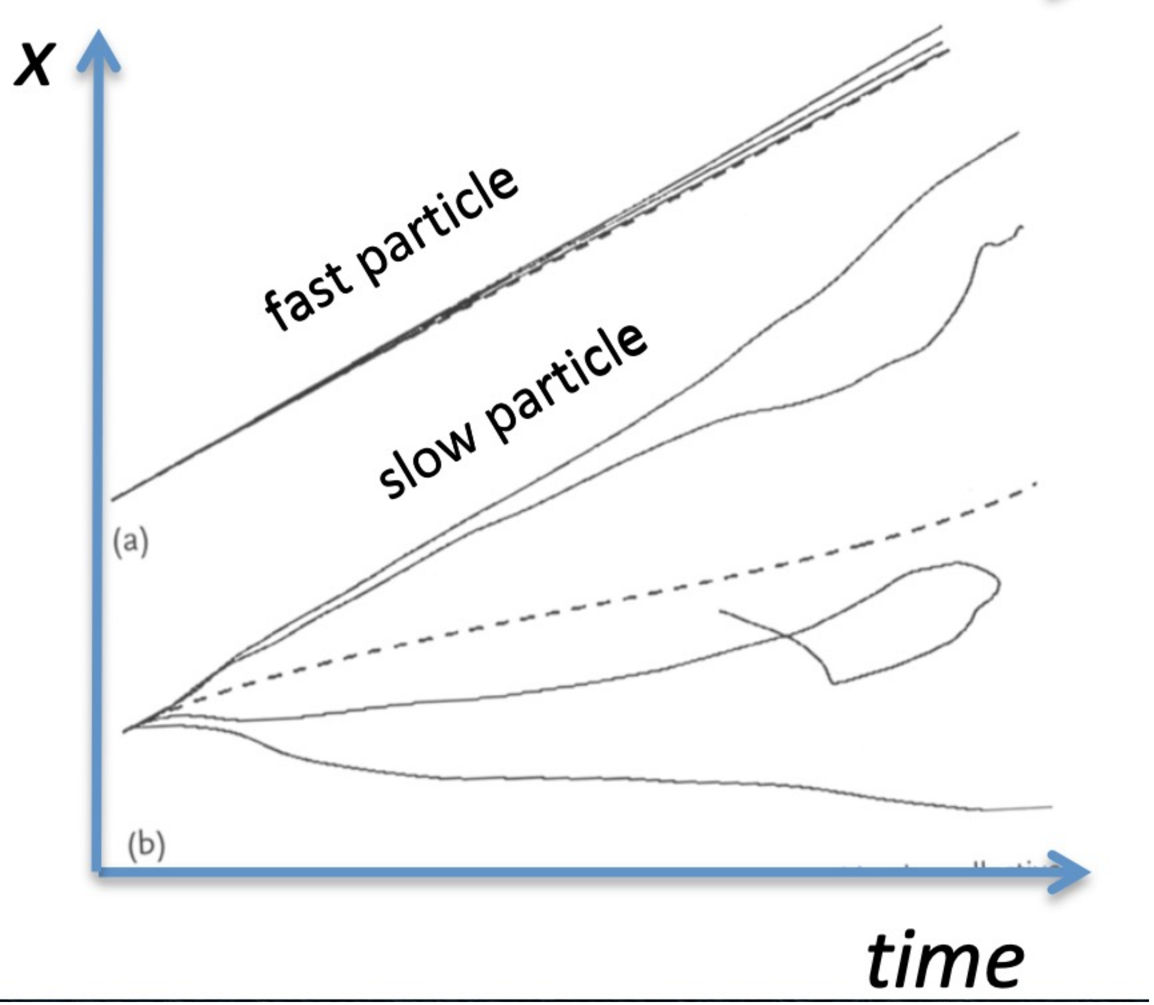
Electric field : large vs small scales fluctuations



Belmont et al., Collisionless Plasmas in Astrophysics, Wiley-VCH 2014, Fig. 4.2

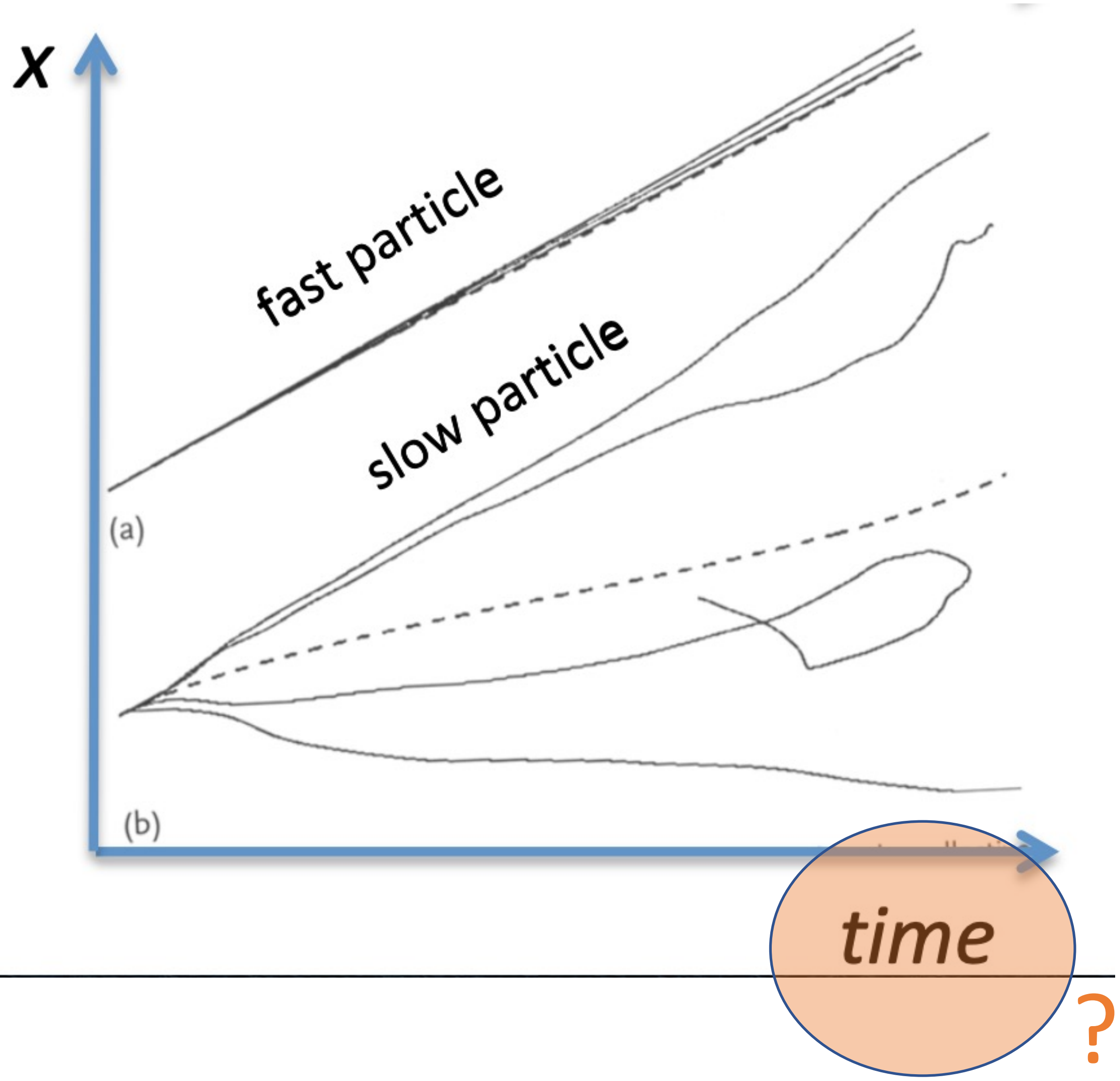
- Charge distribution in the continuous approximation given in the bottom panel
- N >> 1 particles randomly distributed according to this probability distribution
- Electric field calculated in space. Its x component plotted along the x axis on the top panel.

Particles launched in electric field fluctuations



- Nature of the trajectory rather dependent on the particle's energy
- Order of magnitude of deflection of the trajectory per unit time ?
- Roughly: $\alpha = \pi$ for $\Delta t = 1/(n\,\sigma\,v)$
- $\sigma \sim \pi\lambda_L^2$
- $\Delta\alpha/\Delta t \sim \frac{1}{nv\lambda_L^2} \propto 1/v^3$

Particles launched in electric field fluctuations



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- Number of particles within a « Landau sphere » ?

Small angle deflections

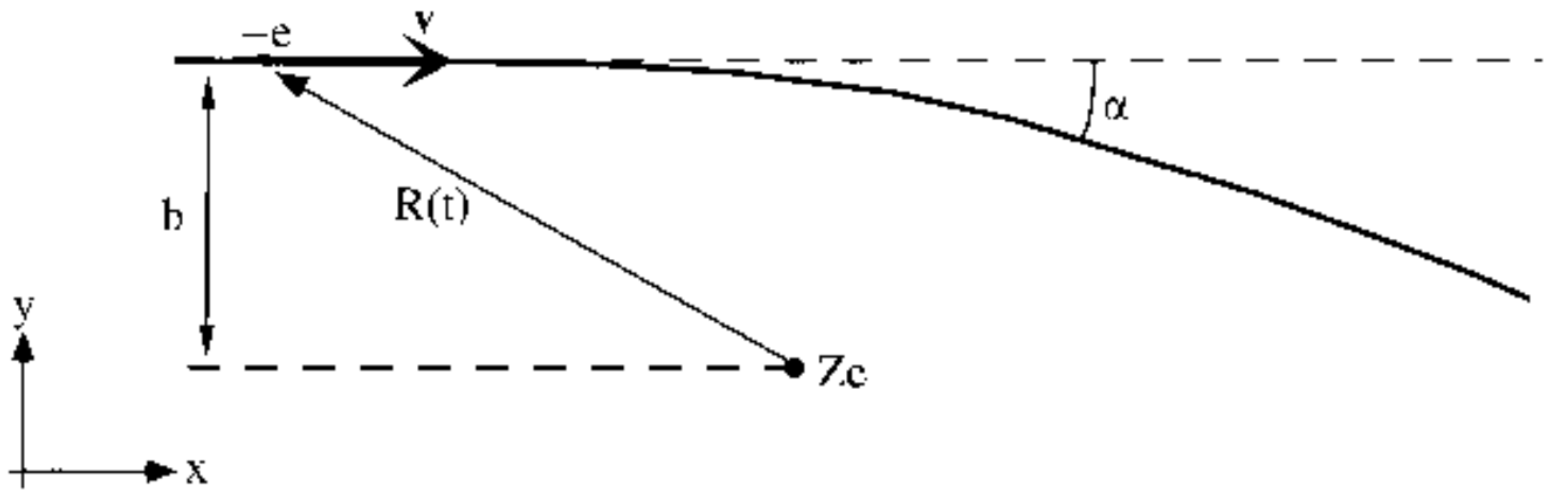

Fig. 1.1. An electron–ion collision.

From Hellander & Sigmar, 2005
Collisional transport in magnetized plasmas

We assume that the trajectory of the electron is unperturbed $r(t) = \sqrt{b^2 + v^2t^2}.$

We compute the velocity deflection over a « full interaction » $\delta\mathbf{v} = \int_{-\infty}^{\infty} \frac{d\mathbf{v}}{dt} dt = \frac{-1}{m_e} \int_{-\infty}^{\infty} \frac{Zq_e^2 \mathbf{r}(t)}{r^3(t)} dt$

Small angle deflections

We obtain $$\delta v_x = \frac{-1}{m_e}\int_{-\infty}^{\infty}\frac{Zq_e^2 vt}{r^3(t)}dt = 0 \equiv \delta v_{\|}$$

and $$\delta v_y = \frac{-1}{m_e}\int_{-\infty}^{\infty}\frac{Zq_e^2 b}{r^3(t)}dt = \frac{-2Zq_e^2}{bm_e v} \equiv \delta v_{\perp}$$

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5. You may demonstrate that a primitive of $(a+bt^2)^{-3/2}$ is $(t/a)\times(a+bt^2)^{-1/2} + const.$

The deflection during a « 1-1 » long range interaction is $$\alpha(b) \simeq \frac{|\delta v_y|}{v} = \frac{Ze^2}{2\pi\varepsilon_0 b m_e v^2} = \frac{b_{min}}{b}$$

Expression valid (as expected) if $b \gg b_{min} \equiv \lambda_L$ (approximation of electrons on a straight line justified)

Orders of magnitude and Coulomb logarithm

The most relevant impact parameter to consider in a plasma is λ_D

Indeed : the number of electrons having an impact parameter b increases (rather fastly) with b…
But for $b \gg \lambda_D$, the deflection angle $\alpha(b)$ quickly goes to zero (because the potential is exponentially cut after this distance : we may roughly speaking assume that the potential is zero after this distance)

Therefore the most typical deflection angle is around $\alpha \sim \lambda_L/\lambda_D$

So, most collisions should be producing small angle deflections if $\frac{\lambda_L}{\lambda_D} \ll 1.$

This is measured through the logarithm of this quantity, $\ln \Lambda = \ln \frac{\lambda_D}{\lambda_L} \simeq 23.4 - 1.15 \ln \frac{n}{1 \text{ cm}^{-3}} + 3.4 \ln \frac{T}{1 \text{ eV}}$

Typically in plasmas the Coulomb logarithm is $\ln \Lambda \sim 15 - 25$.

In a « real plasma », the Coulomb log has to be large, and small angle collisions have to dominate

Isotropization of an electron population on static ions

This is the « elastic problem » : ion stays at rest (mass → ∞), energy of an electron is conserved during an interaction.

The deviation angle produced during the time Δt by the sum of all ions in the « interaction volume » is zero, because of the axial symetry

$$\langle \Delta \mathbf{v}_\perp \rangle = \Delta t \frac{2Znq_e^2}{m_e} \int_0^{2\pi} \mathbf{u}_b(\phi) d\phi \int db = 0$$

The deviation of the square angle produced during the time Δt by the sum of all ions is

$$\langle \Delta \alpha^2 \rangle = \Delta t \frac{4Z^2 n q_e^4}{m_e^2 v^3} \int \frac{2\pi db}{b}$$

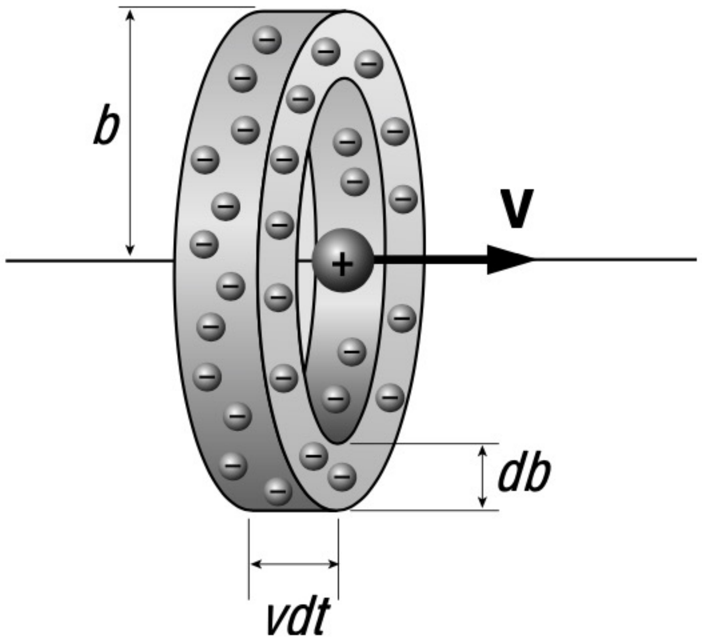


Figure 4.7 Volume d'interaction électrons-ion.

From Rax, 2005 (for our problem the sign of the charges needs to be inverted)

Signature of a diffusion process

Regularization of the integral

The divergence of the integral is due

- At large values of b, to the long range nature of the Coulomb potential. In a plasma, the potential is screened. The integral is regularized by assuming a 0 potential beyond λ_D

- At small values of b, to the divergence of the deflection angle – which is due to our approximate treatment of the problem, were we assumed small α. This is regularized by taking λ_L as a lower bound were our treatment is correct – we somehow assume that there are no particles having impact parameters smaller than the Landau radius.

- We obtain

$$\langle \Delta\alpha^2 \rangle = \frac{8\pi Z^2 q_e^4 \ln\Lambda}{m_e^2 v^3}\Delta t$$

- We can introduce the diffusion coefficient $D_\alpha = \frac{\langle \Delta\alpha^2 \rangle}{2\Delta t} = \frac{4\pi Z^2 q_e^4 \ln\Lambda}{m_e^2 v^3} = \nu_{ei}\left(\frac{v_{th}}{v}\right)^3$

- And the « thermal » electron-ion collision frequency $\nu_{ei} = \frac{4\pi Z^2 q_e^4 \ln\Lambda}{m_e^2 v_{th}^3}$

Associated friction force

The constant diffusion of an electron velocity in direction leads to the existence of a friction force.

The energy of the electron is conserved during the collision (motionless ion). So :

$$\Delta E = \frac{1}{2} m_e \left((v + \Delta v_{\parallel})^2 + \Delta v_{\perp}^2 - v^2 \right) = 0$$

From which, using $\Delta v_{\parallel} \ll v$, and averaging, we get

$$\frac{\langle \Delta v_{\parallel} \rangle}{\Delta t} = -\frac{1}{v} \frac{\langle \Delta v_{\perp}^2 \rangle}{2\Delta t} = -\nu_{ei} \frac{v_{th}^3}{v^2}$$

So that a friction force exists, with an expression

$$\mathbf{f} = \frac{m_e \langle \Delta v_{\parallel} \rangle}{\Delta t} \frac{\mathbf{v}}{v} = -m_e \nu_{ei} \frac{v_{th}^3}{v^3} \mathbf{v}$$

The Dreicer electric field

The coulomb collision mean free path increases with particle's energy (as v^4)

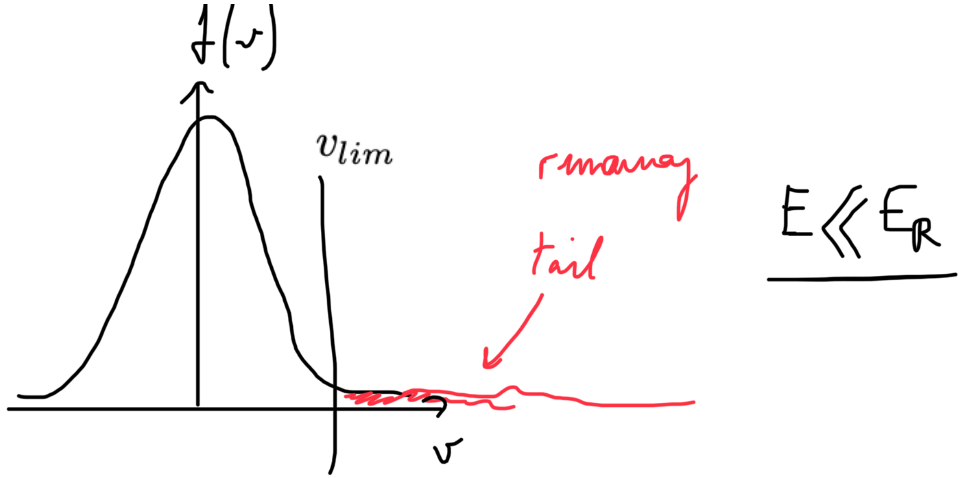
Therefore for particles in an electric field, there exists a minimal value of the velocity such that the energy gained between 2 collisions is larger than the one lost.

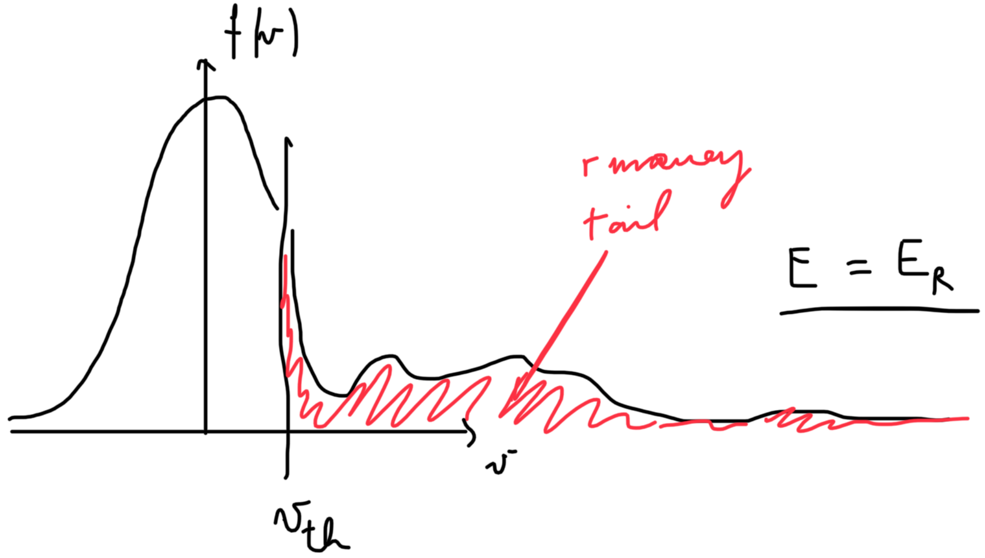
$$eE\ell_{mfp}(v_{lim}) = \frac{1}{2}mv_{lim}^2$$

Drecier field defined as the value of the electric field for which thermal particles become runaway.

$$E_R = \frac{mv_{th}\nu_{ei}}{e}. \qquad \nu_{ei} = \frac{4\pi n q_e^4 \ln \Lambda}{m_e^2 v_{th}^3},$$

Obvious limit for any fluid model since the VDF becomes completely distorted for E = E_R





Solar wind electrons : why so isotropic ?
The Dreicer electric field in the solar wind

In the solar wind, negligible electron inertia gives

$$-neE - \frac{dp_e}{dr} \sim 0$$

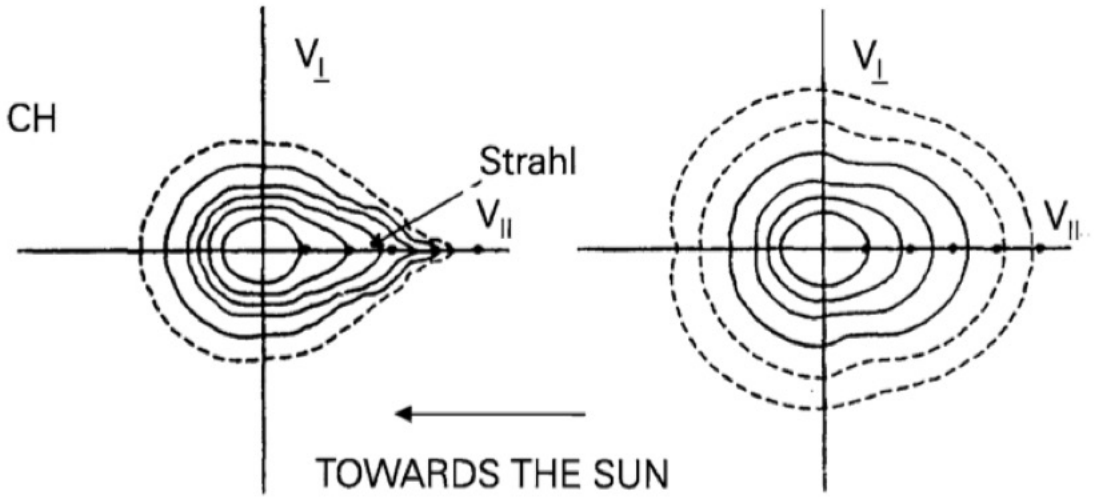
So the electric field is, in order of magnitude

$$eE \sim \frac{kT_e}{L}$$

Reminding that the Dreicer field by definition is

$$eE_R = \frac{kT_e}{\ell_{mfp}}$$

One has in the solar wind $\frac{E}{E_R} \sim K_n \sim 1$



Electric field direction

We expect to see strongly distorted VDF (moreover, in the other direction…)

This is not seen, why ?

Slowing down of a fast ion

Same problem than the isotropization of electron, but this time inelastic problem: what is the energy lost by an ion propagating through a background of cold electrons?

The variation of electron velocities are the same as previously, one obtains the energy transfer from the ion to the electron during a single interaction.

$$\delta\varepsilon = \frac{1}{2} m_e \delta \mathbf{v}_\perp^2 = \frac{2Z^2 q_e^4}{b^2 m_e v^2}$$

We integrate over all electrons in the « interaction volume » as previously defined, and obtain

$$\langle \delta\varepsilon(\delta t) \rangle = \delta t \frac{4\pi Z^2 q_e^4}{m_e v} \ln \Lambda$$

Defining the ion slowing down frequency as $\frac{d\varepsilon}{dt} = -\nu_{ie}\varepsilon,$ one obtains $\nu_{ie} = \frac{8\pi Z^2 q_e^4}{m_e m_i v^3} \ln \Lambda \equiv \frac{m_e}{m_i} \nu_{ei} \ll \nu_{ei}$

Ionization cross-section: Thomson's formula

The same procedure can be used to calculate the energy transfer from a free electron to an electron bound to an atom, in a purely classic approximation.

We have $\delta\varepsilon(E,b) = \frac{q_e^4}{Eb^2}$

Defining the ionization cross section as $d\sigma = 2\pi b db = \frac{\pi q_e^4}{E\delta\varepsilon^2} d\delta\varepsilon.$

We obtain the total ionization cross section by considering energy transfers between the energy of the bound electron (E_I) and the impacting electron energy (that can't transfer more than its own energy). We obtain

$$\sigma_I = \int_{E_I}^{E} \frac{\pi q_e^4}{E\delta\varepsilon^2} d\delta\varepsilon = \pi q_e^4 \frac{E - E_I}{E^2 E_I} = \frac{\pi q_e^4}{E_I^2}\left(\frac{E_I}{E} - \frac{E_I^2}{E^2}\right)$$

Ionization cross-section: Thomson's formula

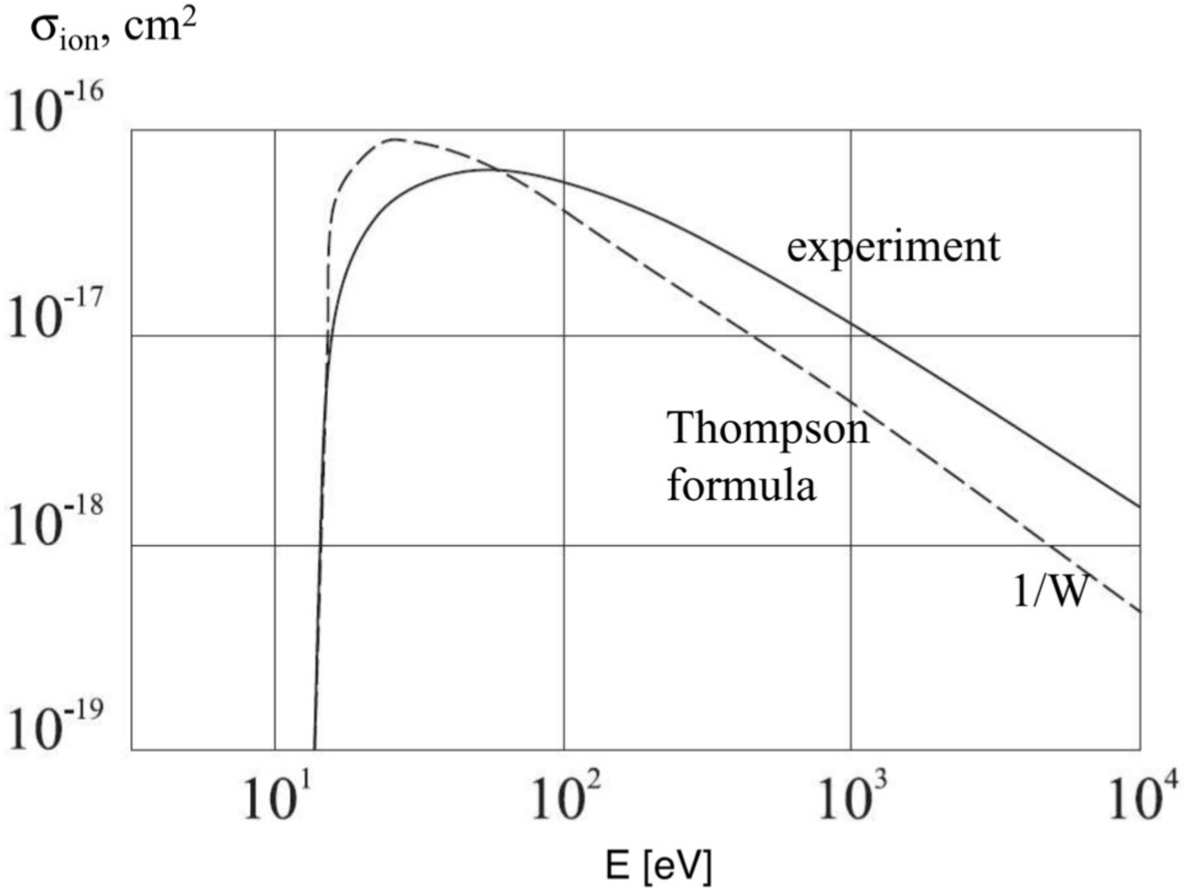
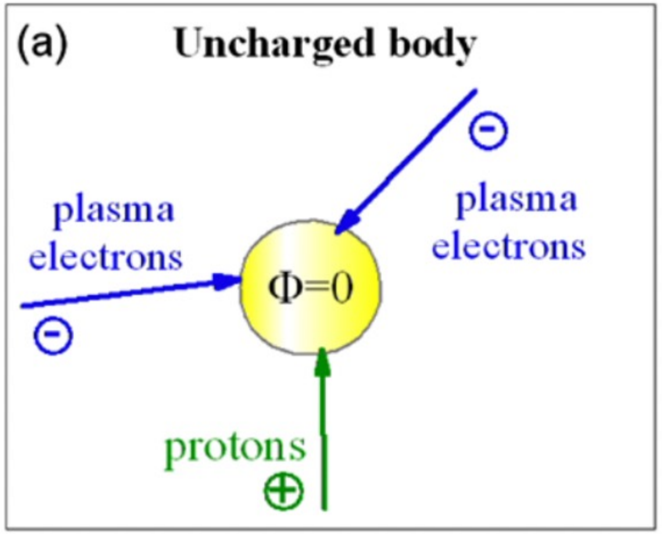
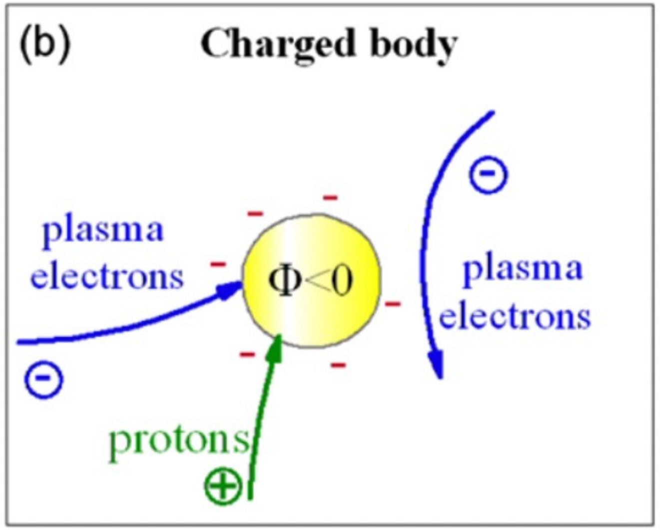


FIGURE 5 – Ionzation cross section as a function of the incident electron's energy and comparison of the classical formula to experiment.

Charge/Potential of a macroscopic object in a plasma





The charge of an object is governed by the current incoming on its surface

$$\frac{dQ}{dt} = I_e + I_i + I_{ph} + I_{sec} + \ldots$$

We assume a planar object of surface S, with normal axis z. The object has potential φ. The particles having a charge with the same sign as φ will be repelled, so that only that with velocity larger than $\sqrt{2q\varphi/m}$ can reach the surface

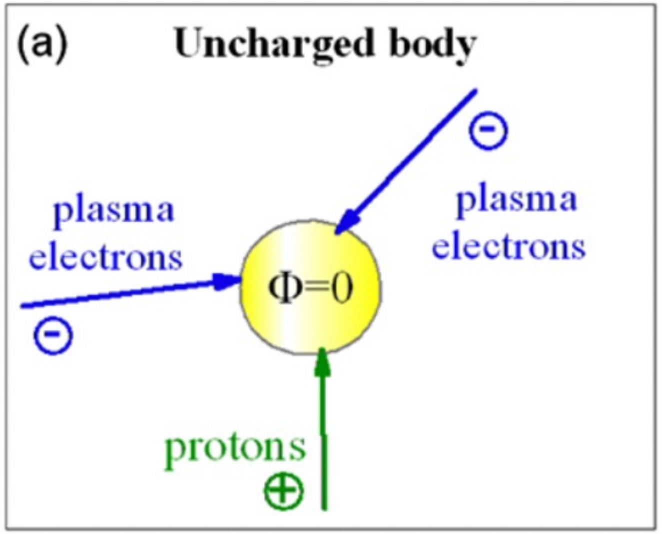
$$I_\alpha(\varphi) = q_\alpha S \int_{\sqrt{2q_\alpha\varphi/m_\alpha}}^{\infty} \frac{n_\alpha}{\sqrt{2\pi}v_{th,\alpha}} e^{-v_z^2/2v_{th,\alpha}^2} v_z dv_z = I_{\alpha,0} \exp\left(-\frac{q_\alpha\varphi}{kT_\alpha}\right)$$

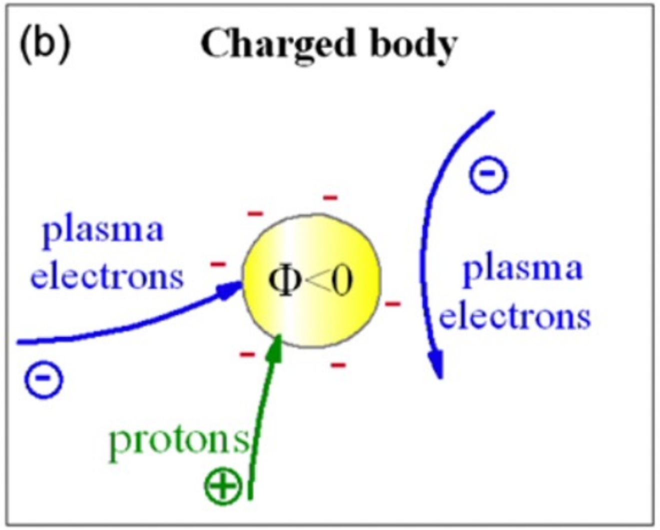
All the particles with charge of opposite sign than φ can reach the surface :

$$I_\alpha(\varphi) = q_\alpha S \int_0^{\infty} \frac{n_\alpha}{\sqrt{2\pi}v_{th,\alpha}} e^{-v_z^2/2v_{th,\alpha}^2} v_z dv_z = I_{\alpha,0}$$

with $I_{\alpha,0} = q_\alpha n_\alpha v_\alpha S$ and $v_\alpha = (kT_\alpha/2\pi m_\alpha)^{1/2}$

Object in an ion/electron plasma



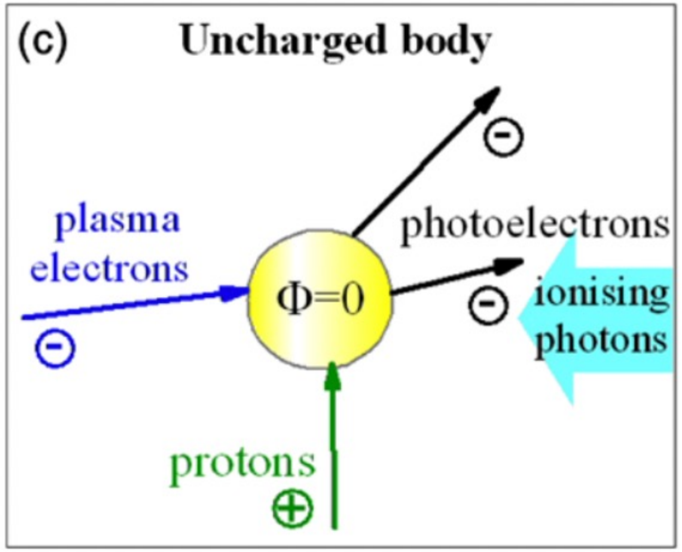


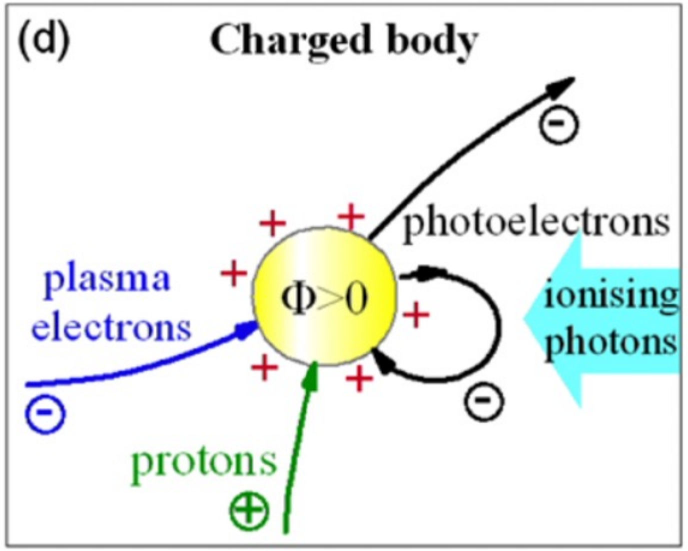
- At $\varphi = 0$, the electron current is much larger than the ion current (because of the small electron mass)
- Therefore the object will be charged negatively at equilibrium : $\varphi_{eq} < 0$
- The equilibrium potential is reached for dQ/dt = 0
- Using the previous expressions for the current, we obtain

$$\varphi_{eq} = \frac{kT}{e} \ln\left(\frac{I_{i,0}}{I_{e,0}}\right) = -\frac{kT}{2e} \ln\left(\frac{m_i}{m_e}\right)$$

- The object's charge depend on the plasma(electron) temperature only.
- The expression for the ion saturation current may be in reality quite different from our simple approximation… (sound speed replace the value of v_i – Bohm's criterion).

Object illuminated by UV: photo-electric effect





Under the action of solar UV, photo-electrons are produced that will escape the surface and tend to charge it positively. If $\varphi = 0$, the photo-emission is depending on the material photoelectric yield, and on the intensity of ionizing radiation.

$$I_{ph}(\varphi = 0) = j_{ph,0} S$$

With $j_{ph,0} \sim 50\ \mu A/m^2$ at 1AU. In comparison, the electron current density from the solar wind at 1 AU is $j_e \sim 0.5\ \mu A/m^2$. Therefore the object will tend to charge positively under the action of the photoelectric effect.

If $\varphi > 0$, one has $I_{ph}(\varphi) = j_{ph,0} S \exp\left(-\frac{e\varphi}{kT_{ph}}\right)$

With the photo-electron temperature $T_{\rm ph} \sim 3$ eV

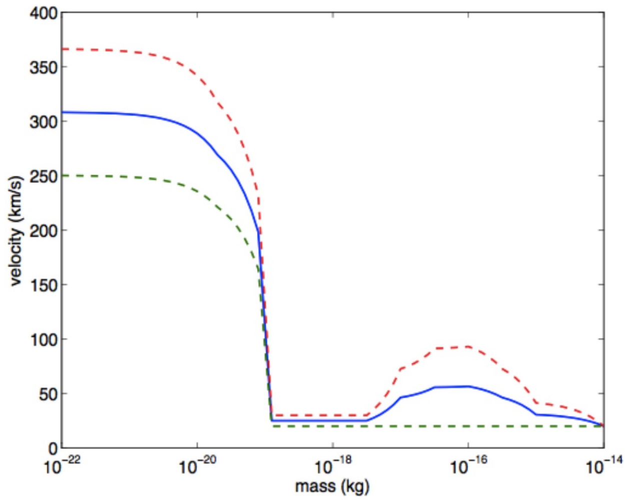
In such condition the potential of the object is $\varphi_{eq} = \frac{kT_{ph}}{e} \ln\left(\frac{j_{ph,0}}{e n_e v_e}\right)$

Dynamics of dust grains in the interplanetary medium

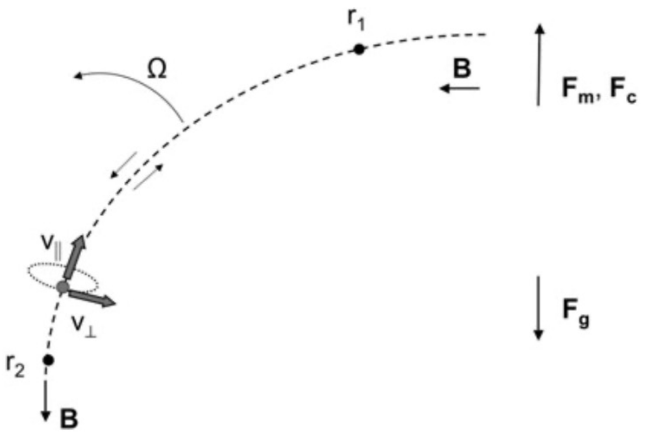
The charge of a dust grain in the interplanetary medium is $Q \sim 4\pi\varepsilon_0 a\varphi_{eq} > 0$, with a the dust radius.

This charge is roughly independent of the distance to the Sun (cf previous equation for φ_{eq}, in which the current density from the solar wind and the illuminating photon flux both decrease as $1/r^2$).

So the charge on mass ratio decreases as $\sim 1/r^2$: small particles have much larger charge to mass ratio than large particles.



Velocity vs mass

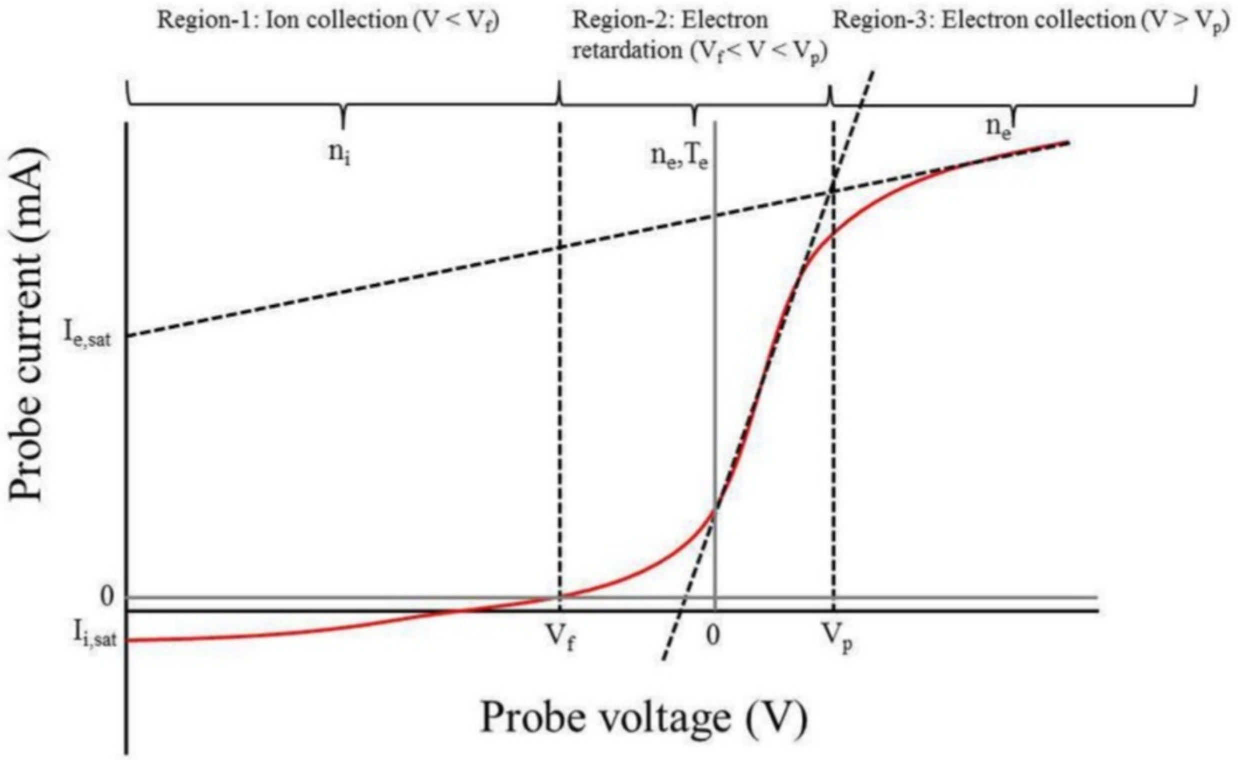


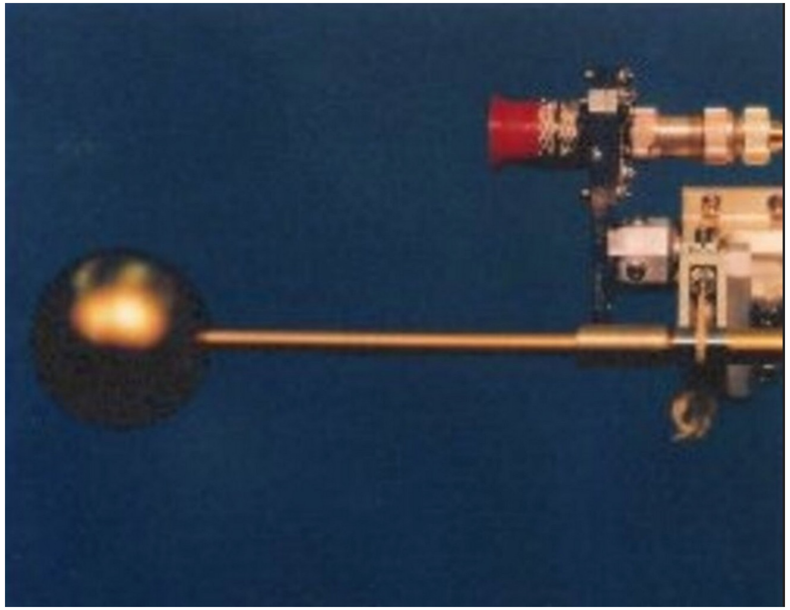
Nano-dust trajectory

Principle of the Langmuir probe

The working principle is to bias the probe to a known potential V, and to measure the plasma current I(V) flowing through the probe.

From the caracteristic curve, and the expressions for the currents obtained before, we can derive the plasma density and temperature.





Langmuir probe on the CASSINI spacecraft (journey to Saturn)