Space plasmas

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Outline of the course

- 3 first lectures by Arnaud Zaslavsky: an introduction to plasma physics, collective behaviour in a plasma, macroscopic object in a plasma, particle dynamics in electromagnetic fields.

- 5 next lectures by Olga Alexandrova: MHD description of plasmas

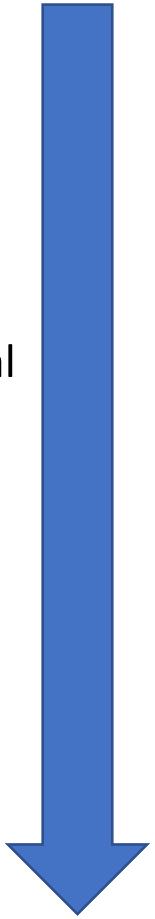
- 2 last lectures by Arnaud Zaslavsky: Kinetic description of plasmas

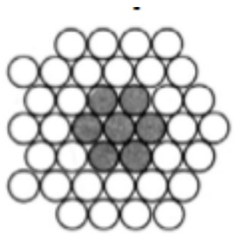
Some bibliography:

- F. Chen, *Introduction to Plasma Physics and Controled Fusion*
- W. Baumjohann & R.A. Treumann, *Basic Space Plasmas Physics*
- N. Meyer-Vernet, *Basics of the Solar Wind*
- J.-M. Rax, *Physique des Plasmas*
- G. Belmont, L. Rezeau, C. Riconda & A. Zaslavsky, *Introduction to Plasma Physics / Introduction à la physique des plasmas.*

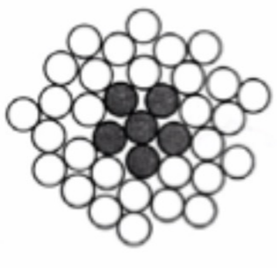
The fourth state of matter (?)

Internal energy

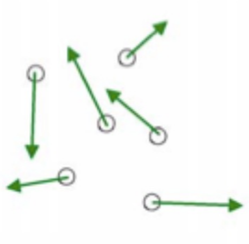




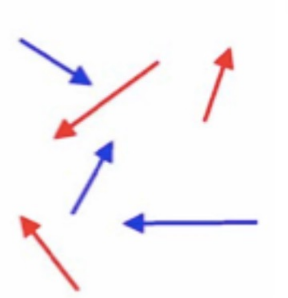
- Solid : atoms have fixed position with respect to each other + vibrations



- Liquid : atoms do not have fixed position, but strong interactions with neighbors (incompressibility) still prevail



- *Gas : free atoms (compressible) + short interactions (collisions)*

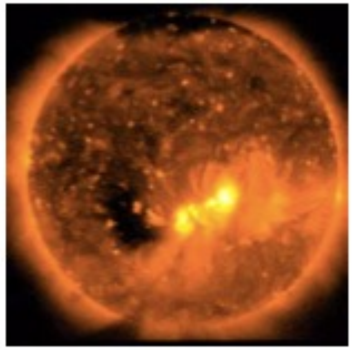


- *Plasma : Free electrons + ions (+ neutrals…). Collisions + long range interaction (Lorentz force) induce collective behaviours.*

(not strictly speaking a « phase » since no phase transition from gas to plasma, but a continuous increase of the degree of ionization with increase of T)

Producing a plasma from a gas

- Heating: $kT \sim W_{ionization}$



- Photo-ionizing: $h\nu \sim W_{ionization}$

- Electrifying: $eE\ell_{mfp} \sim W_{ionization}$

Plasmas in the universe

- Plasma state is an « electrified gas », that is, a gas in which a sufficient fraction of atoms or molcules are dissociated into positive ions and electrons for Lorentz forces to play a significant role in the dynamics.

- 99 % of the matter in the universe under plasma state: probably not a very accurate evaluation, but stellar interiors and atmospheres, and interstellar nebulae – and much of the interstellar hygrogen in general is under plasma state.

Table 1: An overview of parameters of ionised gases in the Universe (adapted from J.A. Irwin, Astrophysics - Decoding the Cosmos, Wiley 2007)

Location	n_e [m^{-3}]	$n_m + n_H$ [m^{-3}]	T [K]	Reference
Solar corona	10^{14}	$\simeq 0$	10^6	Koutchmy 1994, Adv. Space Res. 14(4), 29
Interplanetary space (1 AU)	$10^6 - 10^8$	$\simeq 0$	$10^4 - 10^5$	Schwenn 1991, dans "Physics of the Inner Heliosphere", Springer
Stellar interiors	10^{33}	$\simeq 0$	$10^{7.5}$	
Planetary nebulae	$10^9 - 10^{11}$	$\simeq 0$	10^4	Zhang et al. 2004, MNRAS 351, 935
H II regions	$10^8 - 10^9$	$\simeq 0$	$10^3 - 10^4$	Feng-Yao Zhu et al 2019, ApJ 881, 14
Interstellar medium	$10^3 - 10^7$	$10^4 - 10^{11}$	10^2	compare with lecture on the ISM
Intergalactic space	< 10	$\simeq 0$	$10^5 - 10^6$	

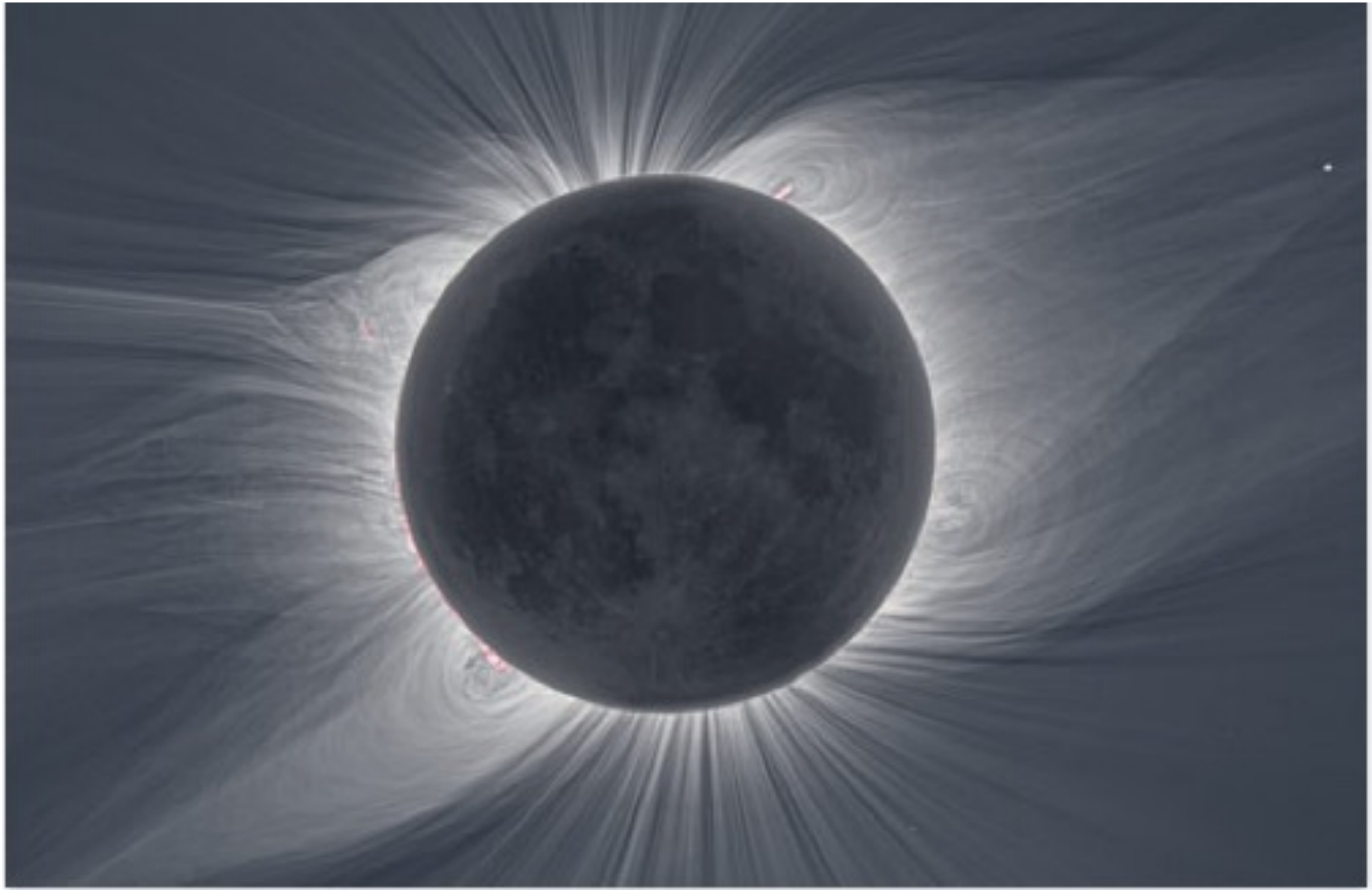
number densities: n_e free electrons, n_m molecules, n_H neutral hydrogen

HII region in the Large Magellanic Cloud

Cygnus A radio galaxy

radio and X-ray image: relativistic electrons (radio, red) and hot plasma (X-rays, blue)

Heart of the Heliosphere: the Sun

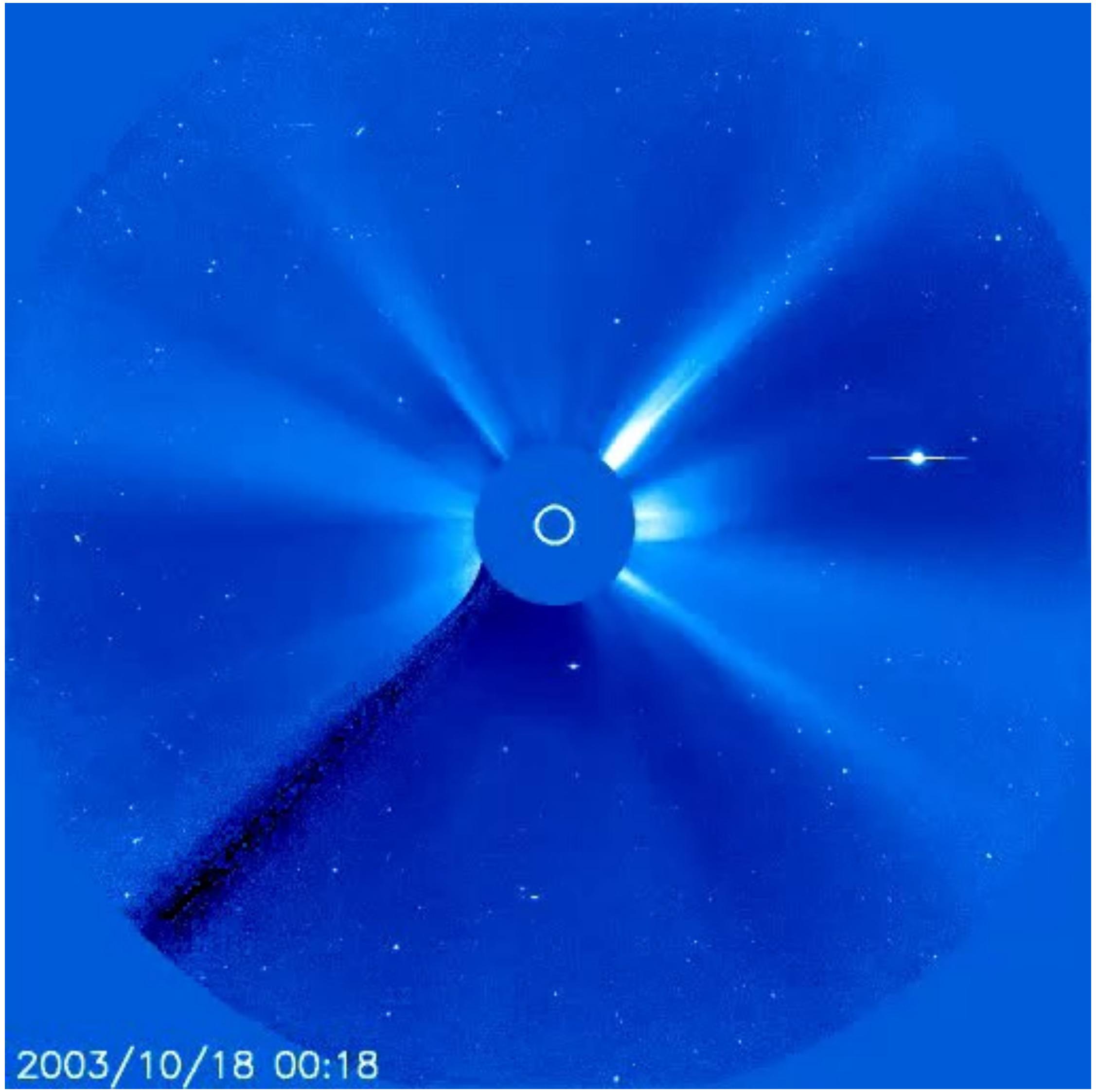


The solar corona in white-light during a total eclipse.

(Compton scattering)

A medium structured by the magnetic field.

A dynamic atmosphere: solar wind and flares

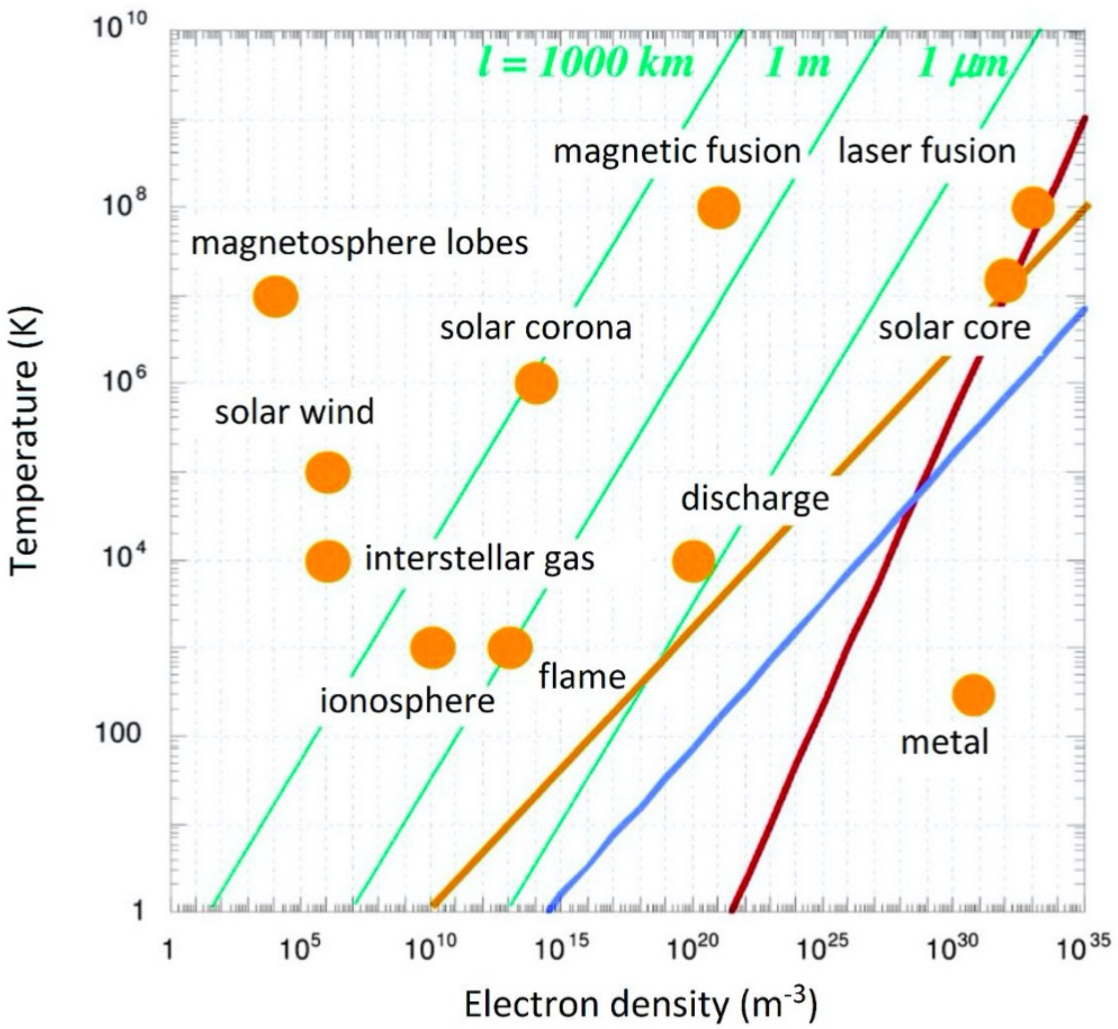


The interplanetary medium seen from the earth neighborhood.

SOHO/LASCO C2 coronograph (ESA/NASA SOlar and Heliospheric Observatory)

(White light)

Some plasmas in a (n,T) diagram



Ionization and recombination

- Plasma state consists in a dynamic « chemical » equilibrium between ionization and recombination processes

$$A \rightleftarrows A^+ + e^-$$

- Two main agents to trigger this reaction: electrons, or photons

$$A + e^- \rightarrow A^+ + e^- + e^-$$

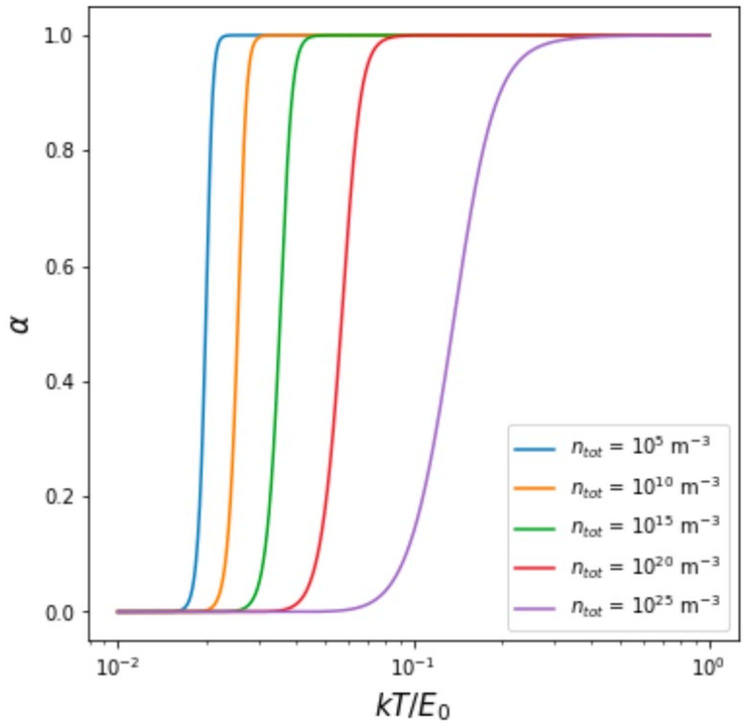
$$A + h\nu \rightleftarrows A^+ + e^- .$$

- Reaction rates given by the usual law : $\frac{dn_A}{dt} = -kn_A n_e$ (first example)
- The reaction constant is related to the interaction cross section by $k = \langle \sigma v \rangle \simeq \sigma v_{the}$

Thermal equilibrium: Saha's equation

- In thermal equilibrium, the ionization degree is given by Saha's equation (here for hydrogen atoms)

$$\frac{n_e n_i}{n_a} = \frac{e^{-W/kT}}{\lambda_B^3} \equiv K(T), \qquad \lambda_B = \frac{\hbar}{m v_{th}} = \frac{\hbar}{(mkT/2\pi)^{1/2}}.$$



The degree of ionization gets important at temperature relatively lower than the ionization energy, at least for sparse media (because recombination is not efficient)

Efficient for studying stellar surfaces / external parts

Not usueful for most plasma states, which are not in equilibrium

Example study : chapman's ionization layer

- We consider the photo-ionization reaction, caracterized by a cross section σ_{ph}

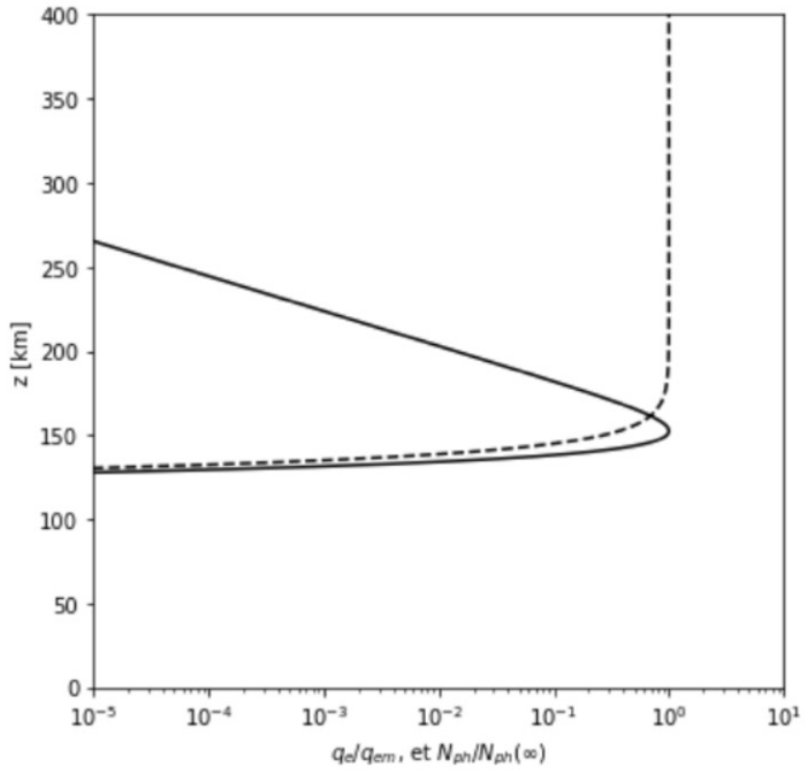
$$A + h\nu \rightleftarrows A^+ + e^-.$$

- We consider the gas to be illuminated by a photon flux density $N_{ph}(z)$

- Then $\frac{dN_{ph}}{dz} = n_A(z)\sigma_{ph}N_{ph}$ and $N_{ph} = N_\infty \exp - \int_z^\infty n_A(z)\sigma_{ph}dz$

- The electron production rate per unit volume is $Q_e = \frac{dN_{ph}}{dz} = n_A(z)\sigma_{ph}N_{ph}$ since one electron is produced per photon absorbed

- We obtain $Q_e = Q_{max} \exp\left(1 - y - \exp -y\right)$ with $y = \frac{z - z_m}{H}$, assuming the atmosphere is in hydrostatic isothermal equilibrium (H = kT/mg, $z_m = H \ln \sigma_{ph} H n(0)$)



Chapman's ionization layer : recombination

We can consider two main recombination processes:

- Radiative recombination $X + h\nu \rightleftarrows X^+ + e^-$

The cross section of these processes are $\sigma_{ph} \sim 10^{-22}\ \mathrm{m}^2, \qquad \sigma_{rad} \sim 10^{-24}\ \mathrm{m}^2$

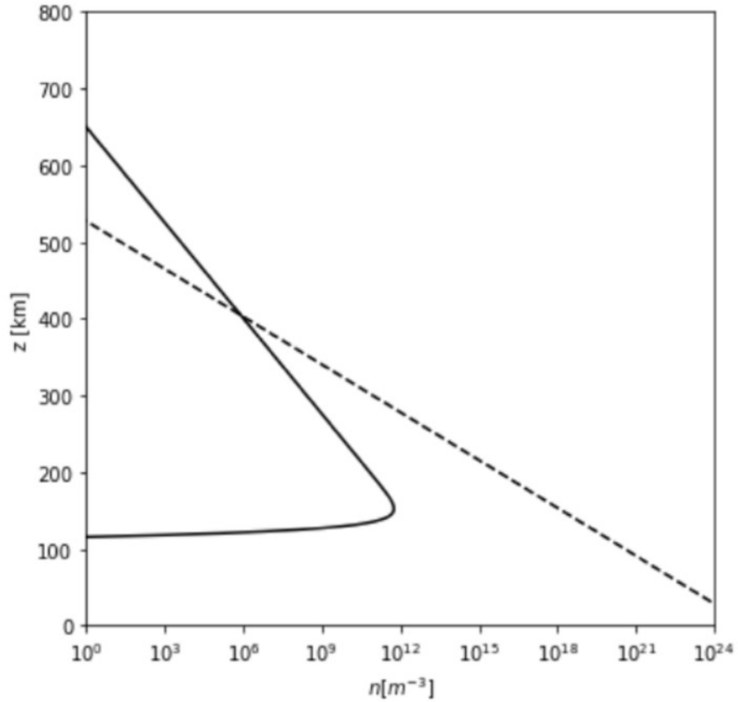
- Dissociative recombination $XY^+ + e^- \to X^* + Y^*$

The cross section of this process is $\sigma_{dis} \sim 10^{-18}\ \mathrm{m}^2$

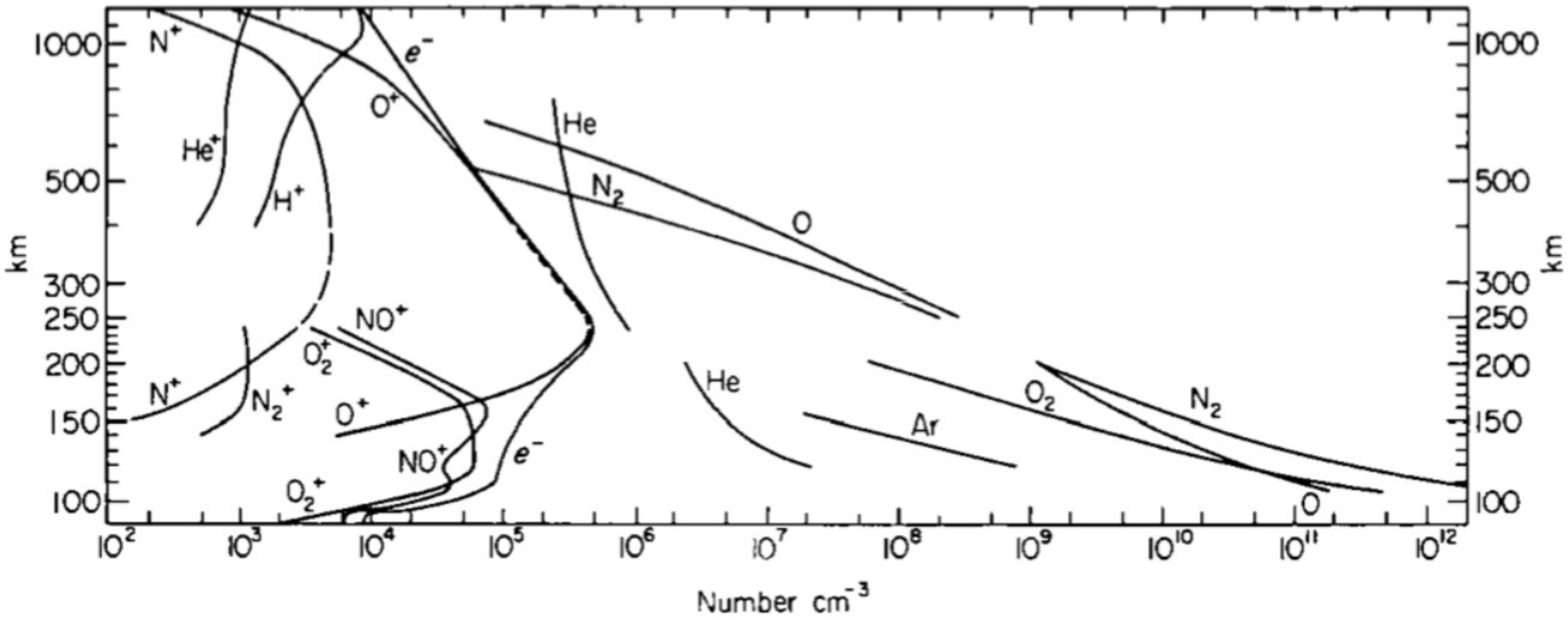
- In presence of diatomic molecules (terrestrial ionosphere…) the second dominates. We have a recombination rate $R = k_{dis} n_{XY^+} n_e = k_{dis} n_e^2$ with $k_{dis} \simeq \sigma_{dis} v_{the}$
- The electron density in the ionization layer is then given by $n_e(z) = \sqrt{Q_e(z)/k_{dis}}$

Chapman's ionization layer : model vs observations

Our simple model:



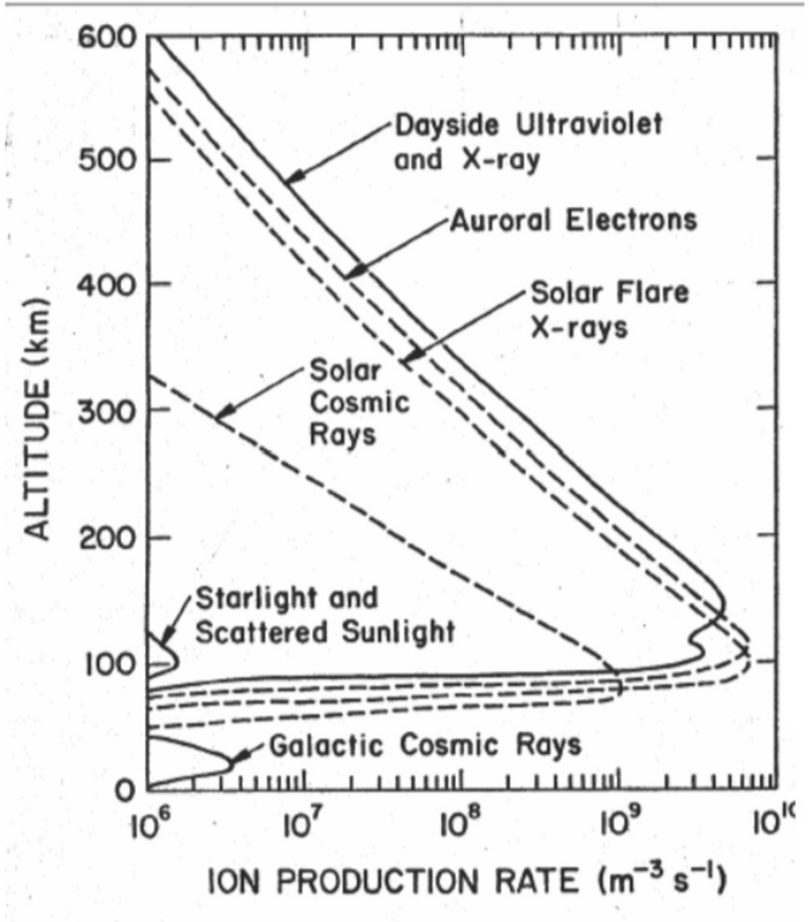
Measurements in the terrestrial ionosphere:



Origin of the airglow ?

Seen from the international space station

Other ionization processes in the ionosphere



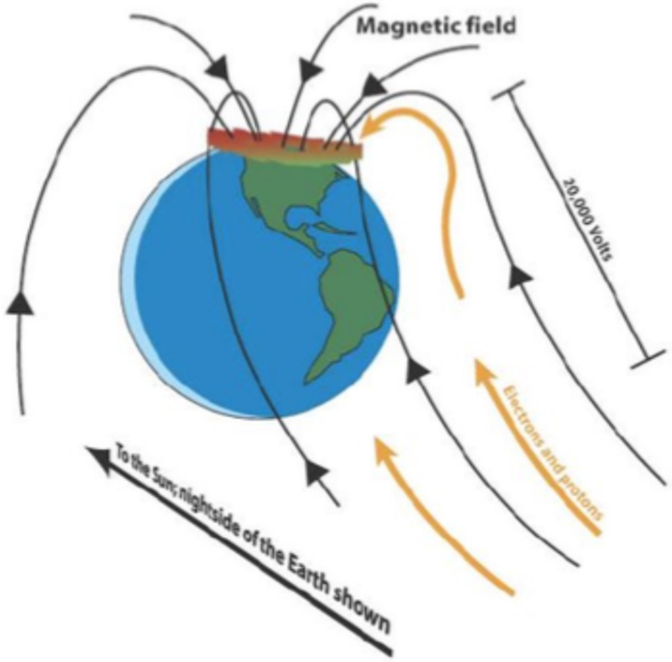
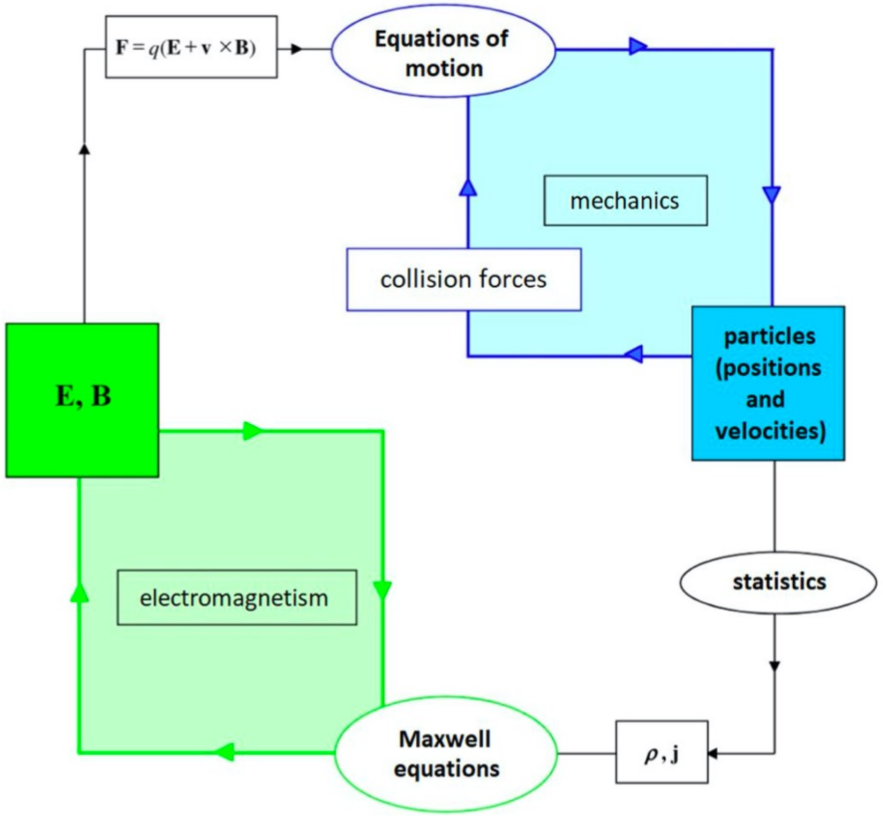


Photo credit: Fabrice Mottez

Plasma physics: collective phenomena

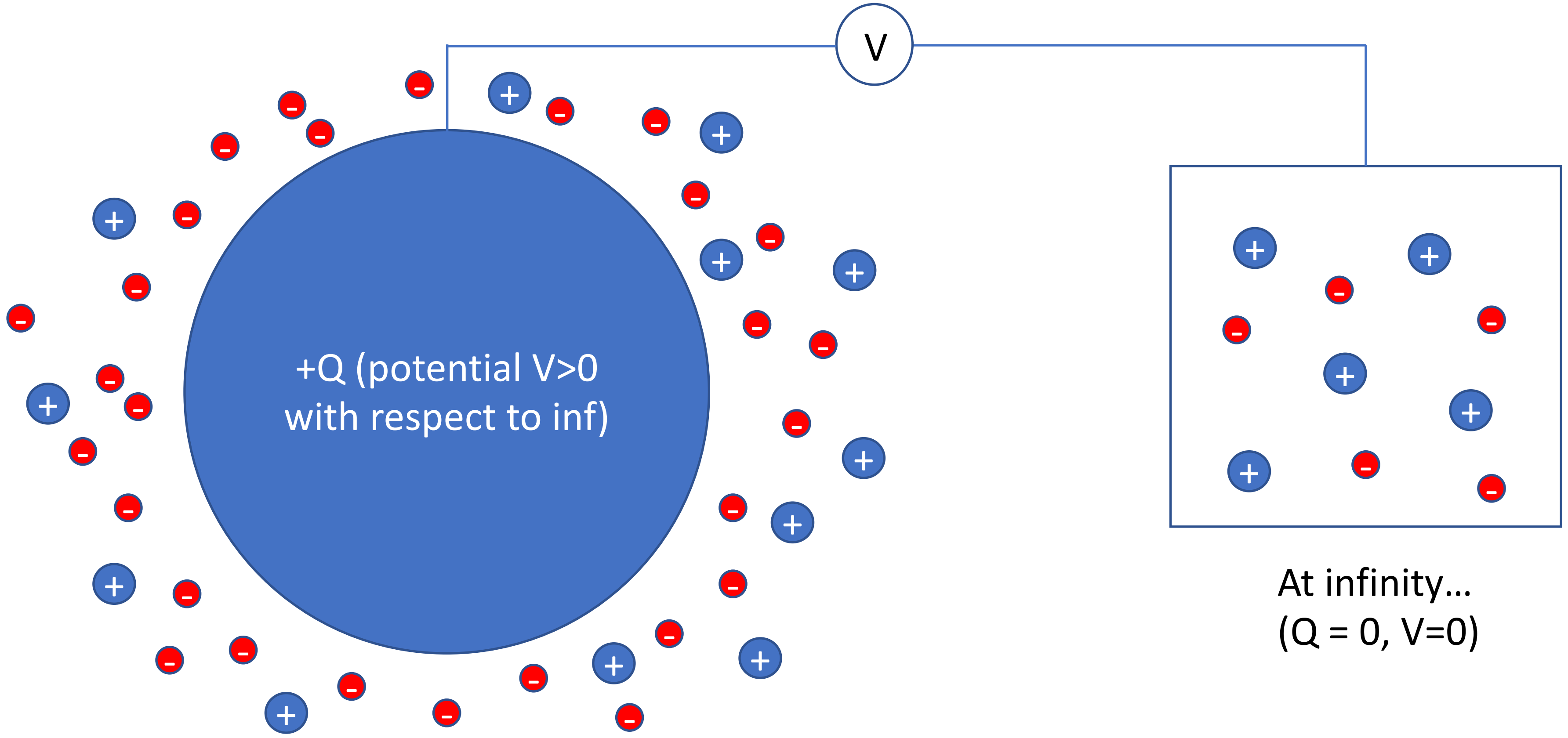
- As we shall see in a forthcoming lecture, collisions may not be very effective in a plasma, and the physics is not only controled by particle-particle interactions, but also by interactions of particles with a mean electromagnetic field.



Collective plasma phenomena, general scheme

The Debye screening

- Question : what is the capacitance of a sphere of radius a in vacuum ?
- More complicated question: what is the capacitance of a sphere of radius a in a plasma ?



The Debye screening

- Space distribution of charges influenced by the presence of the biased sphere. The total charge sphere + plasma stays neutral overall.

$$n_e(r) = n_0 \exp(e\varphi(r)/kT), \qquad n_i(r) = n_0 \exp(-e\varphi(r)/kT)$$

- The potential is given by Poisson's equation

$$\Delta\varphi(r) = -\frac{2en_0}{\varepsilon_0} \sinh\left(\frac{e\varphi(r)}{kT}\right).$$

- We assume $\frac{e\varphi}{kT} \ll 1$, and obtain the potential $\varphi(r) = \frac{Va}{r} e^{-(r-a)/\lambda_D}$

- The potential is screened over a distance $\lambda_D^2 = \frac{\varepsilon_0 kT}{2n_0 e^2}.$

- The charge in the sheath is $Q_{sheath} = e\int_a^\infty (n_i - n_e) 4\pi r^2 dr = -4\pi\varepsilon_0 a(1 + \frac{a}{\lambda_D})V.$

Debye screening, plasma parameter

- The charge on the sphere is $-Q_{sheath}$, we deduce the capacitance

$$C = 4\pi\varepsilon_0 a(1 + \frac{a}{\lambda_D})$$

- Potential of a point charge in a plasma (Yukawa potential) : $\varphi(r) = \frac{q}{4\pi\varepsilon_0 r} e^{-r/\lambda_D}$

- Electroneutrality : non-zero space charge in a plasma is screened over a (short) distance λ_D. A plasma can always be assumed to be quasi-neutral when its behaviour is considered over distances (gradient scales) much larger than this distance.

- The calculation made above (and quasi-neutrality) assumed that $\Gamma = \frac{1}{n\lambda_D^3} \ll 1$

- Γ is called the plasma parameter. The previous condition is easily fulfilled in all (even very sparse) astrophysical ionized media (solar wind ?)

Quasi-neutrality condition: example of an ionized atmosphere.

- Consider the case of an ionized atmosphere, ions of mass m_i and charge +e, electrons of mass m_e, both at temperature T.
- Density profiles for the species : $n_j(z) = n_{0,j}\exp(-\frac{z}{H_j})$, with $H_e \gg H_i$
- Neutrality problem: an electric field arise.
- Taking into a steady-state electric field E_{PR} in the force balance, while imposing everywhere quasi-neutrality (assumption $\frac{\lambda_D}{H} \ll 1$), we obtain

$$-2kT\frac{dn}{dz} - n(m_e + m_i)g = 0 \Rightarrow n(z) = n_0 e^{-z/H}, \qquad H = \frac{2kT}{m_i g}$$

- And the Panekoek-Rossland electric field $E_{PR} = mg/2e$
- Stellar atmospheres, ionospheres…

Collective phenomena: time response

- Consider the problem of the time evolution of an electron density perturbation in the otherwise quasi-neutral plasma.

- The initial perturbation is assumed invariant per translation on the y and z axis : problem dependent only on the x coordinate

- Dynamics of the electron fluid given by

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e u_e}{\partial x} = 0$$

$$n_e m_e \frac{\partial u_e}{\partial t} + u_e \frac{\partial n_e u_e}{\partial x} u_e = -e n_e E.$$

- Ions assumed motionless (they are too heavy to respond quickly, we can check afterwards)

- Electric field given by the Gauss equation $$\frac{\partial E}{\partial x} = -\frac{e(n_e - n_0)}{\varepsilon_0}$$

The plasma oscillation

- The linearized equation system is

$$\frac{\partial \delta n_e}{\partial t} + n_0 \frac{\partial \delta u_e}{\partial x} = 0$$

$$n_0 m_e \frac{\partial \delta u_e}{\partial t} = -e n_0 \delta E.$$

$$\frac{\partial \delta E}{\partial x} = -\frac{e \delta n_e}{\varepsilon_0}$$

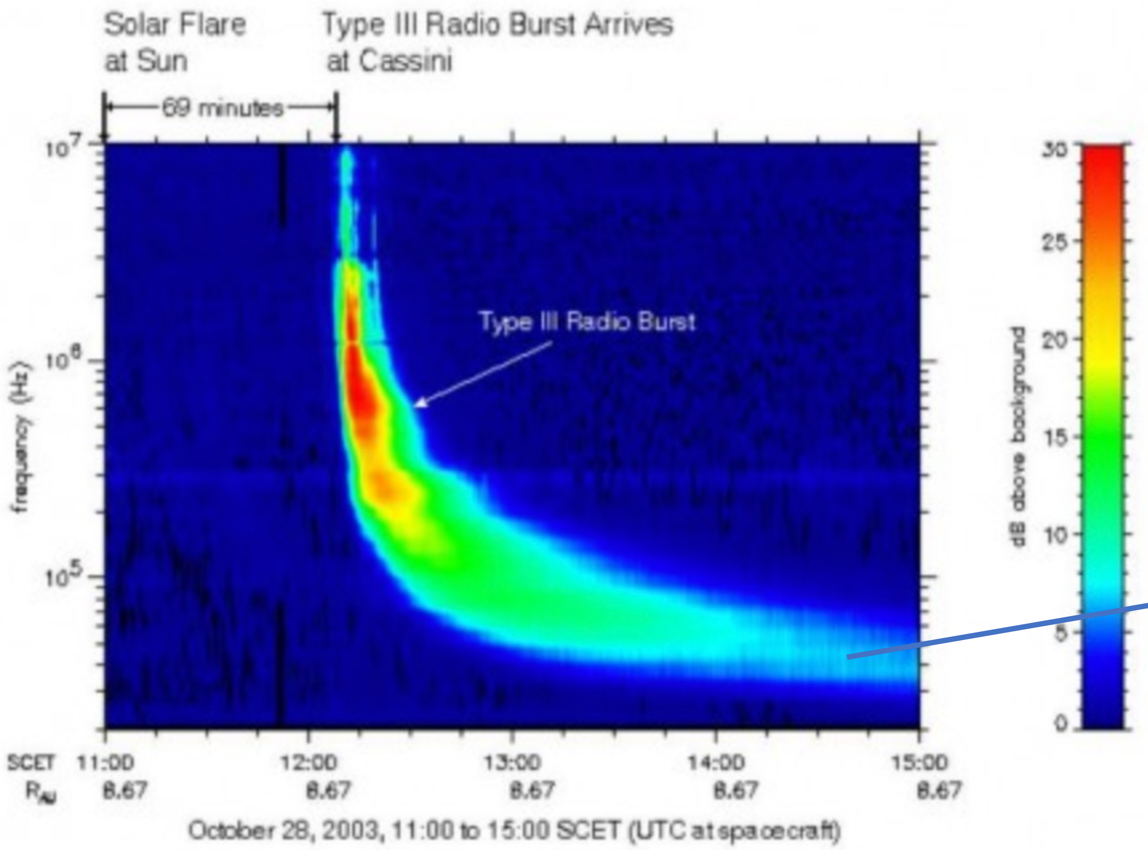
- From which we obtain that the electron perturbation (as well as the electric field) oscillate at the plasma frequency :

$$\frac{\partial^2}{\partial t^2} \delta n_e + \omega_p^2 \delta n_e = 0.$$

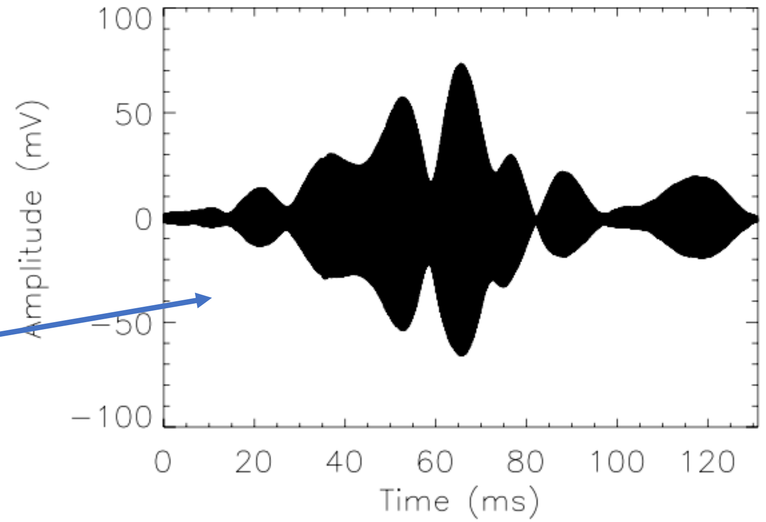
$$\omega_p = v_e / \lambda_D = \sqrt{\frac{n_0 e^2}{m_e \epsilon_0}}.$$

The plasma oscillation : space observations, type III bursts

A type III radio burst



In-situ :



Propagation of a transverse EM wave in a plasma

- Consider the problem of the propagation of a transverse wave in a plasma. The wave equation is

$$\Delta \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{j}}{\partial t},$$

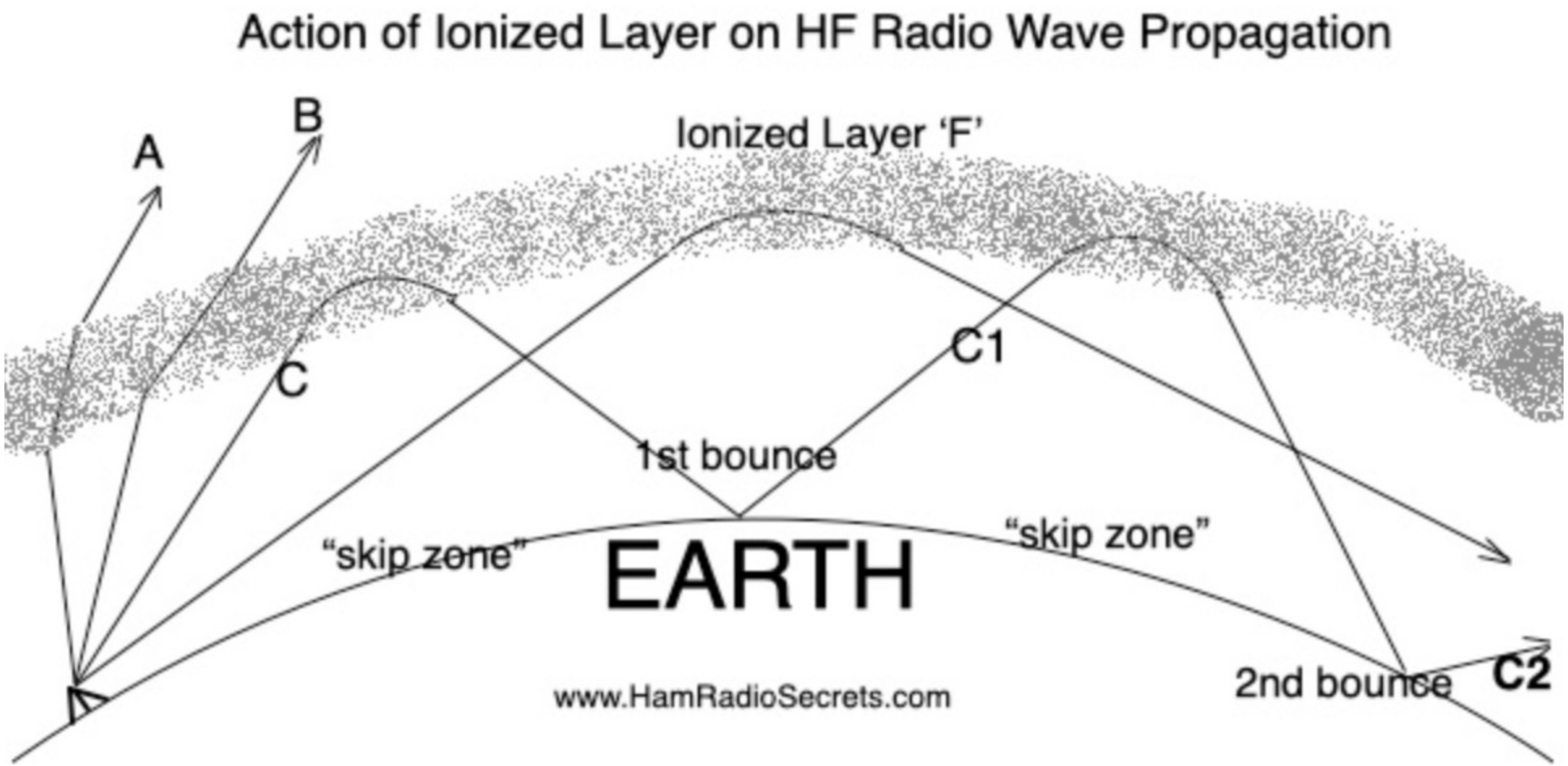
- And the linearized source term is $\mathbf{j} = n_0 e \mathbf{u}_e$ with $\frac{\partial \mathbf{u}_e}{\partial t} = -\frac{e}{m_e}\mathbf{E},$

- So that $\Delta \mathbf{E} - \frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\mathbf{E} = 0$

- The dispersion relation is $\omega^2 = \omega_p^2 + k^2 c^2$

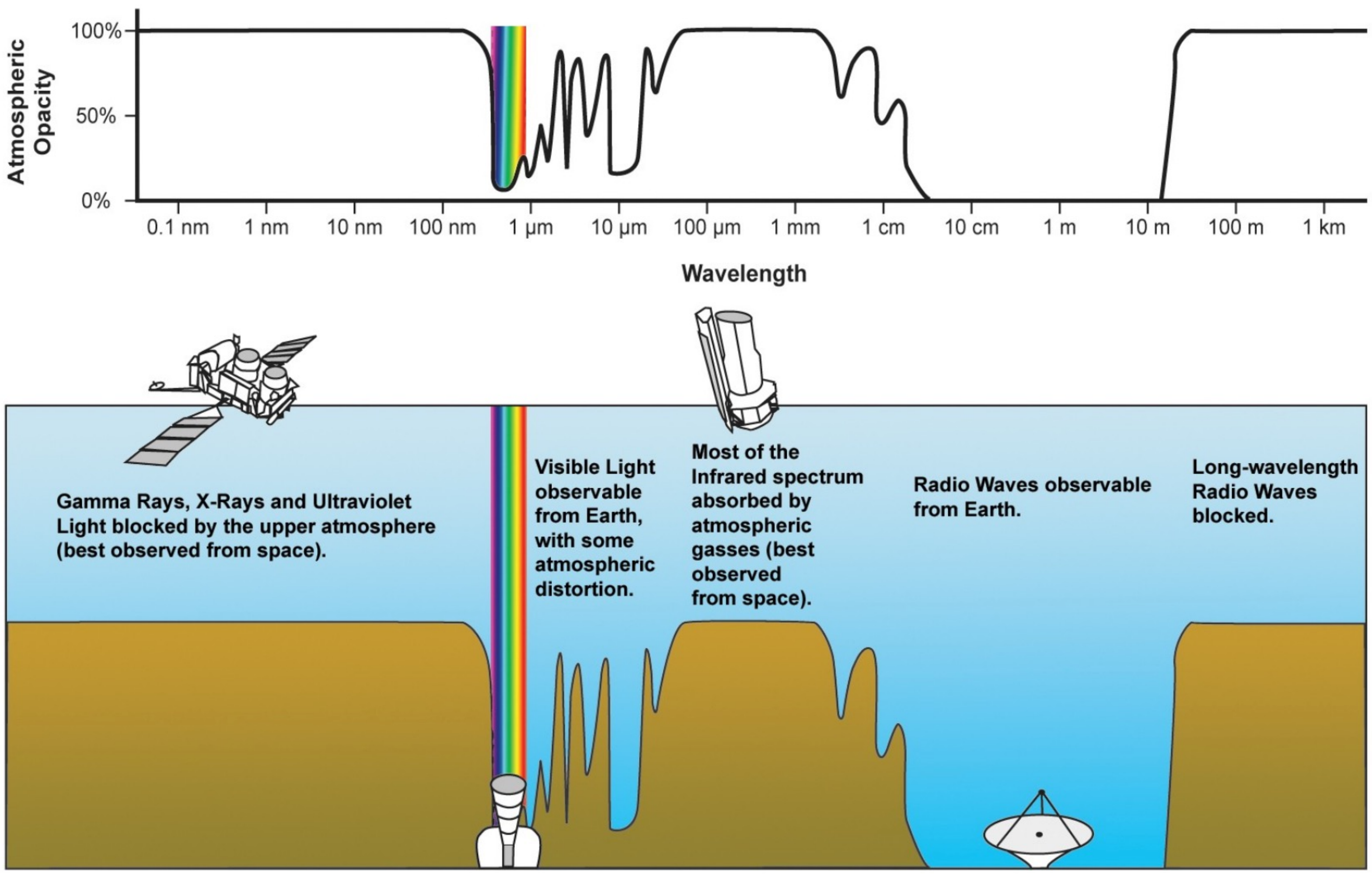
- And the refractive index is $n = \frac{kc}{\omega} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} < 1.$

Long distance radio communication

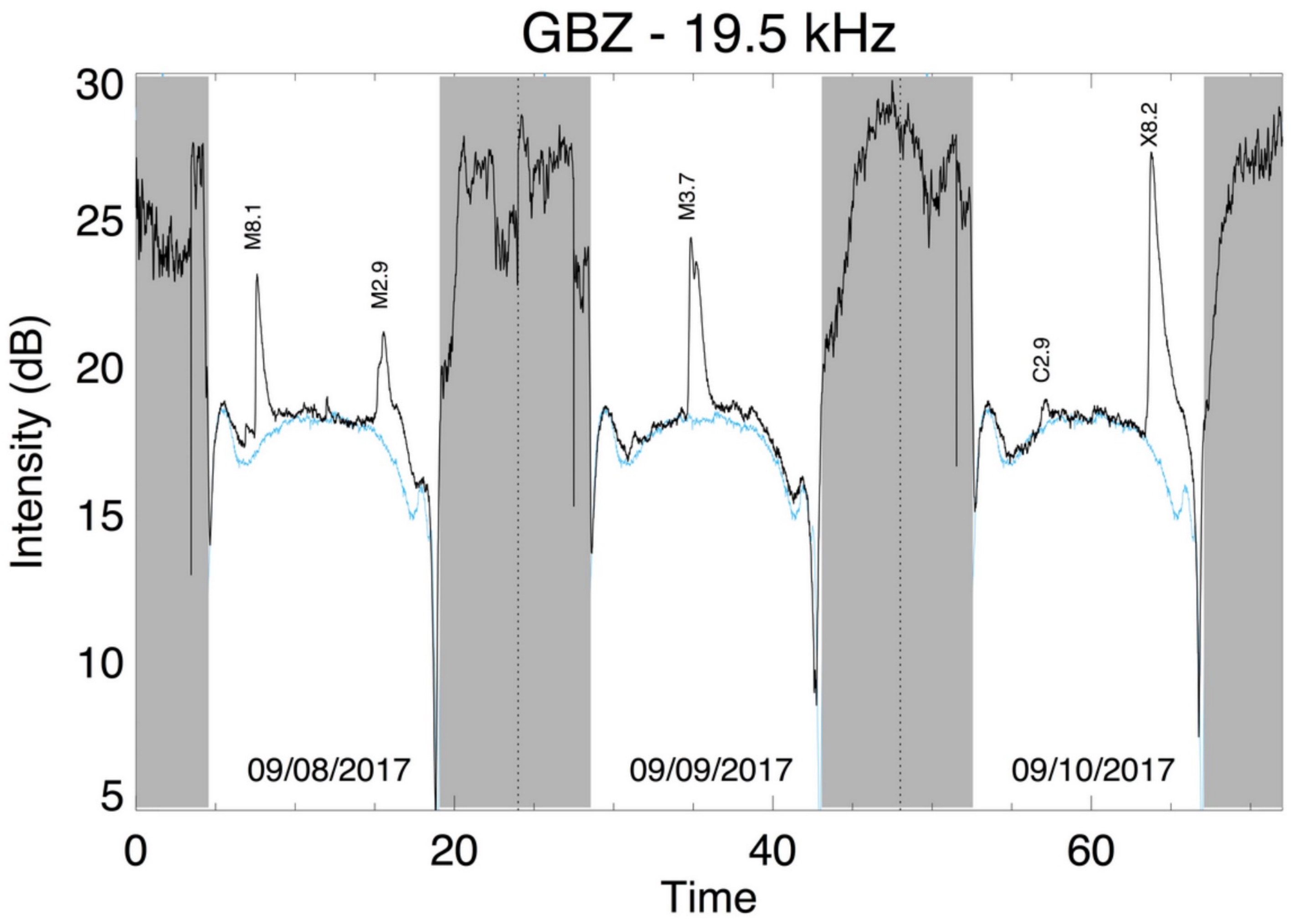


1901 : Marconi « crosses the Atlantic » with a radio transmission at 300 kHz

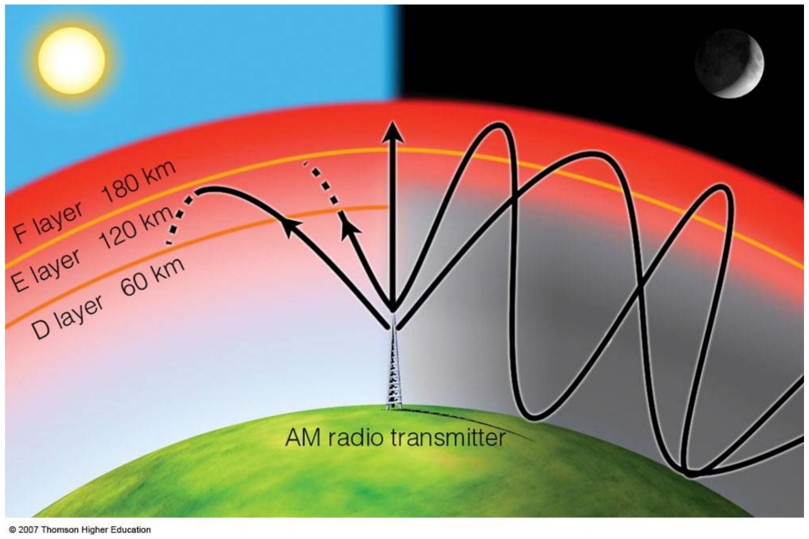
Atmospheric opacity



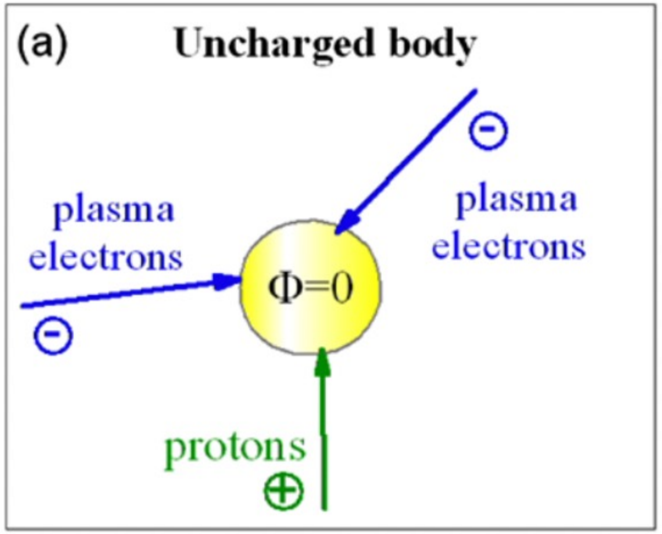
Ionospheric wave transmission

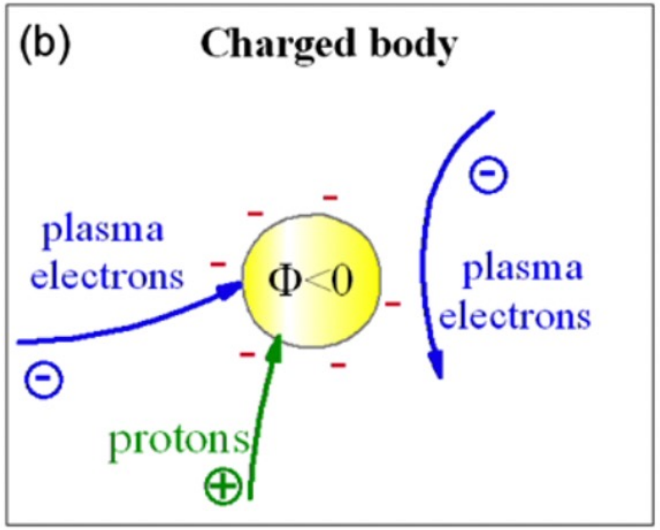


Monitoring the solar activity by observing VLF signals (the solar activity influences the ionospheric ionization degree)



Charge/Potential of a macroscopic object in a plasma





The charge of an object is governed by the current incoming on its surface

$$\frac{dQ}{dt} = I_e + I_i + I_{ph} + I_{sec} + \ldots$$

We assume a planar object of surface S, with normal axis z. The object has potential φ. The particles having a charge with the same sign as φ will be repelled, so that only that with velocity larger than $\sqrt{2q\varphi/m}$ can reach the surface

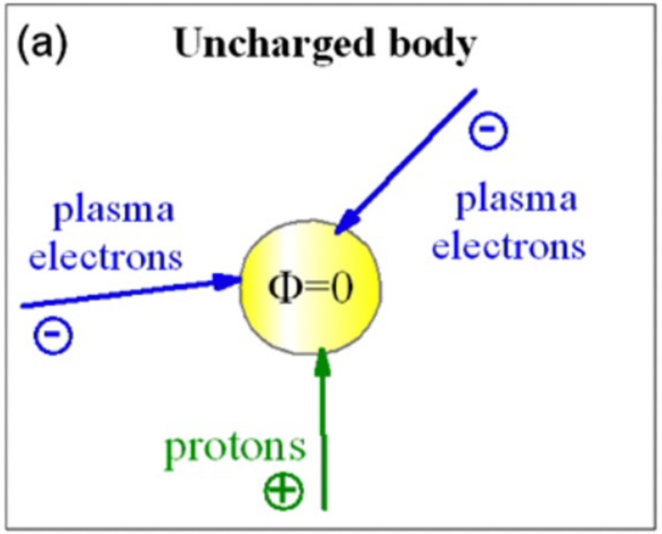
$$I_\alpha(\varphi) = q_\alpha S \int_{\sqrt{2q_\alpha\varphi/m_\alpha}}^{\infty} \frac{n_\alpha}{\sqrt{2\pi}v_{th,\alpha}} e^{-v_z^2/2v_{th,\alpha}^2} v_z dv_z = I_{\alpha,0} \exp\left(-\frac{q_\alpha\varphi}{kT_\alpha}\right)$$

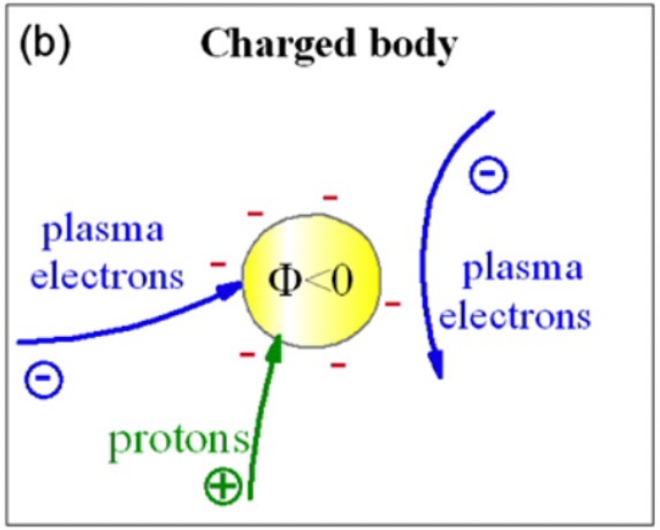
All the particles with charge of opposite sign than φ can reach the surface :

$$I_\alpha(\varphi) = q_\alpha S \int_0^{\infty} \frac{n_\alpha}{\sqrt{2\pi}v_{th,\alpha}} e^{-v_z^2/2v_{th,\alpha}^2} v_z dv_z = I_{\alpha,0}$$

with $I_{\alpha,0} = q_\alpha n_\alpha v_\alpha S$ and $v_\alpha = (kT_\alpha/2\pi m_\alpha)^{1/2}$

Object in an ion/electron plasma



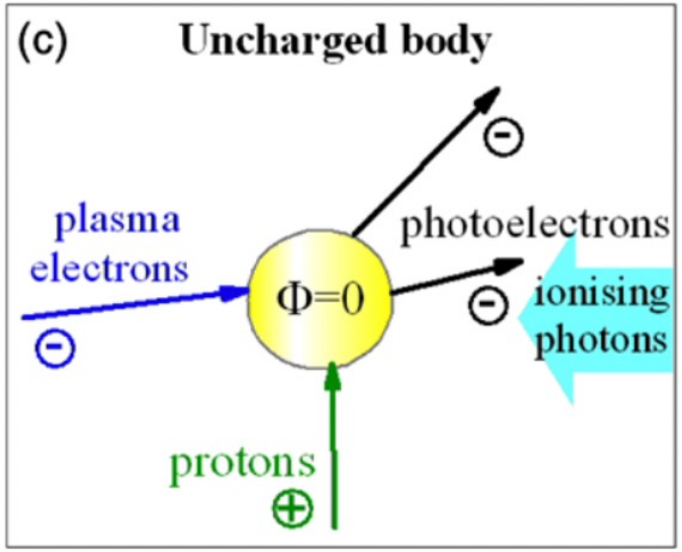


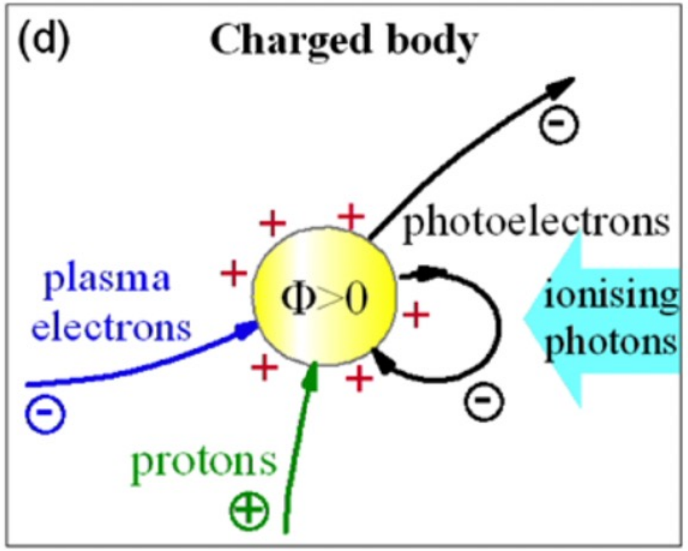
- At $\varphi = 0$, the electron current is much larger than the ion current (because of the small electron mass)
- Therefore the object will be charged negatively at equilibrium : $\varphi_{eq} < 0$
- The equilibrium potential is reached for dQ/dt = 0
- Using the previous expressions for the current, we obtain

$$\varphi_{eq} = \frac{kT}{e} \ln\left(\frac{I_{i,0}}{I_{e,0}}\right) = -\frac{kT}{2e} \ln\left(\frac{m_i}{m_e}\right)$$

- The object's charge depend on the plasma(electron) temperature only.
- The expression for the ion saturation current may be in reality quite different from our simple approximation… (sound speed replace the value of v_i – Bohm's criterion).

Object illuminated by UV: photo-electric effect





Under the action of solar UV, photo-electrons are produced that will escape the surface and tend to charge it positively. If $\varphi = 0$, the photo-emission is depending on the material photoelectric yield, and on the intensity of ionizing radiation.

$$I_{ph}(\varphi = 0) = j_{ph,0} S$$

With $j_{ph,0} \sim 50\ \mu A/m^2$ at 1AU. In comparison, the electron current density from the solar wind at 1 AU is $j_e \sim 0.5\ \mu A/m^2$. Therefore the object will tend to charge positively under the action of the photoelectric effect.

If $\varphi > 0$, one has $I_{ph}(\varphi) = j_{ph,0} S \exp\left(-\frac{e\varphi}{kT_{ph}}\right)$

With the photo-electron temperature $T_{\mathrm{ph}} \sim 3$ eV

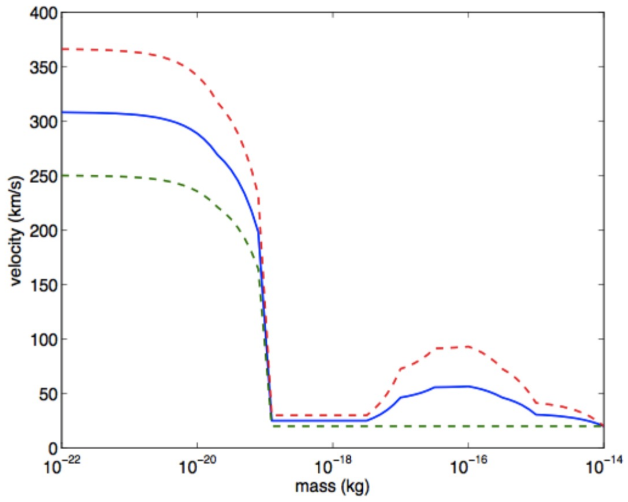
In such condition the potential of the object is $\varphi_{eq} = \frac{kT_{ph}}{e} \ln\left(\frac{j_{ph,0}}{e n_e v_e}\right)$

Dynamics of dust grains in the interplanetary medium

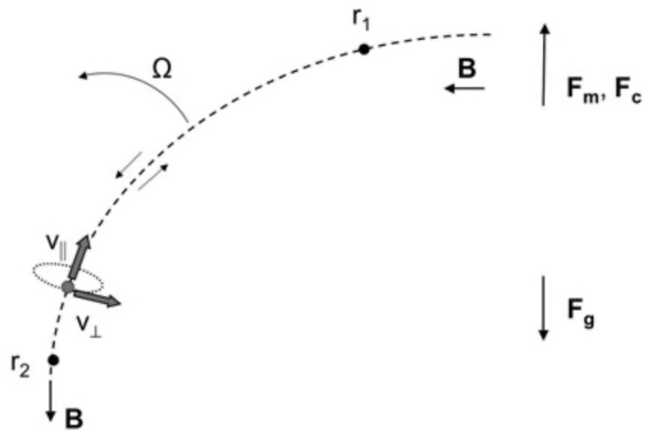
The charge of a dust grain in the interplanetary medium is $Q \sim 4\pi\varepsilon_0 a\varphi_{eq} > 0$, with a the dust radius.

This charge is roughly independent of the distance to the Sun (cf previous equation for φ_{eq}, in which the current density from the solar wind and the illuminating photon flux both decrease as $1/r^2$).

So the charge on mass ratio decreases as $\sim 1/r^2$: small particles have much larger charge to mass ratio than large particles.



Velocity vs mass

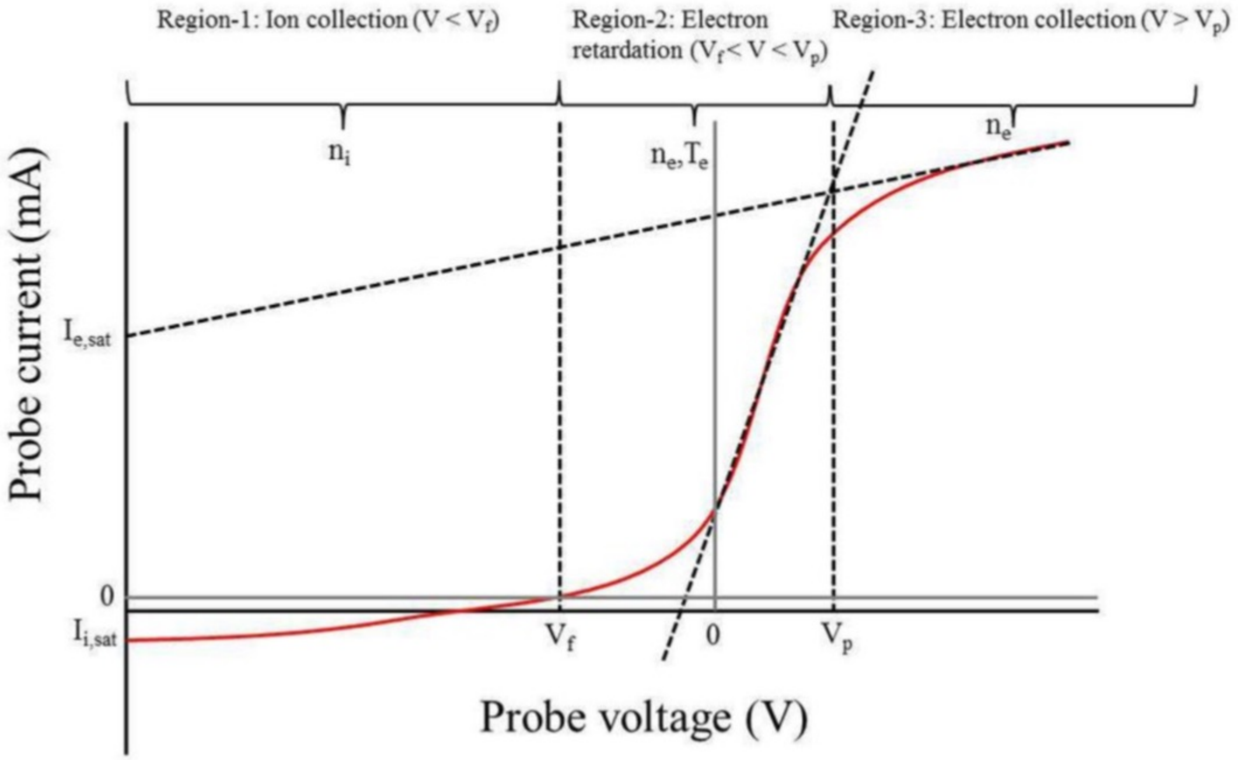


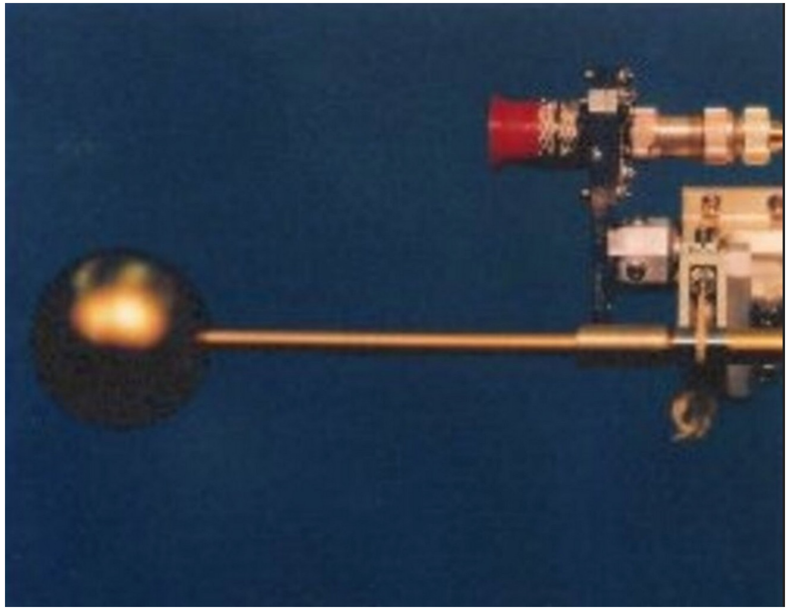
Nano-dust trajectory

Principle of the Langmuir probe

The working principle is to bias the probe to a known potential V, and to measure the plasma current I(V) flowing through the probe.

From the caracteristic curve, and the expressions for the currents obtained before, we can derive the plasma density and temperature.





Langmuir probe on the CASSINI spacecraft (journey to Saturn)

Dynamics of charged particles in a magnetic field

Equation of motion of a particle of charge q and mass m $\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{v} \times \mathbf{B}.$

First we look at the most simple case : what if the magnetic field is constant (along the z axis) ?

$$\frac{dv_\perp}{dt} + i\frac{qB}{m}v_\perp = 0, \qquad \Rightarrow \qquad v_\perp(t) = v_\perp(0)\exp\left(-i\omega_c t\right)$$

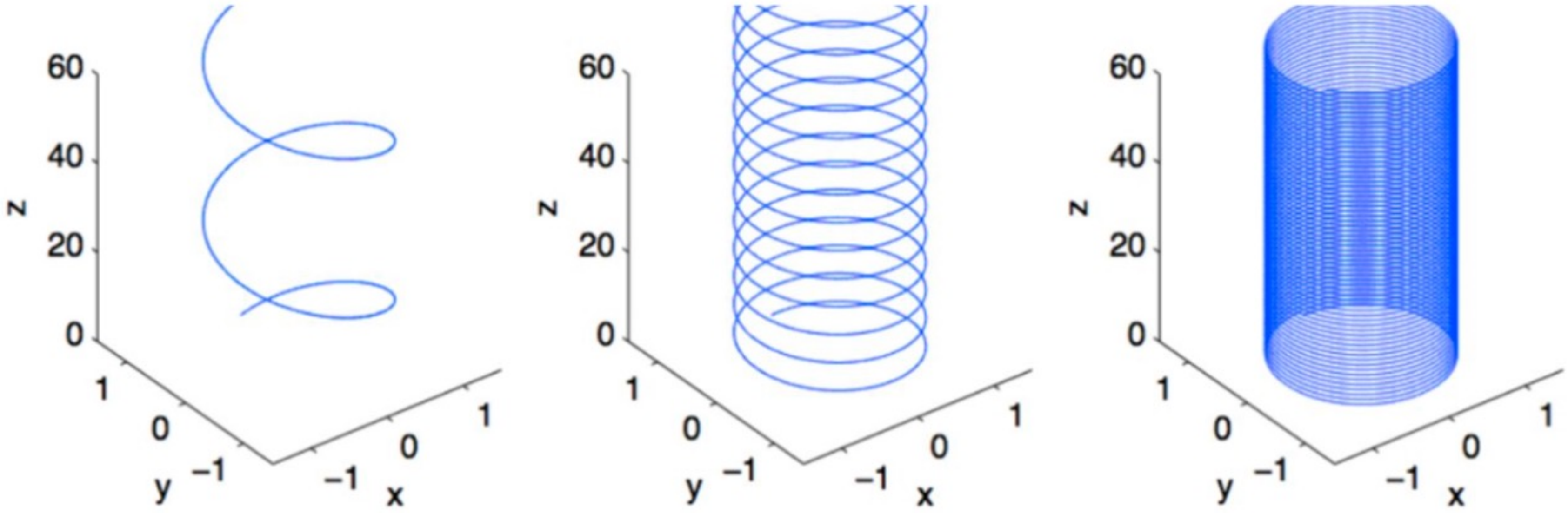
Where ω_c is the cyclotron frequency (angular frequency, to be precise). Can be positive or negative depending on the particle's charge : positive ions rotate clockwise, and electrons anti-clockwise.

The trajectory in the perpendicular plane is given by $r_\perp(t) = r_\perp(0) + \frac{iv_\perp(0)}{\omega_c}\exp\left(-i\omega_c t\right)$

Where we can introduce the Larmor radius $\rho = |\frac{v_\perp}{\omega_c}|$, which is the radius of the perpendicular trajectory.

Pitch-angle

$\theta = 10°$ $\theta = 45°$ $\theta = 80°$



The motion parallel to the magnetic field is $v_z = const.$ and $z(t) = v_z t + z_0$

Therefore the motion of the particle is an helix, with a pitch $p = 2\pi v_z/\omega_c$

We generally introduce the pitch-angle of the particle such that $\tan\theta = \left|\frac{v_\perp}{v_z}\right| = \frac{2\pi\rho}{p}$

Diamagnetic behaviour of a plasma

One can remark that the current produced by the current loop of a particle rotating in the magnetic field **B** produces a small magnetic field that opposes **B.**

Modeling the particle as a small current loop of radius ρ, one can associate to it a magnetic moment

$$\boldsymbol{\mu} = I\mathbf{S} = -\frac{|q\omega_c|\rho^2}{2}\mathbf{u}_z = -\frac{\mathscr{E}_\perp}{B}\mathbf{b}$$

Reminding the Ampère's law in a magnetized medium : $\nabla \times \mathbf{B} = \mu_0(j_{ext} + \nabla \times \mathbf{M}) = \nabla \times (\boldsymbol{B_0} + \mu_0 \boldsymbol{M})$

With the medium magnetization vector $\mathbf{M} = n\boldsymbol{\mu}$

So the magnetic field B in the plasma checks $\mathbf{B} = \mathbf{B}_0 + \mu_0\mathbf{M} \simeq \left(1 - \frac{nkT}{B_0^2/2\mu_0}\right)\mathbf{B}_0 \simeq (1-\beta)\,\mathbf{B}_0$

Where we summed the contribution on the electrons and ions in the calculation of the magnetization, assumed ions and electrons of equal temperatures. We also assumed that $\beta \ll 1$. Conclusion: the thermal agitation decreases the effective value of the B field in the plasma.

Magnetization current

The plasma particles carry a current equal to $\mathbf{j}_{\mathbf{plasma}} = \nabla\times\mathbf{M}$

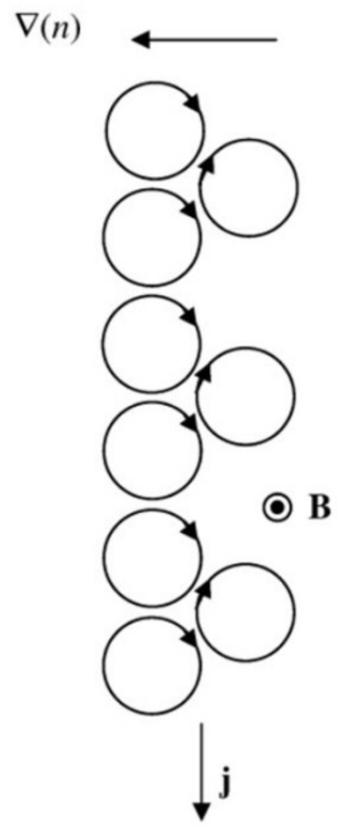
And we have just seen that $\mathbf{M} = -\frac{2nkT}{B_0^2}\mathbf{B}$ and we know that $\nabla\times\varphi\boldsymbol{B} = \nabla\varphi\times\mathbf{B}$
(if we assume that B is constant in space for simplicity)

So there exist a current in the plasma if there is a gradient of the modulus of the magnetization vector,

$$\boldsymbol{J_M} = -\nabla\left(\frac{2nkT}{B_0^2}\right)\times\mathbf{B}$$

This is called the magnetization current.

Interpretation ?



Motion of a particle in a magnetic and an electric field

In the plane perpendicular to the magnetic field, the equation of motion is $\frac{dv_\perp}{dt} + i\omega_c v_\perp = \frac{q}{m} E_\perp,$

Where all variables are complex. The solution of this equation is

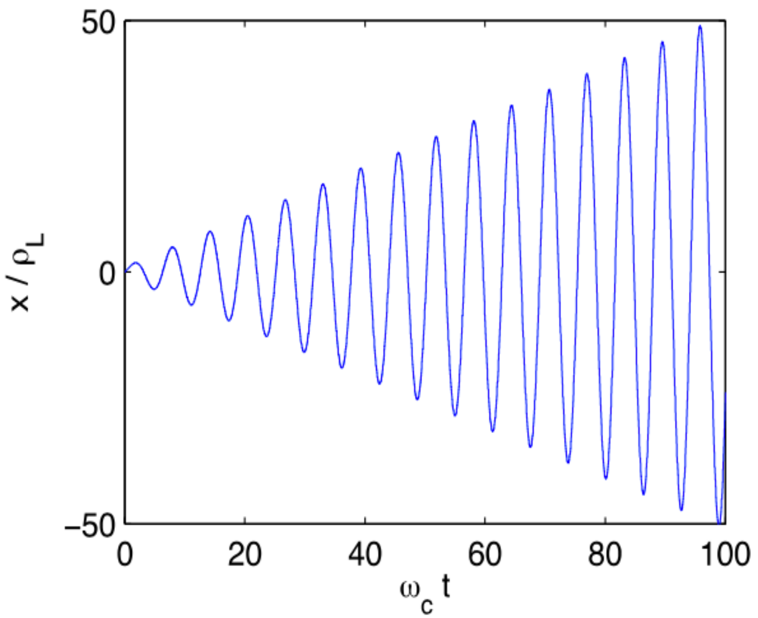
$$v_\perp(t) = e^{-i\omega_c t}\left(v_\perp(0) + \frac{q}{m}\int_0^t E_\perp(t')e^{i\omega_c t'}dt'\right)$$

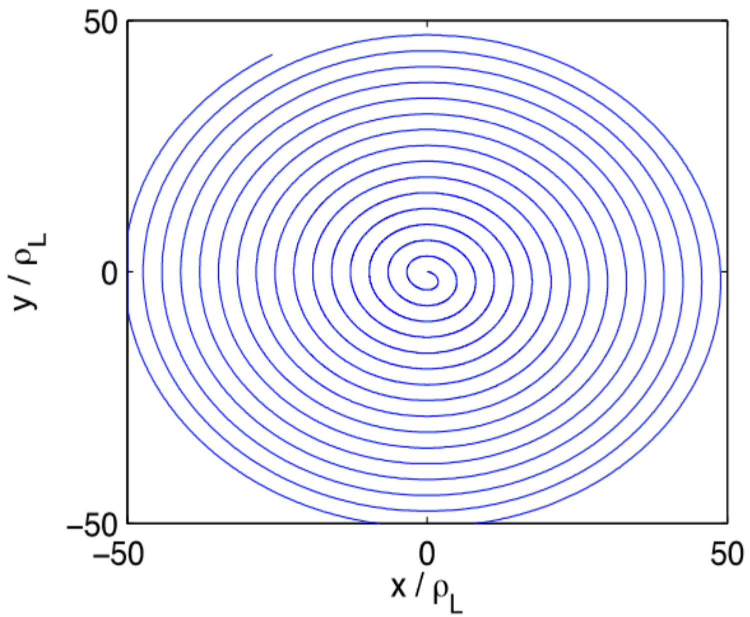
Resonant behaviour :

$E_\perp = E_0 \cos \omega_c\, t$

The Larmor radius increases linearly with time

(wave-particle interaction)





Motion of a particle in a magnetic and an electric field

Integrating twice by part the previous integral, we get:

$$\int_0^t E_\perp(t')e^{i\omega_c t'}dt' = \left[E_\perp(t')\frac{e^{i\omega_c t'}}{i\omega_c}\right]_0^t + \left[\frac{dE_\perp(t')}{dt'}\frac{e^{i\omega_c t'}}{\omega_c^2}\right] + I(t)$$

Where the last term is

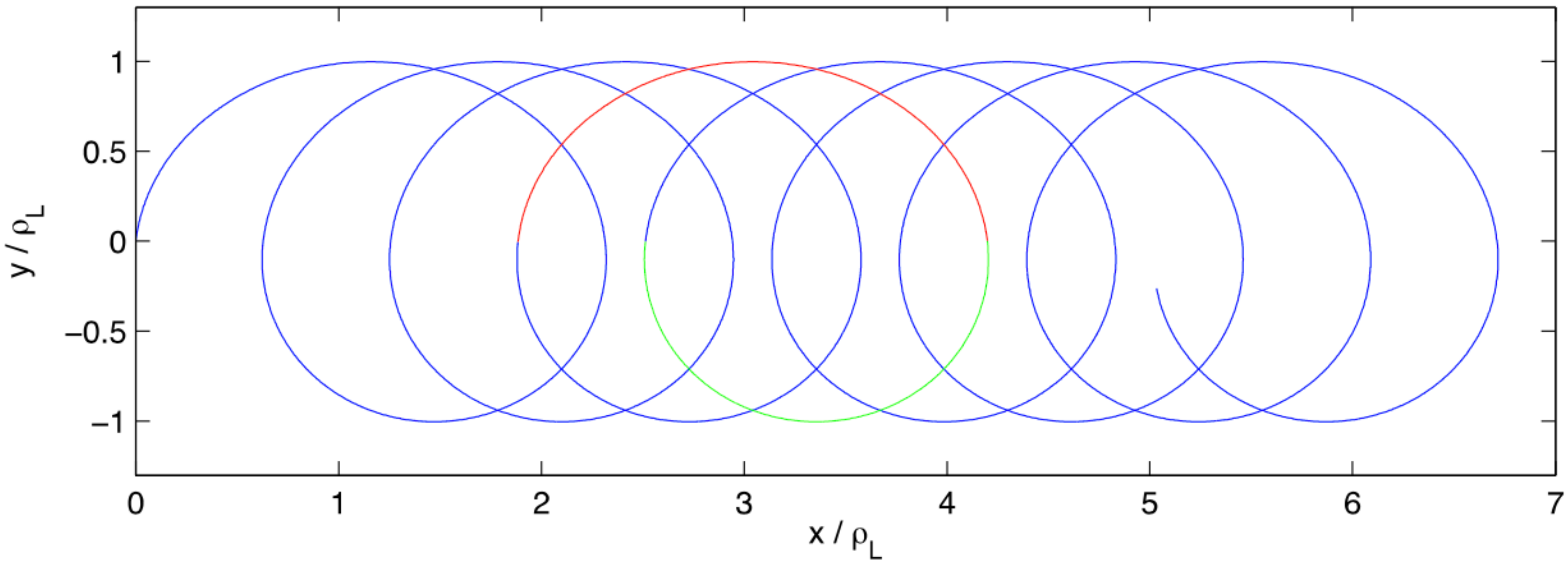
$$I(t) = \int_0^t \frac{d^2E_\perp(t')}{dt'^2}\frac{e^{i\omega_c t'}}{\omega_c^2}dt'$$

This equation can be written as $v_\perp(t) = V_0 e^{-i\omega_c t} + \frac{qE_\perp}{im\omega_c} + \frac{q}{m\omega_c^2}\frac{dE_\perp(t)}{dt}$, + I(t)

Assuming that the electric field is varying on a timescale T : $\frac{dE}{dt} \sim \frac{E}{T}$, we see that if the second term is of the order 1, the second is of the order $1/\omega_c T$, the third of the order $1/(\omega_c T)^2$ etc.

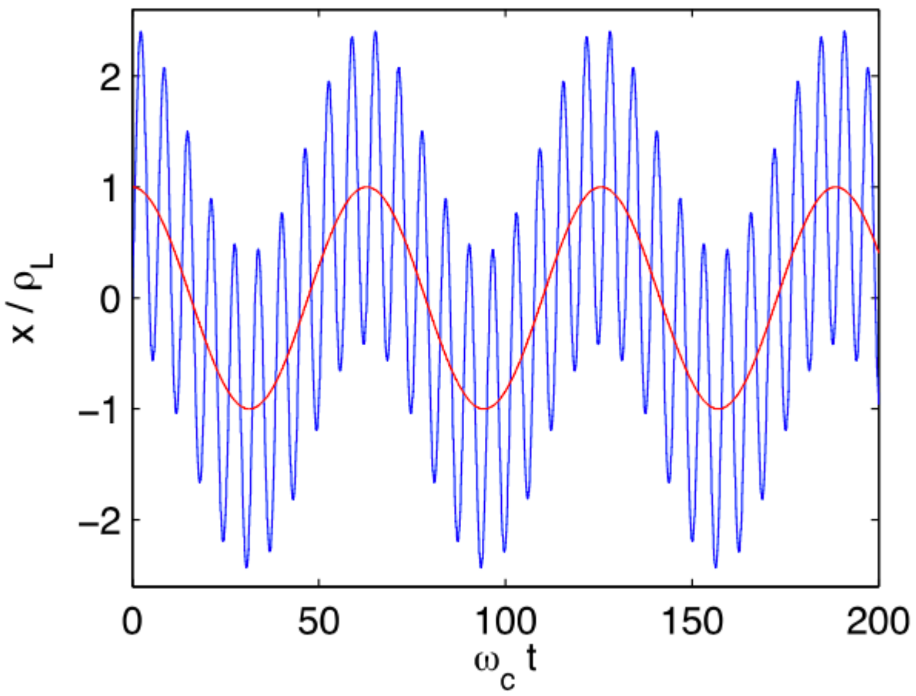
The first term is the cyclotron motion, the second the electric drift, or ExB drift, the third (of order 1 compared to the previous motions) is the polarization drift, and the other ones are neglected.

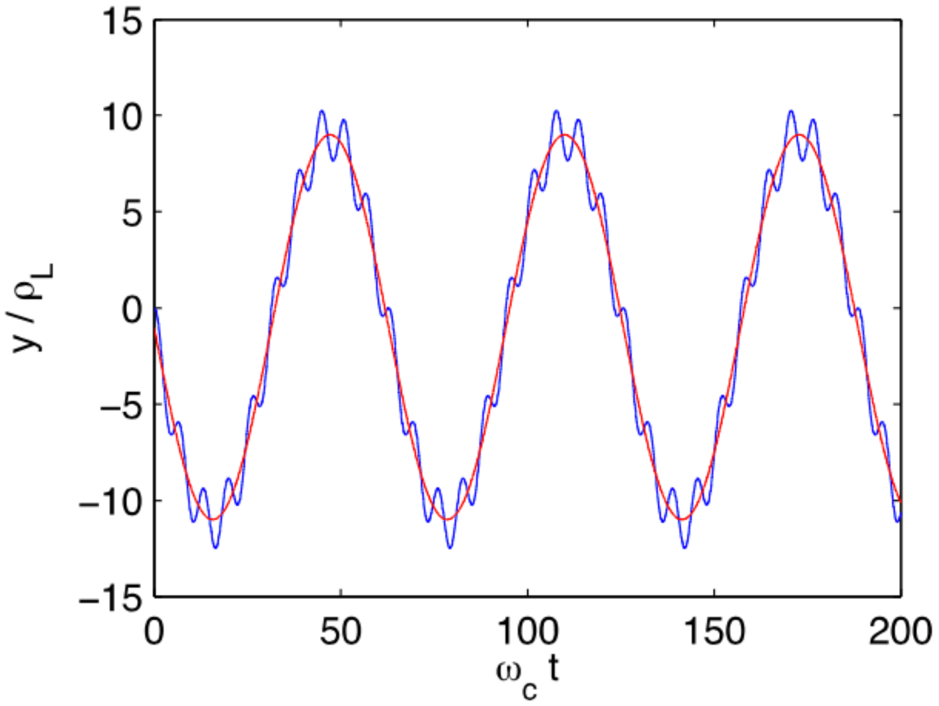
The crossed field drift



- The particle starts at (0,0), assume the particle's charge is positive.
- What is the direction of the magnetic field ?
- Of the electric field ?
- Interpret the sections colored in red and in green
- Is it possible to know if the particle is an ion or an electron, from this trajectory ?

The polarization drift





Identify the different components of the motion of the charge presented above.

The electric field is $E = E_0 \cos \omega t$ with $\frac{\omega}{\omega_c} = 0.1$.

What is the direction of the electric field ?

Drift in a non-electric force field

- Assume a charged particle in a gravitational field, with a component perpendicular to the magnetic field.
- What will be the motion of the charged particle?
- Will the particle « falls » if the magnetic field is completely perpendicular to the gravitational field ?

Slowly varying magnetic field: Adiabatic conservation of the magnetic moment

We have seen that a charged particle in a magnetic field is characterized by the magnetic moment

$$\boldsymbol{\mu} = I\mathbf{S} = -\frac{q\omega_c\rho^2}{2}\mathbf{u}_z = -\frac{\mathscr{E}_\perp}{B}\mathbf{b}$$

If we consider that the magnetic field is varying in time, we can calculate the variation of the perpendicular energy due to this change during a Larmor rotation. This variation is due to the electric field produced, according to Faraday's law, because of the variation of B(t). We assume that the variation of B is slow enough so that we can consider that the Larmor motion is still to a good approximation a circle. Then

$$\delta\mathscr{E}_\perp = \oint q\mathbf{E}\cdot\mathbf{v}_\perp dt = -|q|\oint \mathbf{E}\cdot d\boldsymbol{\ell} = |q|\iint \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$$

We call δB the small variation of B during a cyclotron rotation of period $\delta t = 2\pi/\omega_c$, then to a good approximation:

$$\delta\mathscr{E}_\perp = |q|\frac{\delta B}{\delta t}\iint dS = \frac{q\omega_c\delta B}{2\pi}\pi\rho^2 = \mathscr{E}_\perp\frac{\delta B}{B}$$

And we have finally that $\delta\left(\frac{\mathscr{E}_\perp}{B}\right) = \delta\mu = 0.$ => the magnetic moment is conserved in a slowly varying field (wrt Tc)

Slowly varying magnetic field: Mirror force

The reasoning we did is also valid for space-varying fields, if the variation of the modulus of the field sensed by the particle along its path is small, i.e, if the field modulus varies on a scale L such that

$$v_z/\omega_c \ll L$$

Since the total kinetic energy of the particle in the magnetic field is a constant (no work of the magnetic force), we can write for a magnetic field varying in space B(s), that

$$\mathscr{E} = \frac{1}{2}mv_\parallel^2 - \boldsymbol{\mu} \cdot \mathbf{B}(s) = const.$$

The particle moves along the field line, and the modulus of the magnetic field plays the role of a pseudo potential.

The pseudo-force resulting from this pseudo-potential is $\mathbf{F}_m = -\nabla \mathscr{E}_\perp(s) = -\mu \nabla B$

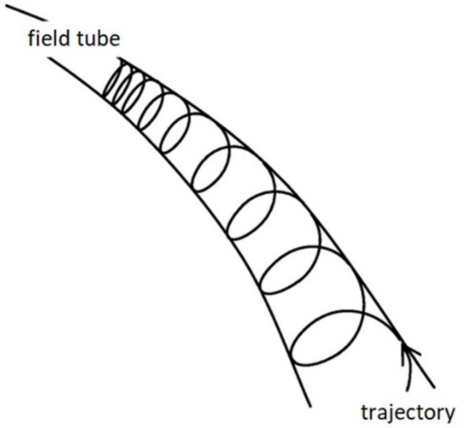
It is called the mirror force, or the diamagnetic force. It plays a very important role in lots of astrophysical magnetized systems, where the slowly varying condition will nearly always be checked, and where resonant effects are, on large scale, negligible.

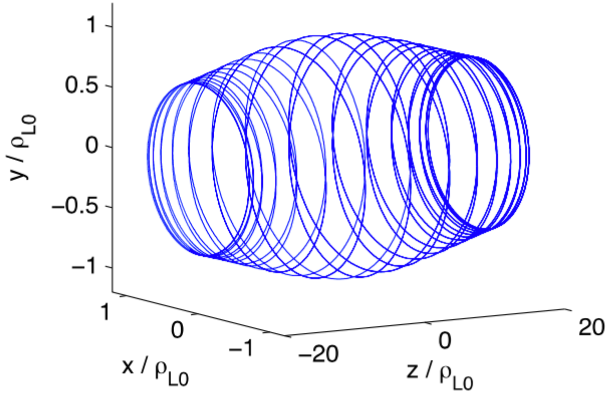
Conservation of magnetic flux through the Larmor ”current ring”

We have seen that the magnetic moment is conserved along the motion of a particle. A consequence, or a reformulation, of this is that the magnetic flux through a surface resting on the contour defined by the cyclotron motion is also conserved. This can be seen from the expression of this flux

$$\Phi = \iint \mathbf{B} \cdot d\mathbf{S} \simeq B(s)\pi\rho(s)^2 = \frac{\pi m^2}{q^2}\frac{v_\perp^2}{B} = \frac{2\pi m}{q^2}\mu = const.$$

This provides an easy and intuitive way to visualize the trajectory of particles in complicated (but slowly varying) fields.

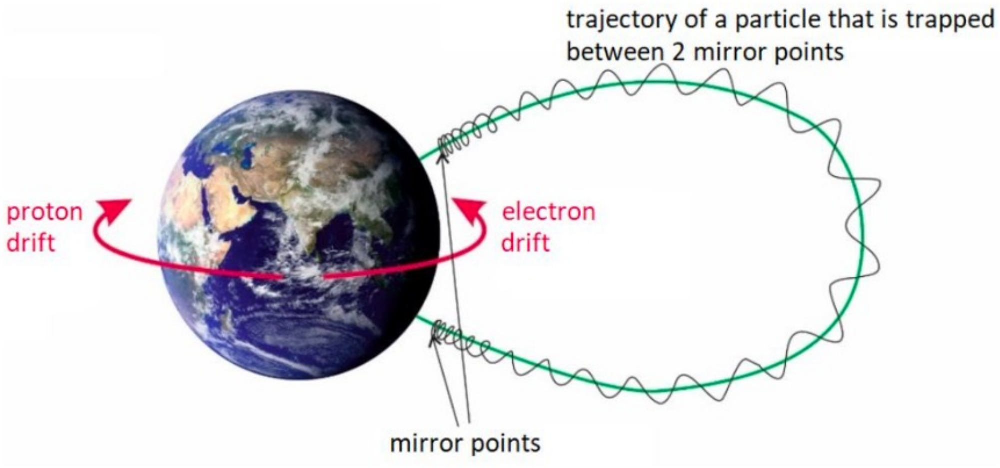




A magnetic bottle

Mirror points

We can summarize the previous result by: the presence of a convergence, or divergence, of the magnetic field lines produces a pseudo-force. The action of this force can lead to the reflexion of a particle at so-called mirror points.



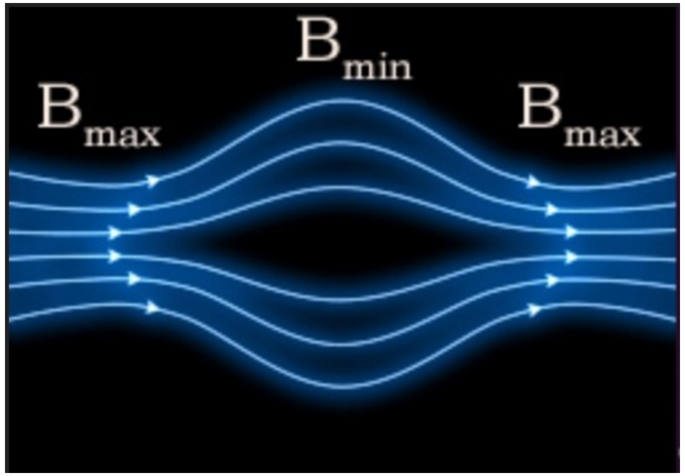
Position of the mirror points (if they exist) given by $\mathscr{E} = \mu B(s_m)$.

Motion of the particles: $v_{\parallel}(s) = \frac{2\mu}{m}\left(B(s_m) - B(s)\right) \Leftrightarrow \left(\frac{v_{\parallel}}{v_{\perp}}\right)^2 = \frac{B(s_m)}{B(s)} - 1$

The evolution of the pitch-angle of the particles along the field line is $\sin^2\theta(s) = \frac{B(s)}{B(s_m)}$

Magnetic confinement and loss cone

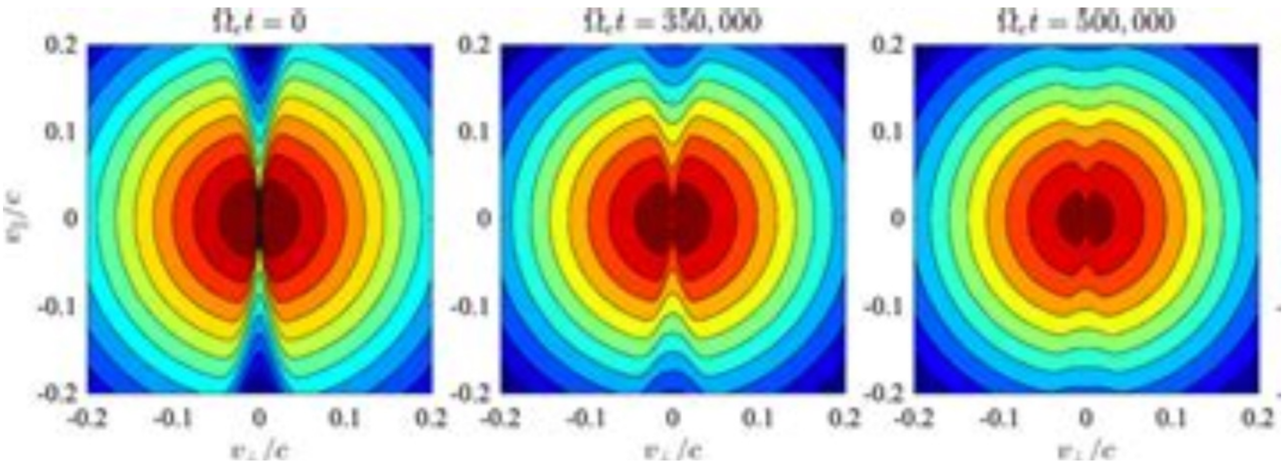
If particles are confined in a magnetic bottle, with max and min values of the field given by B_{max} and B_{min}, and particles initially confined close to the minimum of the field (bottom of the magnetic bottle).



The conditions for particles to be confined in the magnetic bottle is that they have an initial pitch angle such that

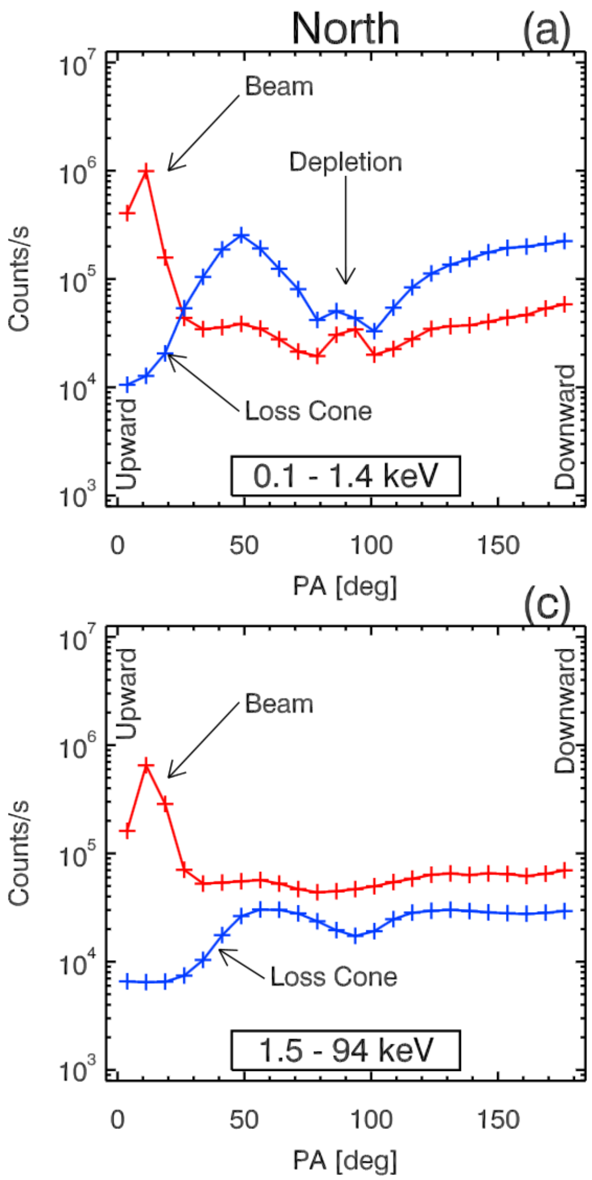
$$\sin^2 \theta_0 \geq \frac{B_{\min}}{B_{\max}} = 1/R_m$$

Rm is called the mirror ratio of the magnetic bottle, and defined the angle of the loss cone.



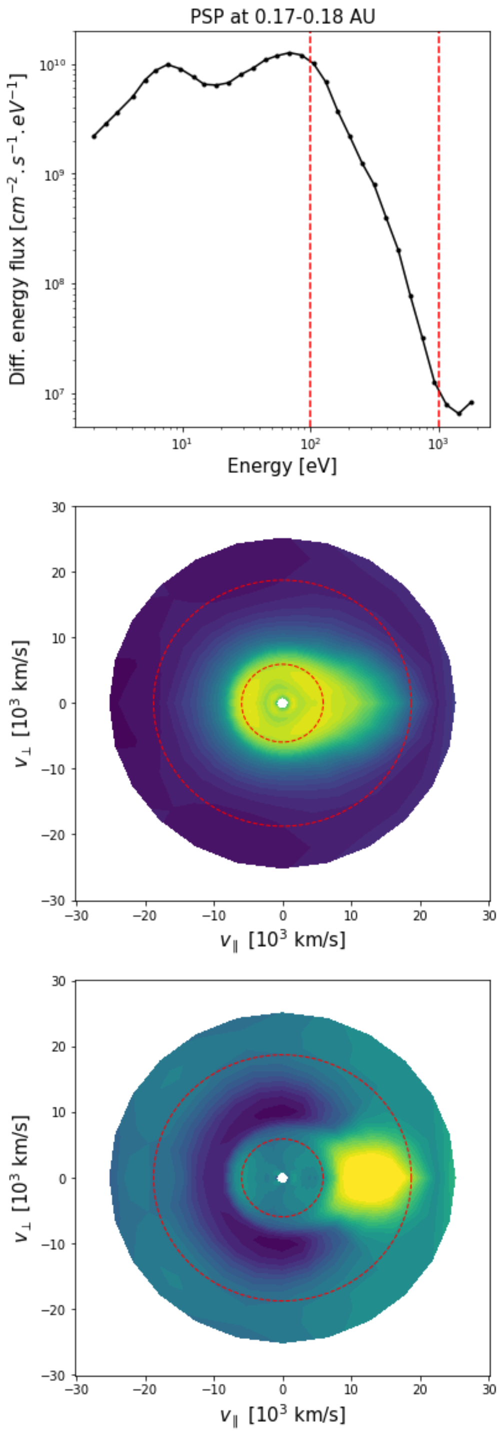
Loss cone and time evolution related to an electrostatic instability

Electron distribution observations



Jupiter's auroral regions

Solar wind at 0.17 AU



Curvature drift

In the presence of curved magnetic field lines, an additional drift motion appear, which is related to the existence of a centrifugal force.

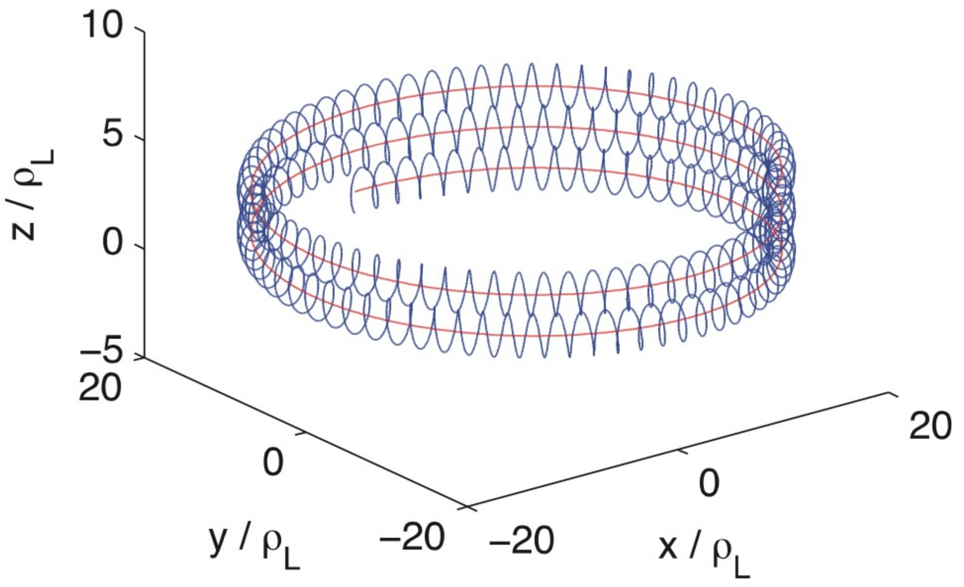
If the curvature radius R is large compared to the Larmor radius (or gyro-radius), and compared to the parallel motion scale of the particle $R \gg v_{\parallel}/\omega_c$, the particle follows to a good approximation the field line.

It undergoes a centrifulgal force $\mathbf{F}_c = \frac{m v_{\parallel}^2}{R}\,\mathbf{u_r}$

As for any applied force, a perpendicular drift is associated, the expression of the speed of which is

$$\mathbf{v_c} = \mathbf{F_c} \times \frac{\mathbf{B}}{qB^2} = \frac{m v_{\parallel}^2}{qBR}\,\mathbf{e_z}$$

The latest part of the equation being in the particular case of our « circular » magnetic field line.



Gradient drift

In the presence of a perpendicular gradient of the magnetic field, the pseudo-force stemming from the conservation of the magnetic moment

$$F_m = -\mu \nabla B$$

Is also responsible for a drift, perpendicular to the main field and to the direction of the gradient. It is called the gradient drift and the expression of its velocity is

$$\mathbf{v}_{\nabla} = -\mu \nabla \mathrm{B} \times \frac{\mathbf{B}}{qB^2}$$

We must note that this drift always complement the curvature drift, since it is not (because of the conservation of magnetic flux) to have a curved magnetic field without having a gradient perpendicular to it.